A Semi-analytical Approach for Dynamic Stress Concentration Factor of Helmholtz Problems with Circular Holes

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Outlines

- Introduction
- Problem statement
- Mathematical formulation
  - Boundary integral formulation
  - Degenerate kernel
  - Linear algebraic system
- Image concept
- Numerical example
- Conclusions

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Introduction

- Finite difference method (FDM)
- Finite element method (FEM)
- Boundary element method (BEM)
- Meshless method
- Boundary integral equation method (BIEM)

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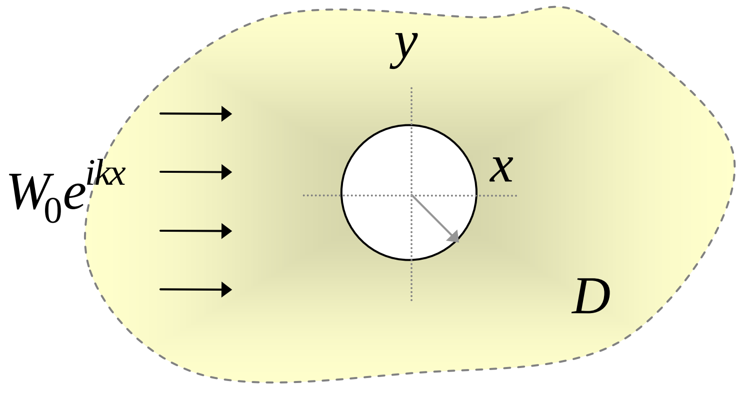
Governing equation



Helmholtz equation

$$\left(\nabla^2 + k^2\right) u(x) = 0, \quad x \in D$$

u : acoustic potential

k : wave number, $k = \omega / c$

ω : angular frequency

c : sound speed

D : domain of interest

∇^2:Laplacian operator

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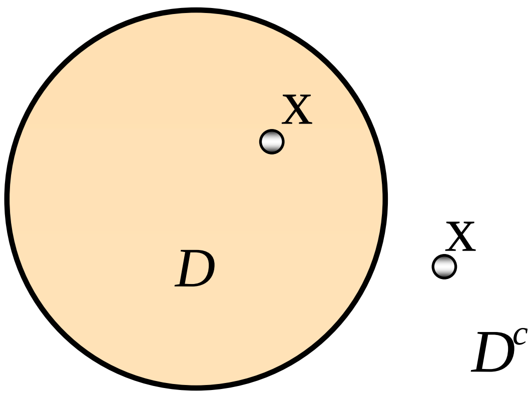
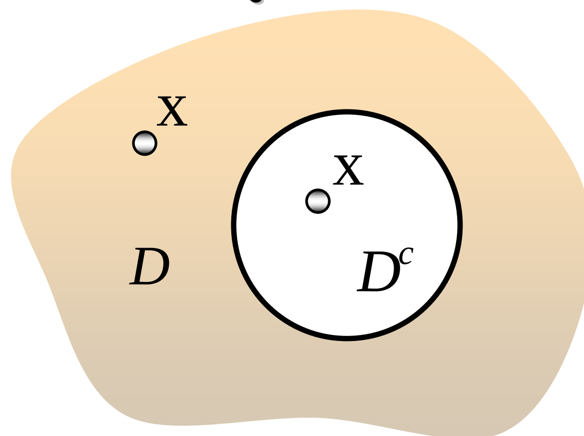
Boundary integral formulation

Interior case



Exterior case



$$U(\mathrm{s},\mathrm{x}) = -i\pi H_0^{(1)}(kr)/2$$

$$T(\mathrm{s},\mathrm{x}) = \frac{\partial U(\mathrm{s},\mathrm{x})}{\partial \mathbf{n}_{\mathrm{s}}}$$

$$t(\mathrm{s}) = \frac{\partial u(\mathrm{s})}{\partial \mathbf{n}_{\mathrm{s}}}$$

$$2\pi u(\mathrm{x}) = \int_B T(\mathrm{s},\mathrm{x})u(\mathrm{s})dB(\mathrm{s}) - \int_B U(\mathrm{s},\mathrm{x})t(\mathrm{s})dB(\mathrm{s}), \quad \mathrm{x} \in D$$

$$0 = \int_B T(\mathrm{s},\mathrm{x})u(\mathrm{s})dB(\mathrm{s}) - \int_B U(\mathrm{s},\mathrm{x})t(\mathrm{s})dB(\mathrm{s}), \quad \mathrm{x} \in D^c$$

Null-field integral equation

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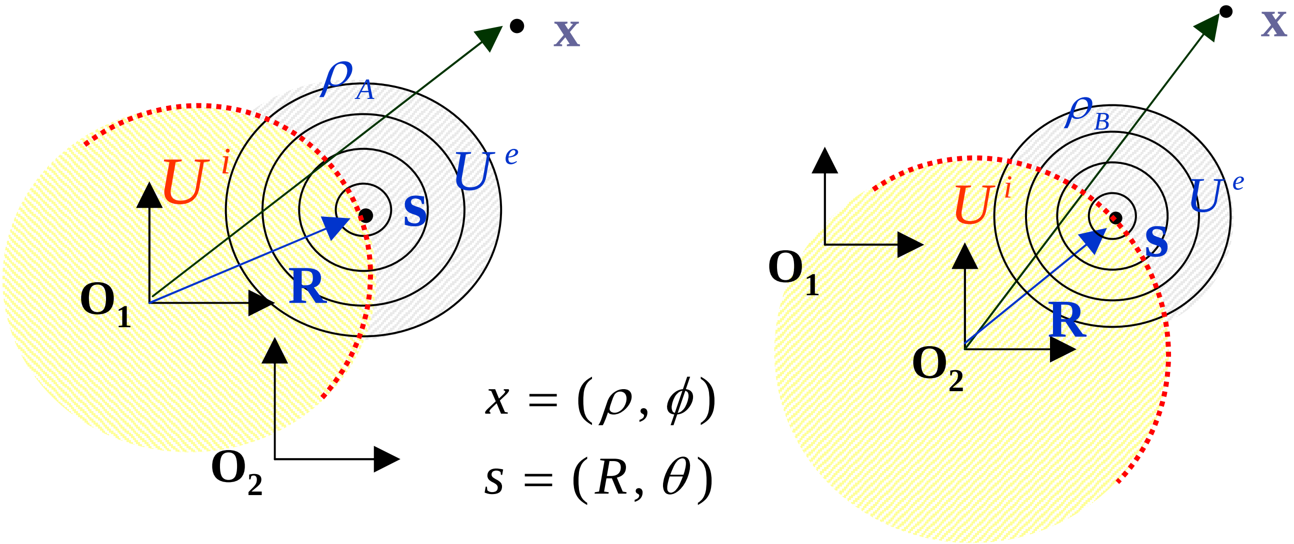
Degenerate kernel

$$U(x,s)=\begin{cases} U^I(x,s)=\sum_j^{\infty} A_j(x)B_j(s), & x<s \\ U^E(x,s)=\sum_j^{\infty} A_j(s)B_j(x), & x>s \end{cases}$$

x (field point)

s (source point)



$x=(\rho,\phi)$

$s=(R,\theta)$

Degenerate kernel

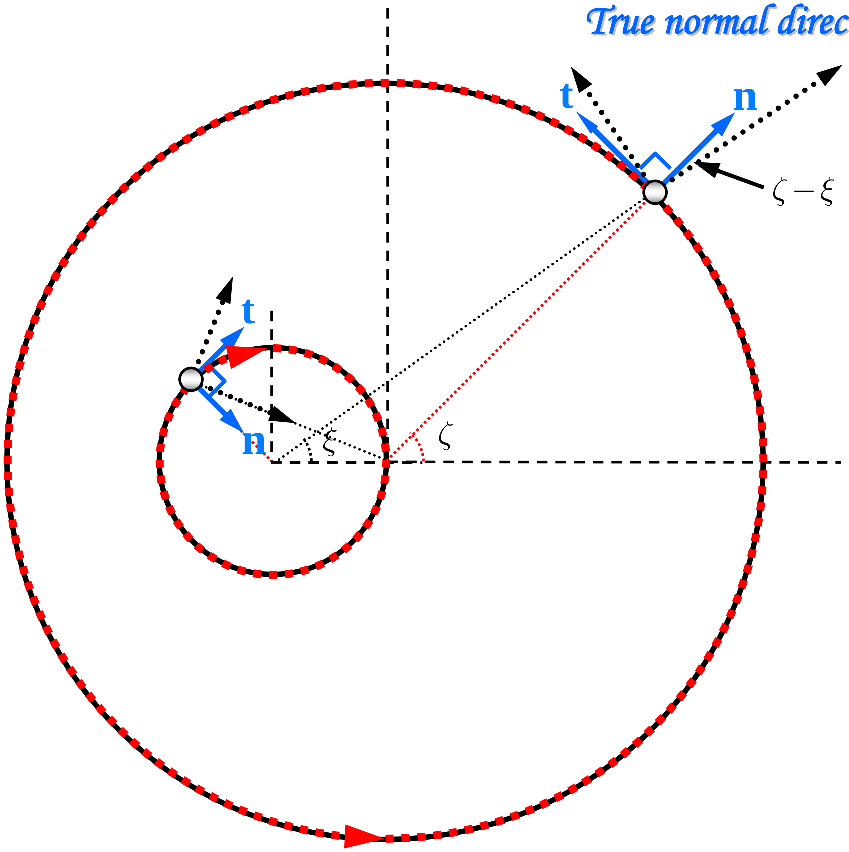
$$G.E.\ (\nabla^2 + k^2)U(x,s) = 2\pi\delta(x-s)$$

Fundamental solution: $U(s,x) = -i\pi H_0^{(1)}(kr)/2$

Degenerate kernels:

$$U(s,x) = \begin{cases} U^I(s,x) = \dfrac{-\pi i}{2}\displaystyle\sum_{m=0}^{\infty} J_m(k\rho)H_m^{(1)}(kR)\cos(m(\theta-\phi)),\ R \geq \rho \\ U^E(s,x) = \dfrac{-\pi i}{2}\displaystyle\sum_{m=0}^{\infty} H_m^{(1)}(k\rho)J_m(kR)\cos(m(\theta-\phi)),\ \rho > R \end{cases}$$

Vector decomposition technique for potential gradient



$$2\pi\frac{\partial u(\mathrm{x})}{\partial \mathbf{n}}=\int_B M_\rho(\mathrm{s},\mathrm{x})u(\mathrm{s})dB(\mathrm{s})-\int_B L_\rho(\mathrm{s},\mathrm{x})t(\mathrm{s})dB(\mathrm{s}),\quad \mathrm{x}\in D$$

$$2\pi\frac{\partial u(\mathrm{x})}{\partial \mathbf{t}}=\int_B M_\phi(\mathrm{s},\mathrm{x})u(\mathrm{s})dB(\mathrm{s})-\int_B L_\phi(\mathrm{s},\mathrm{x})t(\mathrm{s})dB(\mathrm{s}),\quad \mathrm{x}\in D$$

Non-concentric case:

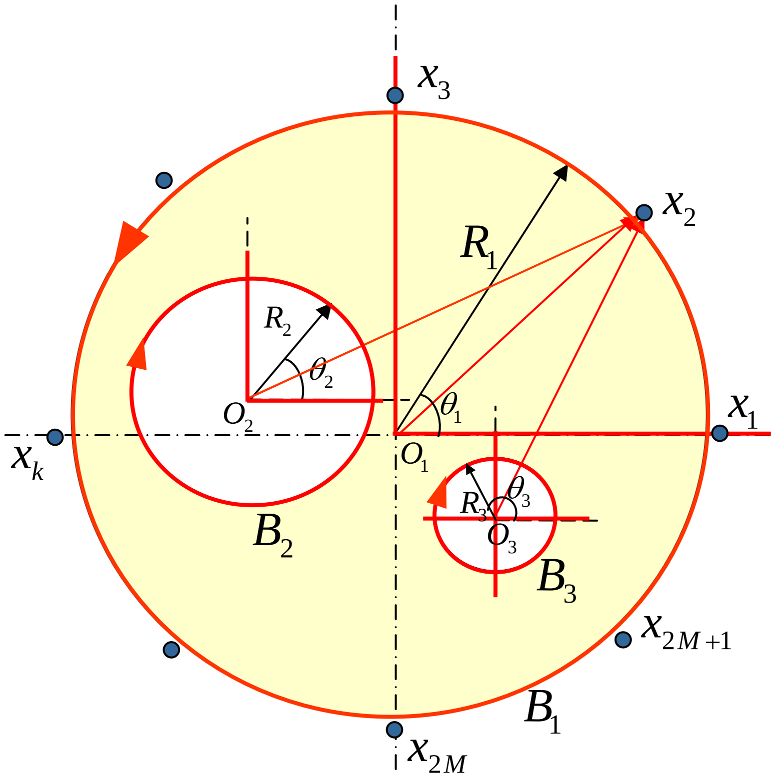
$$L_\rho(\mathrm{s},\mathrm{x})=\frac{\partial U(\mathrm{s},\mathrm{x})}{\partial\rho}\boxed{\cos(\zeta-\xi)}+\frac{1}{\rho}\frac{\partial U(\mathrm{s},\mathrm{x})}{\partial\phi}\boxed{\cos(\frac{\pi}{2}-\zeta+\xi)}$$

$$M_\rho(\mathrm{s},\mathrm{x})=\frac{\partial T(\mathrm{s},\mathrm{x})}{\partial\rho}\boxed{\cos(\zeta-\xi)}+\frac{1}{\rho}\frac{\partial T(\mathrm{s},\mathrm{x})}{\partial\phi}\boxed{\cos(\frac{\pi}{2}-\zeta+\xi)}$$

Special case (concentric case): $\zeta=\xi$

$$L_\rho(\mathrm{s},\mathrm{x})=\frac{\partial U(\mathrm{s},\mathrm{x})}{\partial\rho}\qquad M_\rho(\mathrm{s},\mathrm{x})=\frac{\partial T(\mathrm{s},\mathrm{x})}{\partial\rho}$$

Adaptive observer system



$x = (\rho, \phi)$: Collocation point

R_j : Radius of the jth circle

O_j : Origin of the jth circle

B_j : Boundary of the jth circle

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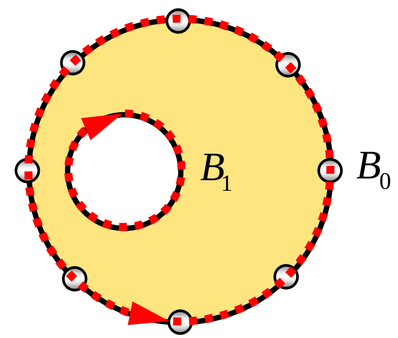
Linear algebraic equation

$$[\mathbf{U}]\{\mathbf{t}\}=[\mathbf{T}]\{\mathbf{u}\}$$



where

Index of collocation circle

$$[\mathbf{U}]=\begin{bmatrix}\mathbf{U}_{00} & \mathbf{U}_{01} & \cdots & \mathbf{U}_{0N}\\ \mathbf{U}_{10} & \mathbf{U}_{11} & \cdots & \mathbf{U}_{1N}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{U}_{N0} & \mathbf{U}_{N1} & \cdots & \mathbf{U}_{NN}\end{bmatrix}$$

Index of routing circle

$$\{\mathbf{t}\}=\begin{Bmatrix}\mathbf{t}_0\\ \mathbf{t}_1\\ \mathbf{t}_2\\ \vdots\\ \mathbf{t}_N\end{Bmatrix}$$

Column vector of Fourier coefficients (Nth routing circle)

Explicit form of each submatrix [$\mathbf{U}_{pk}$] and vector {$\mathbf{t}_k$}

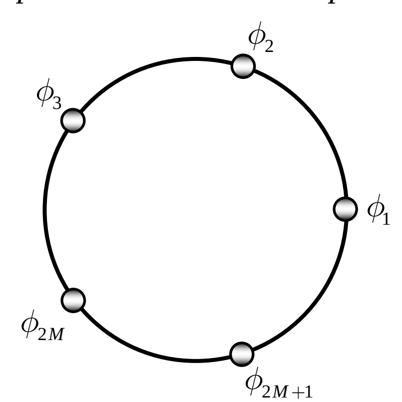
Truncated terms of Fourier series

$$
[\mathbf{U}_{pk}] = \begin{bmatrix}
U_{pk}^{0c}(\phi_1) & U_{pk}^{1c}(\phi_1) & U_{pk}^{1s}(\phi_1) & \cdots & U_{pk}^{Mc}(\phi_1) & U_{pk}^{Ms}(\phi_1) \\
U_{pk}^{0c}(\phi_2) & U_{pk}^{1c}(\phi_2) & U_{pk}^{1s}(\phi_2) & \cdots & U_{pk}^{Mc}(\phi_2) & U_{pk}^{Ms}(\phi_2) \\
U_{pk}^{0c}(\phi_3) & U_{pk}^{1c}(\phi_3) & U_{pk}^{1s}(\phi_3) & \cdots & U_{pk}^{Mc}(\phi_3) & U_{pk}^{Ms}(\phi_3) \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
U_{pk}^{0c}(\phi_{2M}) & U_{pk}^{1c}(\phi_{2M}) & U_{pk}^{1s}(\phi_{2M}) & \cdots & U_{pk}^{Mc}(\phi_{2M}) & U_{pk}^{Ms}(\phi_{2M}) \\
U_{pk}^{0c}(\phi_{2M+1}) & U_{pk}^{1c}(\phi_{2M+1}) & U_{pk}^{1s}(\phi_{2M+1}) & \cdots & U_{pk}^{Mc}(\phi_{2M+1}) & U_{pk}^{Ms}(\phi_{2M+1})
\end{bmatrix}
$$

Number of collocation points

$$
\{\mathbf{t}_k\} = \{ p_0^k \quad p_1^k \quad q_1^k \quad \cdots \quad p_M^k \quad q_M^k \}^T
$$

Fourier coefficients



Flowchart of present method

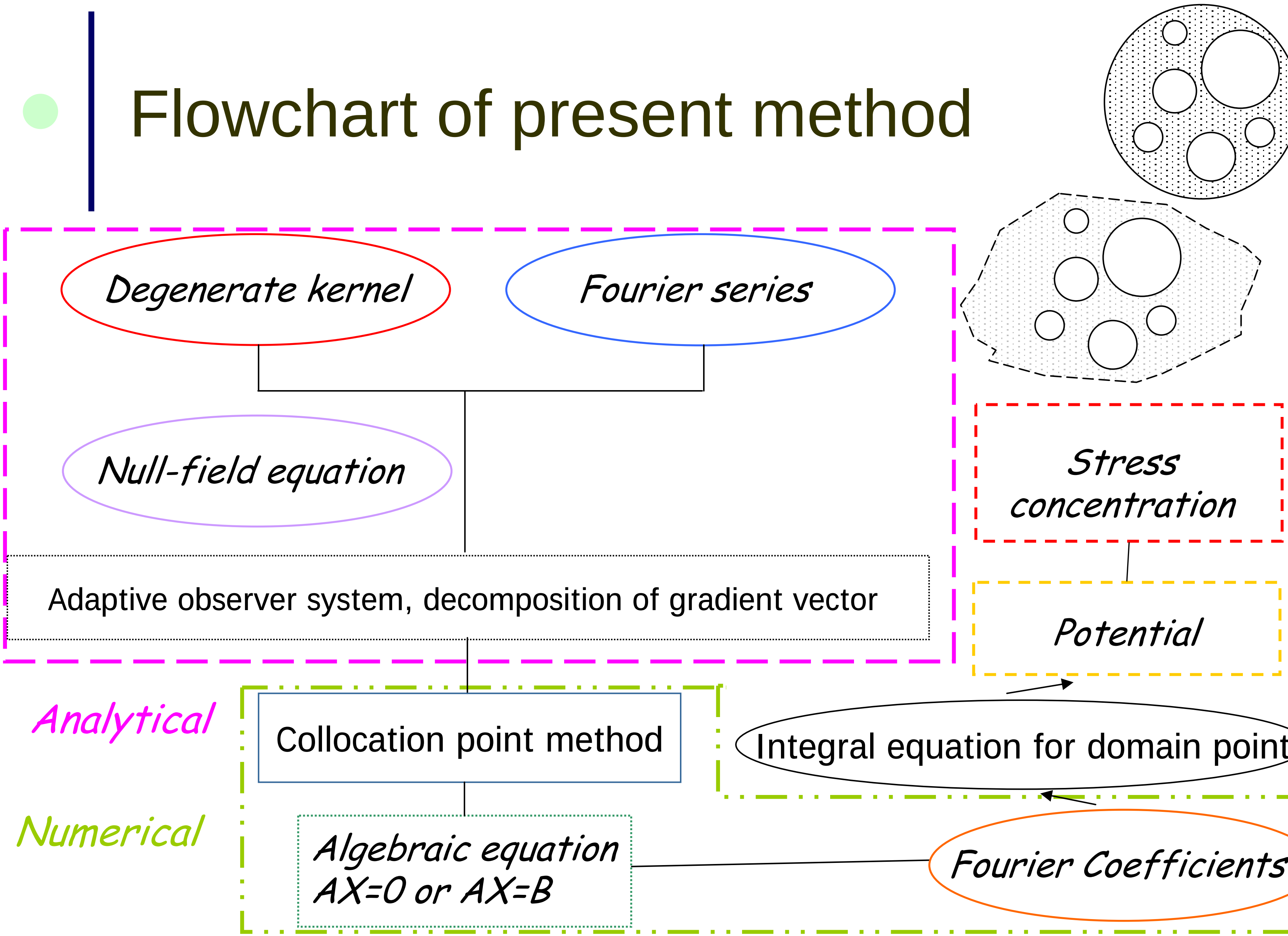


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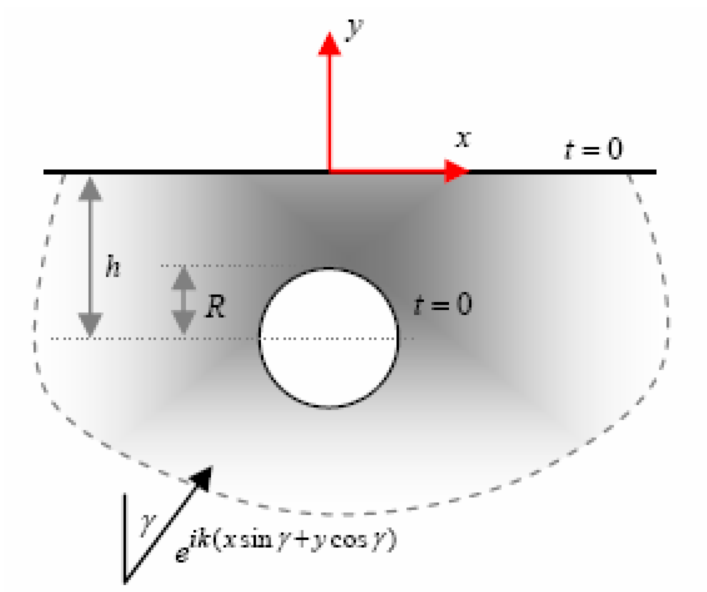
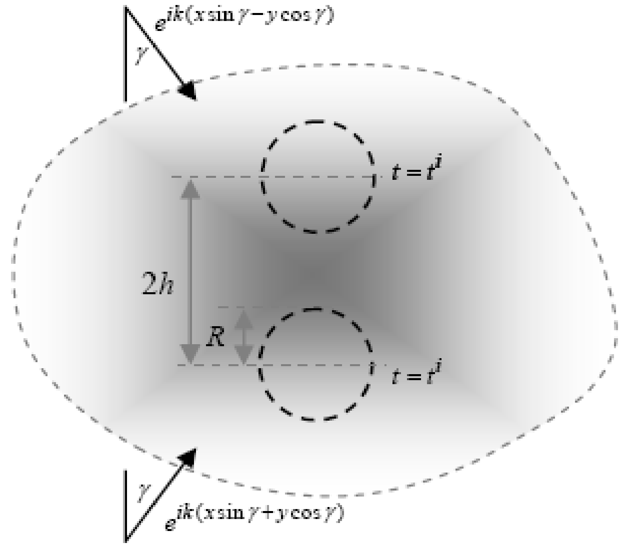
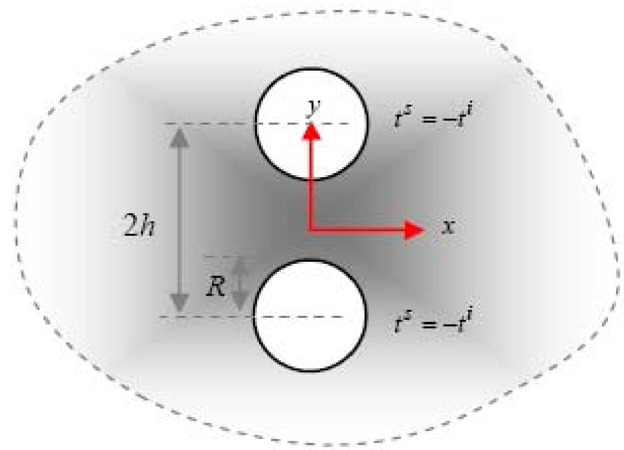
Image concept







Stress concentration

The displacement field :

$$u = v = 0 \qquad w = w(x, y)$$

The total displacement field :

$$w = w^s + w^i$$

$$w^i = W_0 e^{ik(x\sin\gamma + y\cos\gamma)}$$

The stress components :

$$\sigma_{13} = \sigma_{31} = \mu \frac{\partial w}{\partial x}$$

$$\sigma_{23} = \sigma_{32} = \mu \frac{\partial w}{\partial y}$$

The total stress component :

$$\sigma_{31} = \sigma_{31}^s + \sigma_{31}^i$$

$$\sigma_{32} = \sigma_{32}^s + \sigma_{32}^i$$

The shear stress components:

$$\sigma_{rz} = \mu \frac{\partial w}{\partial \mathbf{n}} = \sigma_{31}\cos\phi + \sigma_{32}\sin\phi$$

$$\sigma_{\theta z} = \mu \frac{\partial w}{\partial \mathbf{t}} = -\sigma_{31}\sin\phi + \sigma_{32}\cos\phi$$

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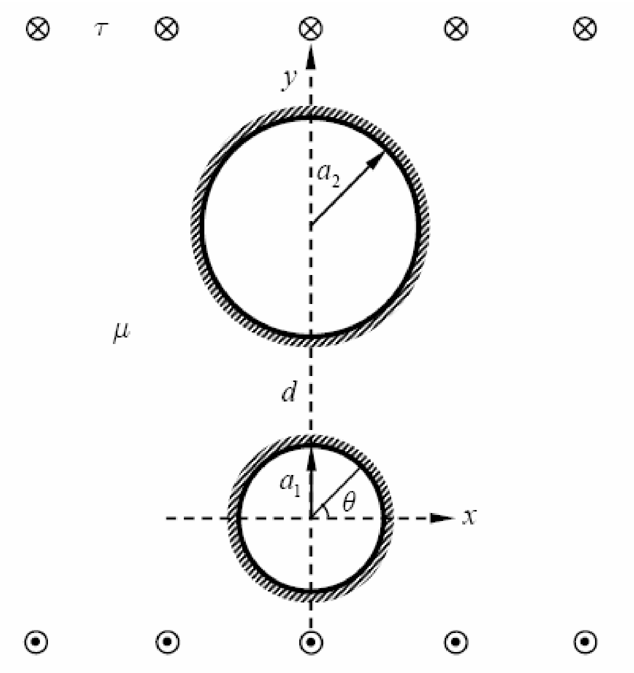
Numerical example

Case1

Two circular cavities lie on the y-axis



Honein's B.C.:

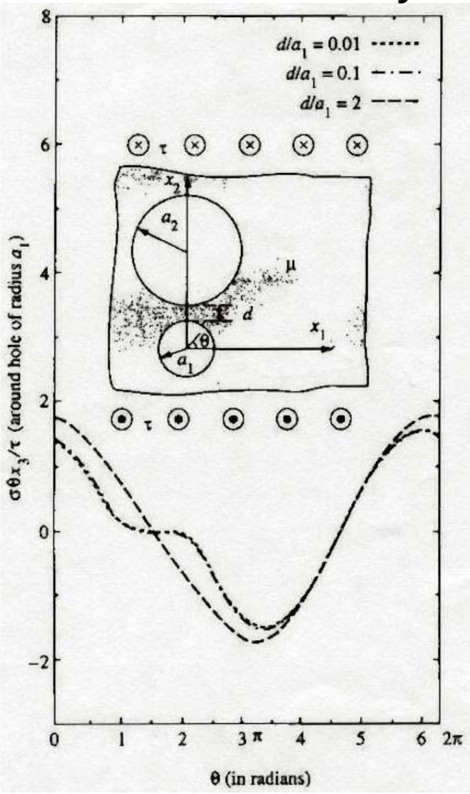
$$w^{\infty} = \frac{\tau y}{\mu}$$

We assume an incident SH-wave:

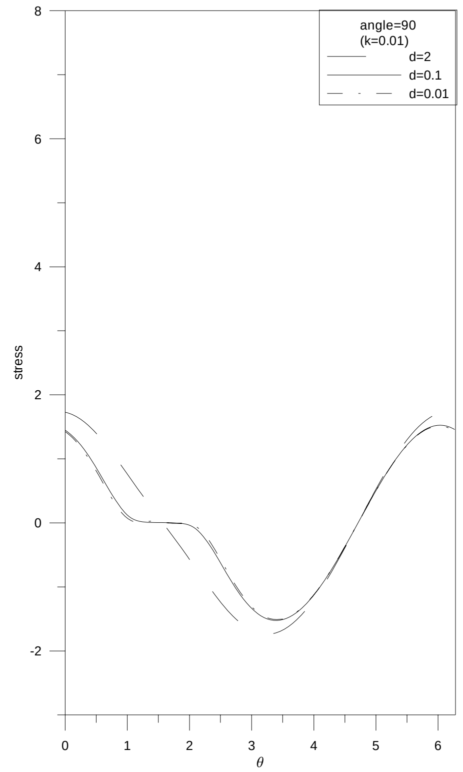
$$w^{i} = \frac{\tau y}{\mu} e^{ikx}$$

Numerical result 1

Honein's data:
Stress around
the smaller cavity



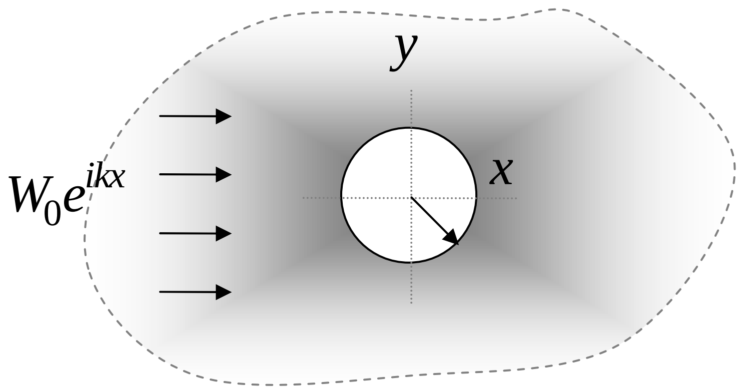
Present method (m=10)



Numerical example

Case2

A circular cylinder in an infinite space



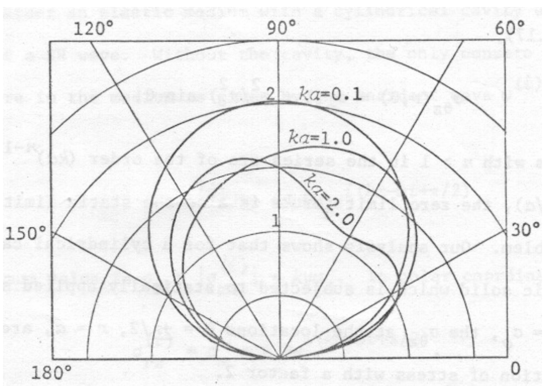
B.C.

t=0 on boundary

Numerical result 2

Pao and Mow's result:

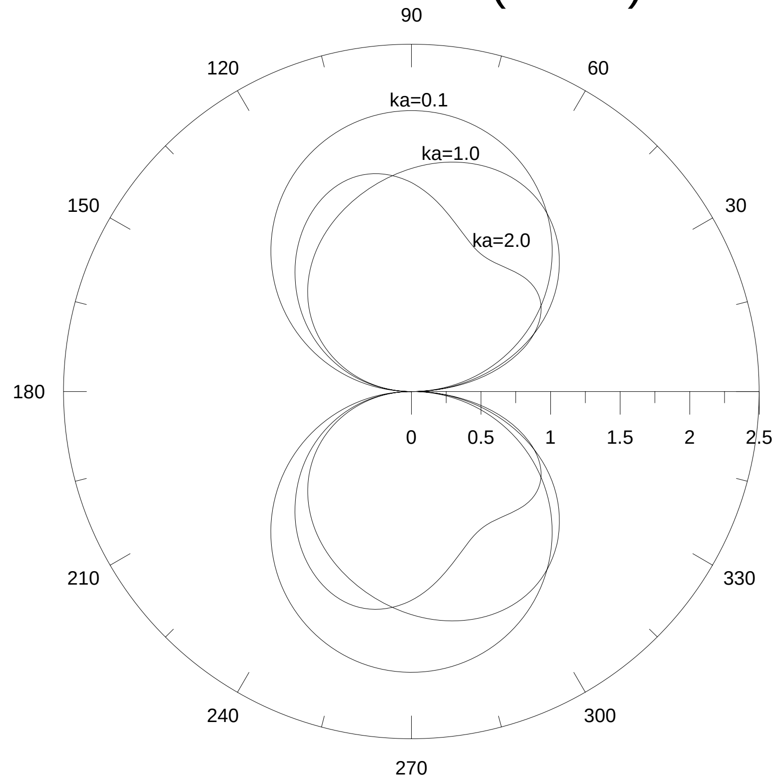
$\sigma^*_{\theta z}$ around the cavity



the nondimensional stress:

$$\sigma^*_{\theta z} = \left| \frac{\sigma_{\theta z}}{\sigma_0} \right|$$

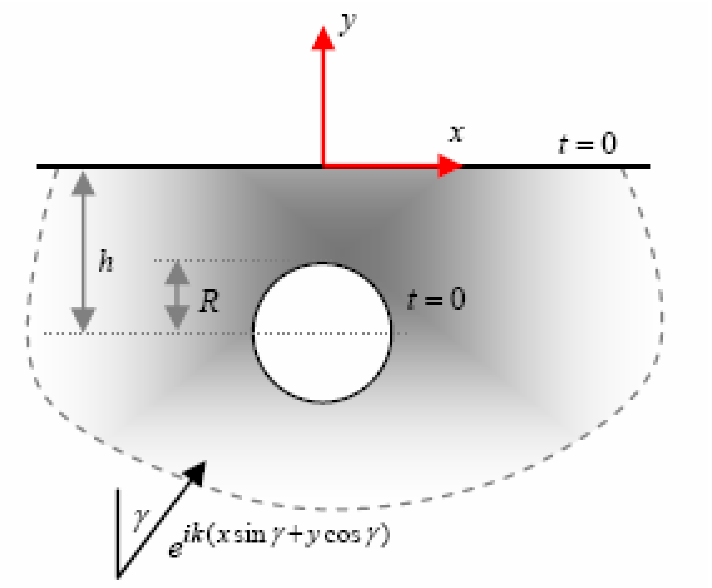
Present method (m=10)



Numerical example

Case3

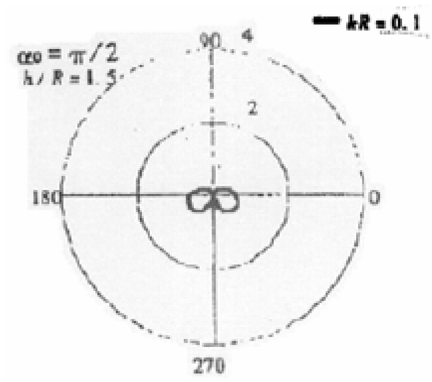
A cavity in the half plane subject to incident SH wave

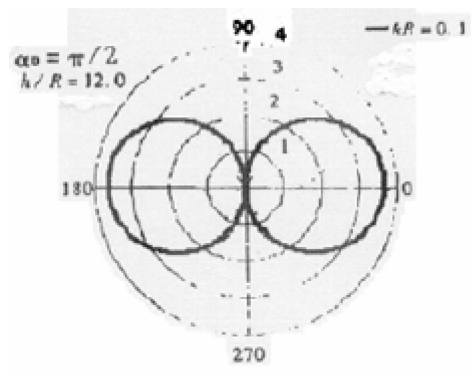


Numerical result 3

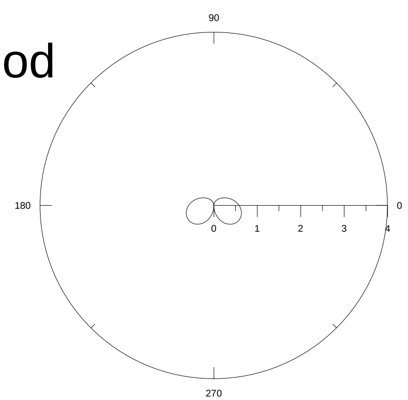
$\gamma = 0$ $\qquad h/R = 1.5$ $\qquad h/R = 12$

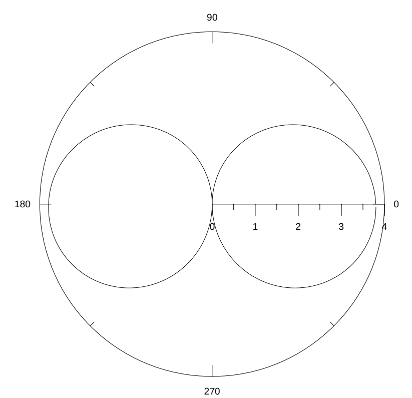
Lin and
Liu's result





Present method
(m=10)





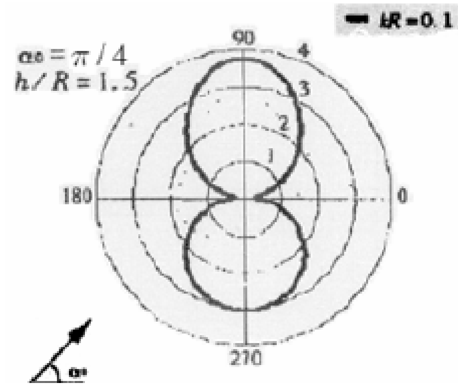
Numerical result 3

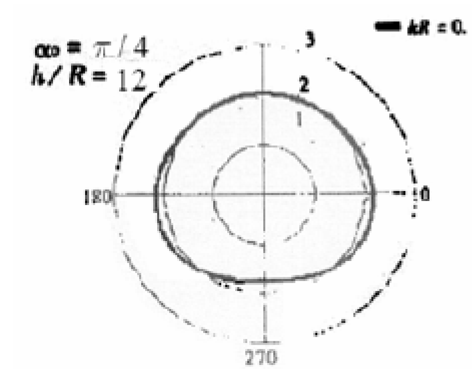
$\gamma = 45^{\circ}$

$h/R = 1.5$

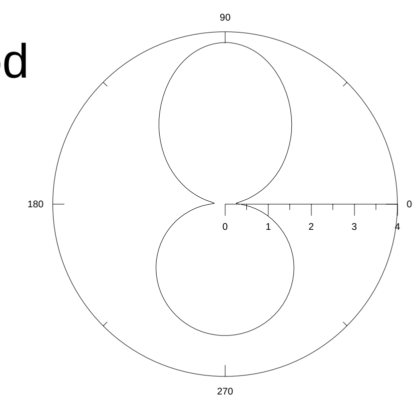
$h/R = 12$

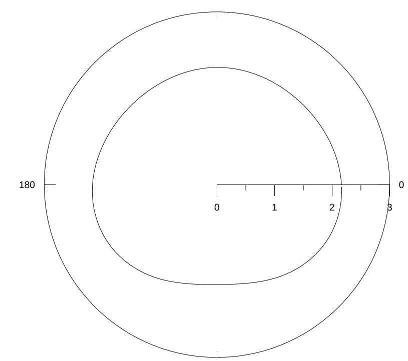
Lin and
Liu's result





Present method
 (m=10)





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Conclusions

- We have successful proposed a BIEM formulation by using degenerate kernels, null-field integral equation and Fourier series in companion with adaptive observer systems and vector decomposition.
- The present method is seen as a “semi-analytical” solution since error only attributes to the truncation of Fourier series.
- Therefore, the developed program of our formulation can be easily applied to solve the problem.

Thanks your kind attentions

You can get more information on our website.

http://msvlab.hre.ntou.edu.tw/