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國立中正大學

數學系暨研究所

Null-field integral equation approach and applications

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中正大學數學系

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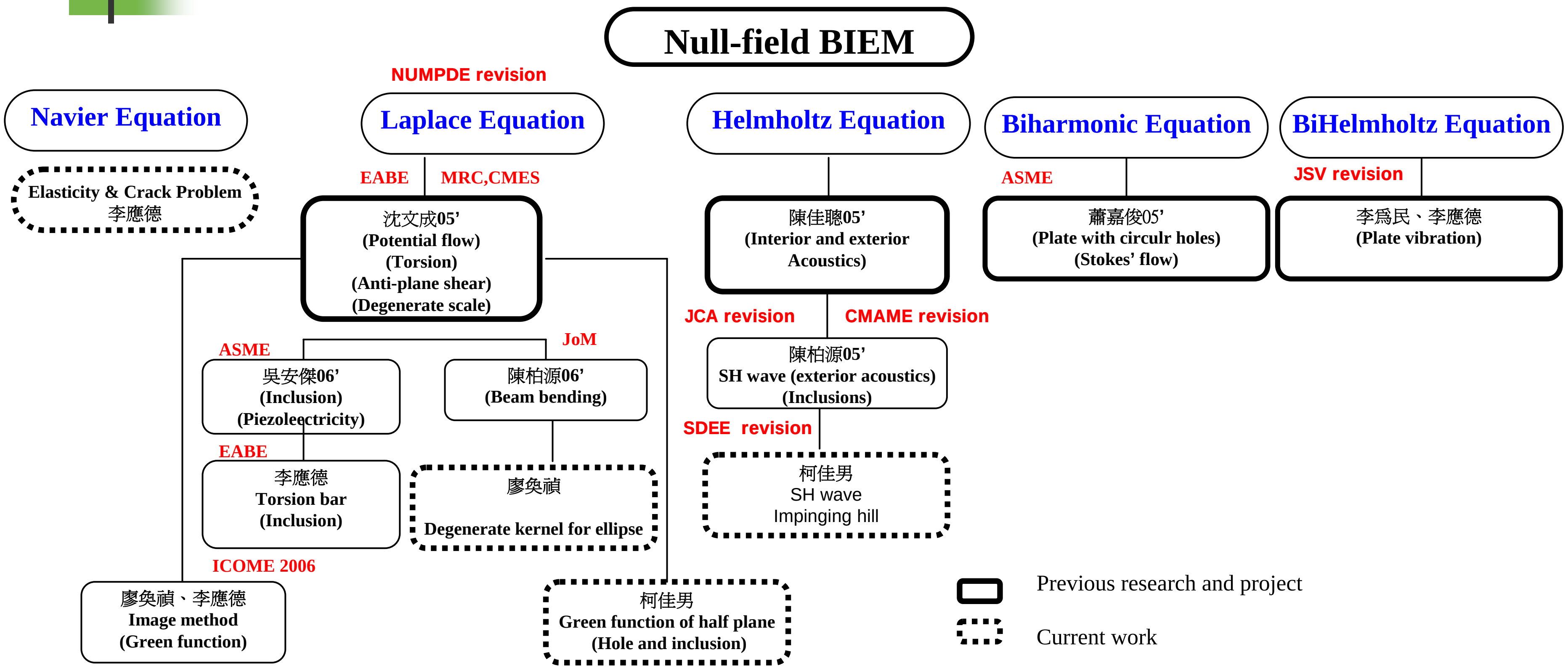
Research topics of NTOU / MSV LAB on null-field BIE (2003-2006)



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哲人日已遠 典型在宿昔
C B Ling (1909-1993)

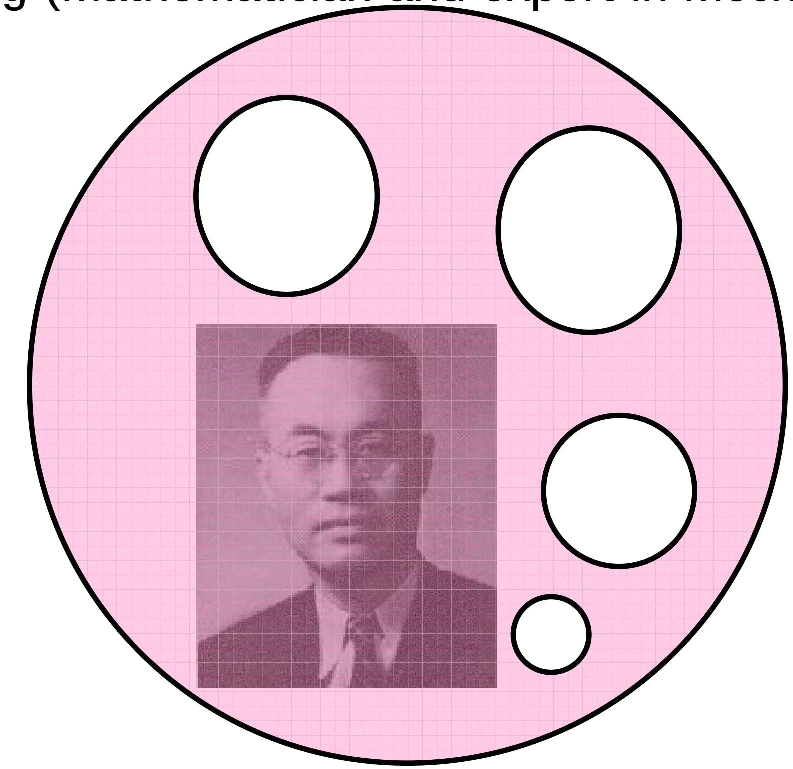
林致平院士(數學力學家)
C B Ling (mathematician and expert in mechanics)

省立中興大學第一任校長

林致平校長
(民國五十年~民國五十二年)

林致平所長(中研院數學所)

林致平院士(中研院)



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PS: short visit (J T Chen) of Academia Sinica 2006 summer `

Outlines

- Motivation and literature review
- Mathematical formulation
  - Expansions of fundamental solution and boundary density
  - Adaptive observer system
  - Vector decomposition technique
  - Linear algebraic equation
- Numerical examples
- Conclusions

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Motivation

Numerical methods for engineering problems

↓

FDM / FEM / BEM / BIEM / Meshless method

↓

BEM / BIEM (mesh required)

↓

- Treatment of singularity and hypersingularity
- Boundary-layer effect
- Convergence rate
- Ill-posed model

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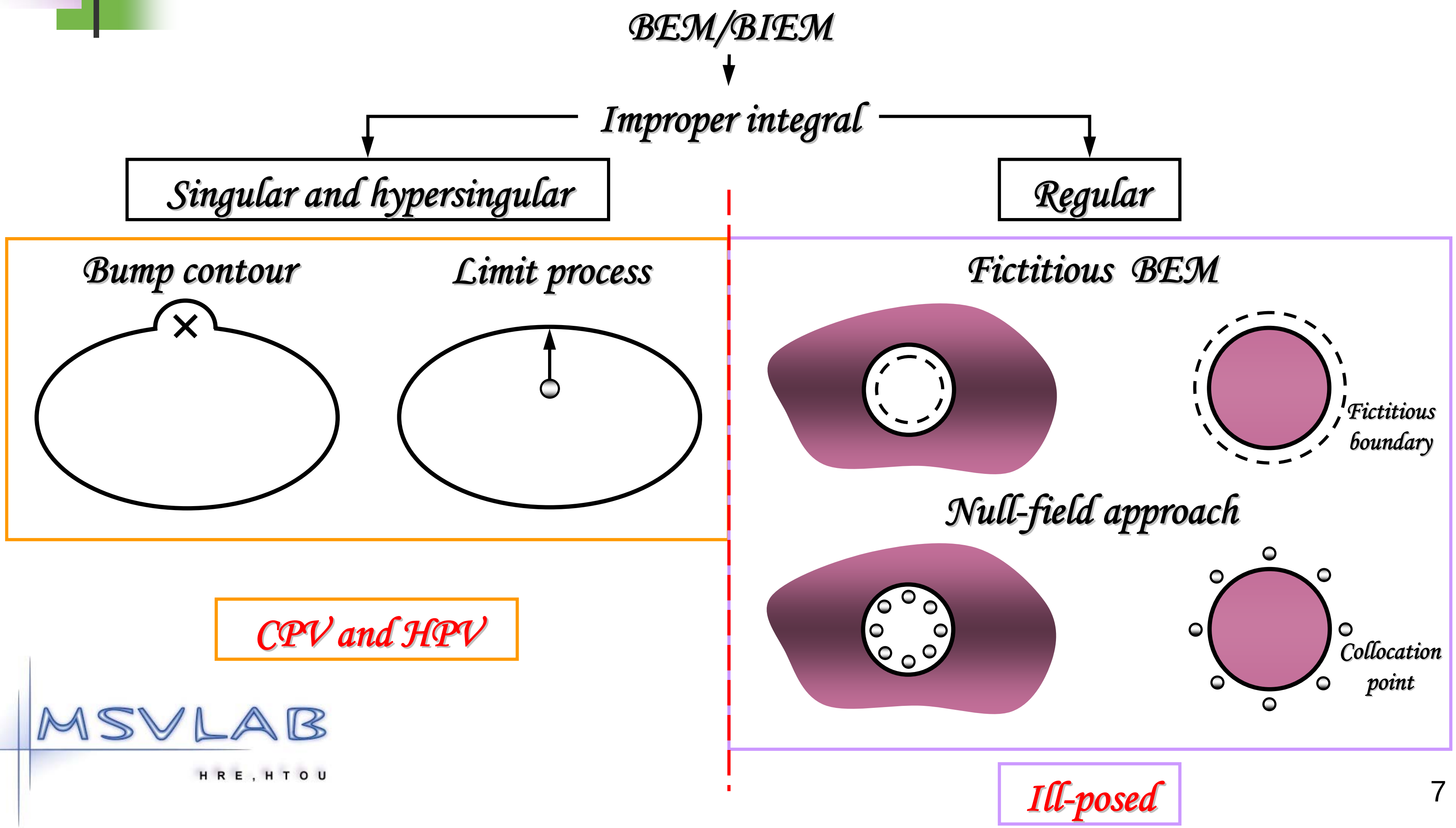
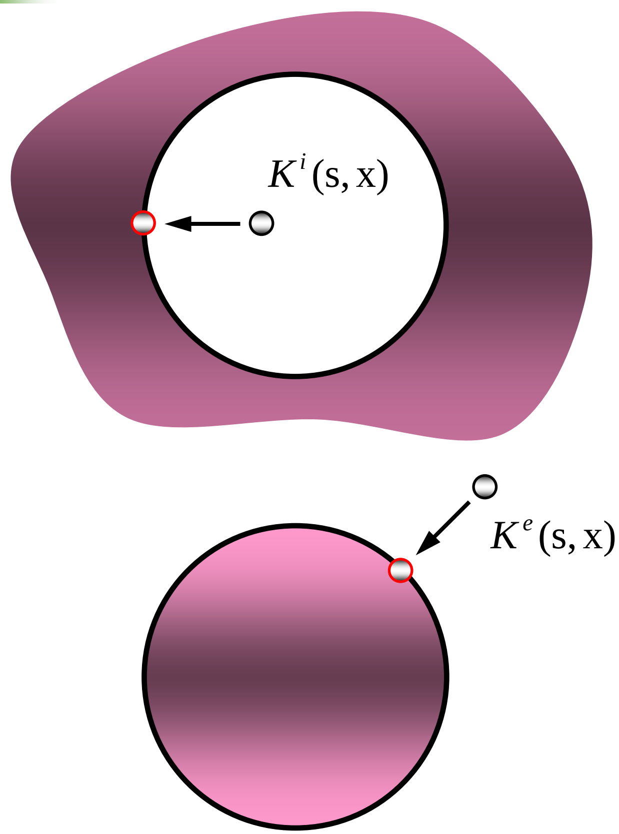
Mesh free for circular boundaries ?

Motivation and literature review



Present approach



$$a(x)b(s) \quad \varphi(x) = \int_B K(s,x)\phi(s)\,dB(s)$$

$$\varphi(x) = \int_B a(x)b(s)\phi(s)\,dB(s)$$

$$O\left(\frac{1}{|x-s|}\right), \quad O\left(\frac{1}{|x-s|^2}\right)$$

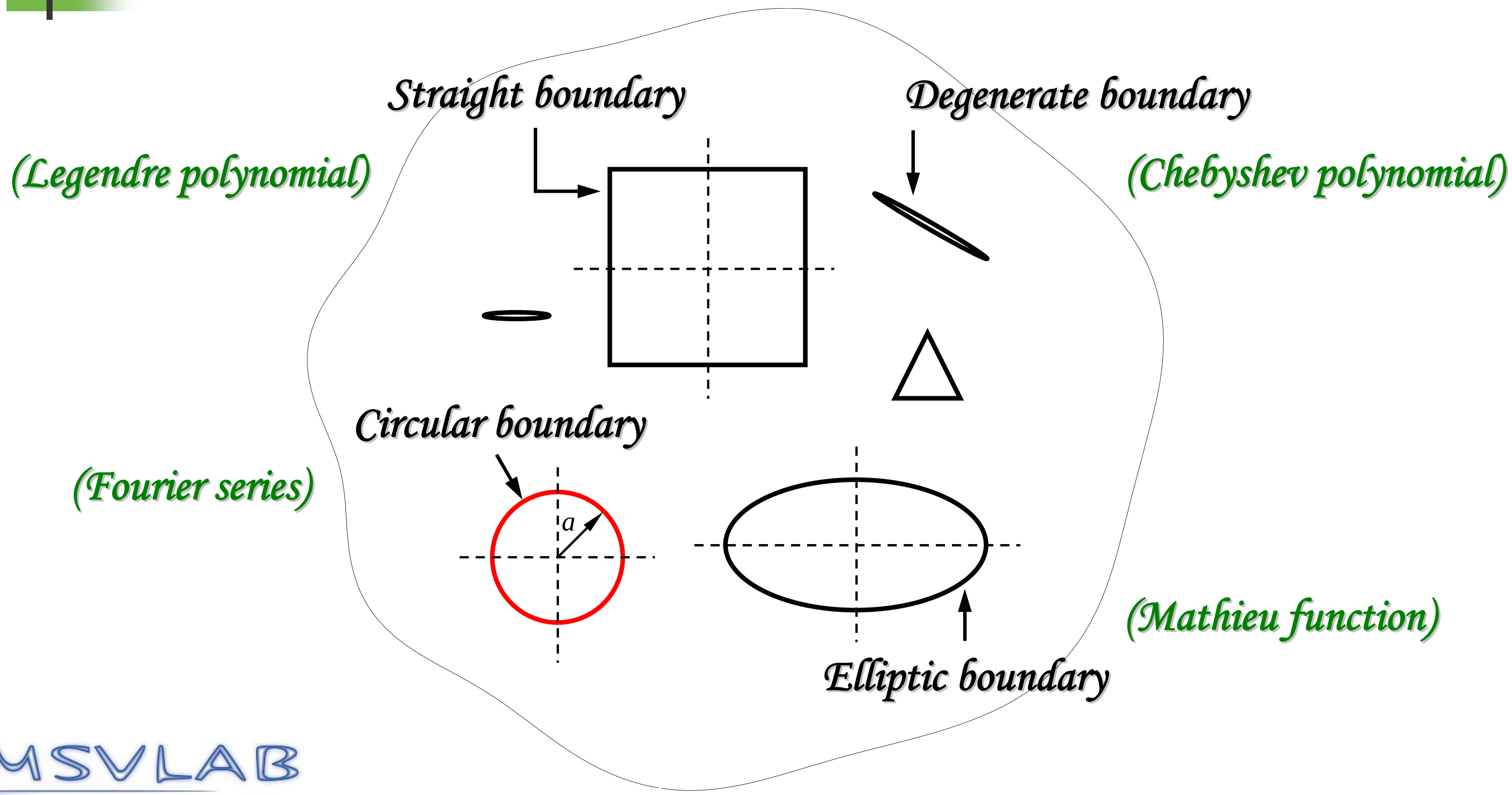
Degenerate kernel → *No principal value*

Fundamental solution → *CPV and HPV*

Advantages of degenerate kernel

1. *No principal value*
2. *Well-posed*
3. *No boundary-layer effect*
4. *Exponetial convergence*

Engineering problem with arbitrary geometries



Motivation and literature review

Analytical methods for solving Laplace problems with circular holes

Conformal mapping	*Bipolar coordinate*	*Special solution*
Chen and Weng, 2001, "Torsion of a circular compound bar with imperfect interface", ASME Journal of Applied Mechanics	*Lebedev, Skalskaya and Uyand, 1979, "Work problem in applied mathematics", Dover Publications*	*Honein, Honein and Hermann, 1992, "On two circular inclusions in harmonic problem", Quarterly of Applied Mathematics*

Limited to doubly connected domain

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Fourier series approximation

- *Ling (1943) - torsion of a circular tube*
- *Caulk et al. (1983) - steady heat conduction with circular holes*
- *Bird and Steele (1992) - harmonic and biharmonic problems with circular holes*
- *Mogilevskaya et al. (2002) - elasticity problems with circular boundaries*

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Contribution and goal

- *However, they didn't employ the null-field integral equation and degenerate kernels to fully capture the circular boundary, although they all employed Fourier series expansion.*
- *To develop a systematic approach for solving Laplace problems with multiple holes is our goal.*

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Outlines (Direct problem)

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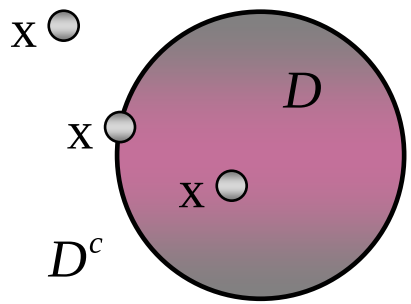
Boundary integral equation and null-field integral equation

Interior case

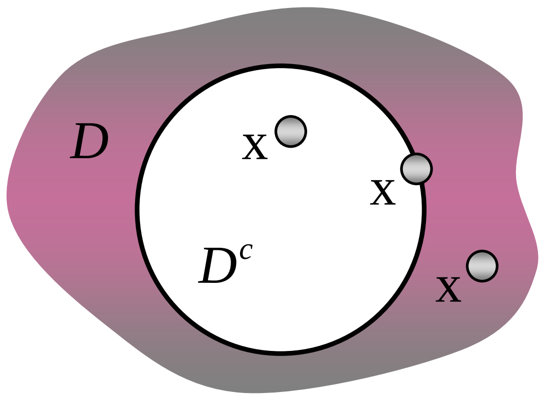


$$U(s,x) = \ln|x-s| = \ln r$$

$$T(s,x) = \frac{\partial U(s,x)}{\partial n_s}$$

$$\psi(s) = \frac{\partial \varphi(s)}{\partial n_s}$$

Exterior case



$$2\pi\varphi(x) = \int_B T(s,x)\varphi(s)dB(s) - \int_B U(s,x)\psi(s)dB(s), \quad x \in D \cup B$$

$$\pi\varphi(x) = C.P.V.\int_B T(s,x)\varphi(s)dB(s) - R.P.V.\int_B U(s,x)\psi(s)dB(s), \quad x \in B$$

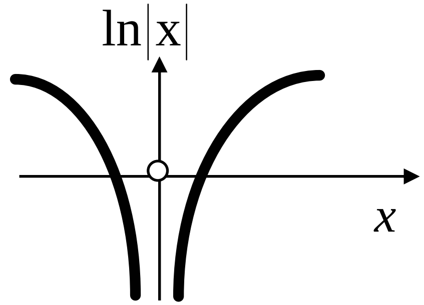
Degenerate (separate) form

$$0 = \int_B T(s,x)\varphi(s)dB(s) - \int_B U(s,x)\psi(s)dB(s), \quad x \in D^c \cup B$$

Definitions of R.P.V., C.P.V. and H.P.V. using bump approach

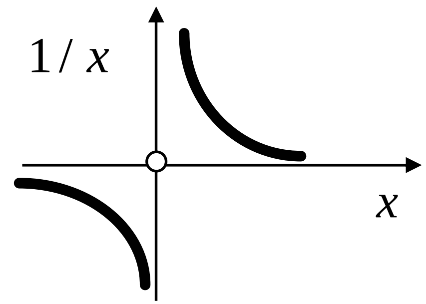
- **R.P.V. (Riemann principal value)**

$$R.P.V.\int_{-1}^{1} \ln|x|\, dx = (x\ln|x| - x)\Big|_{x=-1}^{x=1} = -2$$



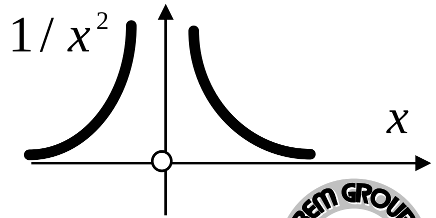
- **C.P.V.(Cauchy principal value)**

$$C.P.V.\int_{-1}^{1} \frac{1}{x}\, dx = \lim_{\varepsilon \to 0} \int_{-1}^{-\varepsilon} + \int_{\varepsilon}^{1} \frac{1}{x}\, dx = 0$$



- **H.P.V.(Hadamard principal value)**

$$H.P.V.\int_{-1}^{1} \frac{1}{x^2}\, dx = \lim_{\varepsilon \to 0} \int_{-1}^{-\varepsilon} + \int_{\varepsilon}^{1} \frac{1}{x^2}\, dx - \frac{2}{\varepsilon} = -2$$



Principal value in who's sense

- Riemann sense (Common sense)
- Lebesgue sense
- Cauchy sense
- Hadamard sense (elasticity)
- Mangler sense (aerodynamics)
- Liggett and Liu's sense
- *The singularity that occur when the base point and field point coincide are not integrable. (1983)*

Two approaches to understand HPV

$$H.P.V.\int_{-1}^{1}\frac{1}{x^2}\,dx=\lim\left[\int_{-1}^{-\varepsilon}+\int_{\varepsilon}^{1}\frac{1}{x^2}\,dx-\frac{2}{\varepsilon}\right]=-2$$

Differential first and then trace operator

$$\lim_{y\to 0}\int_{-1}^{1}\frac{1}{x^2+y^2}\,dx=-2$$

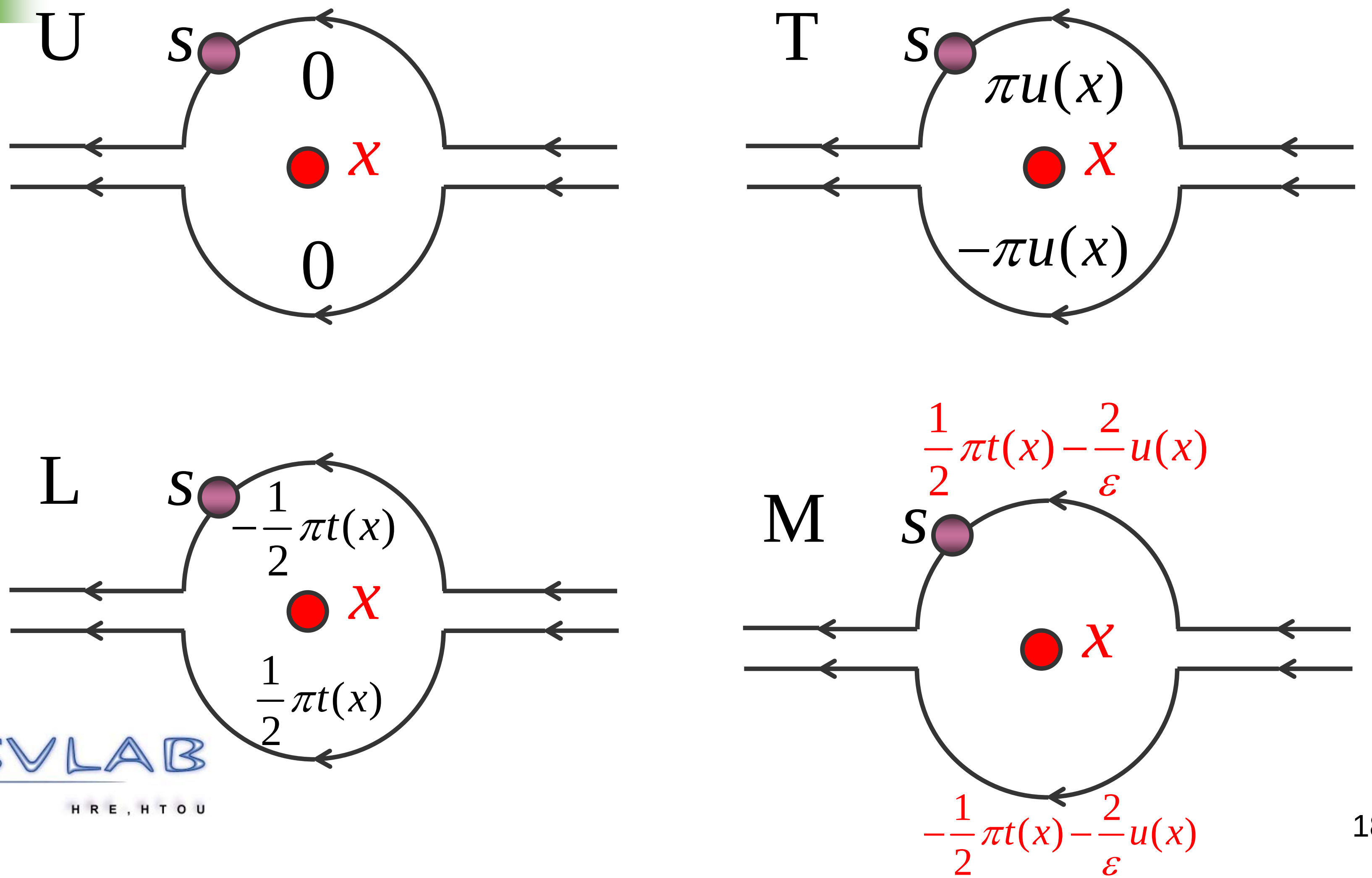
(Limit and integral operator can not be commuted)

Trace first and then differential operator

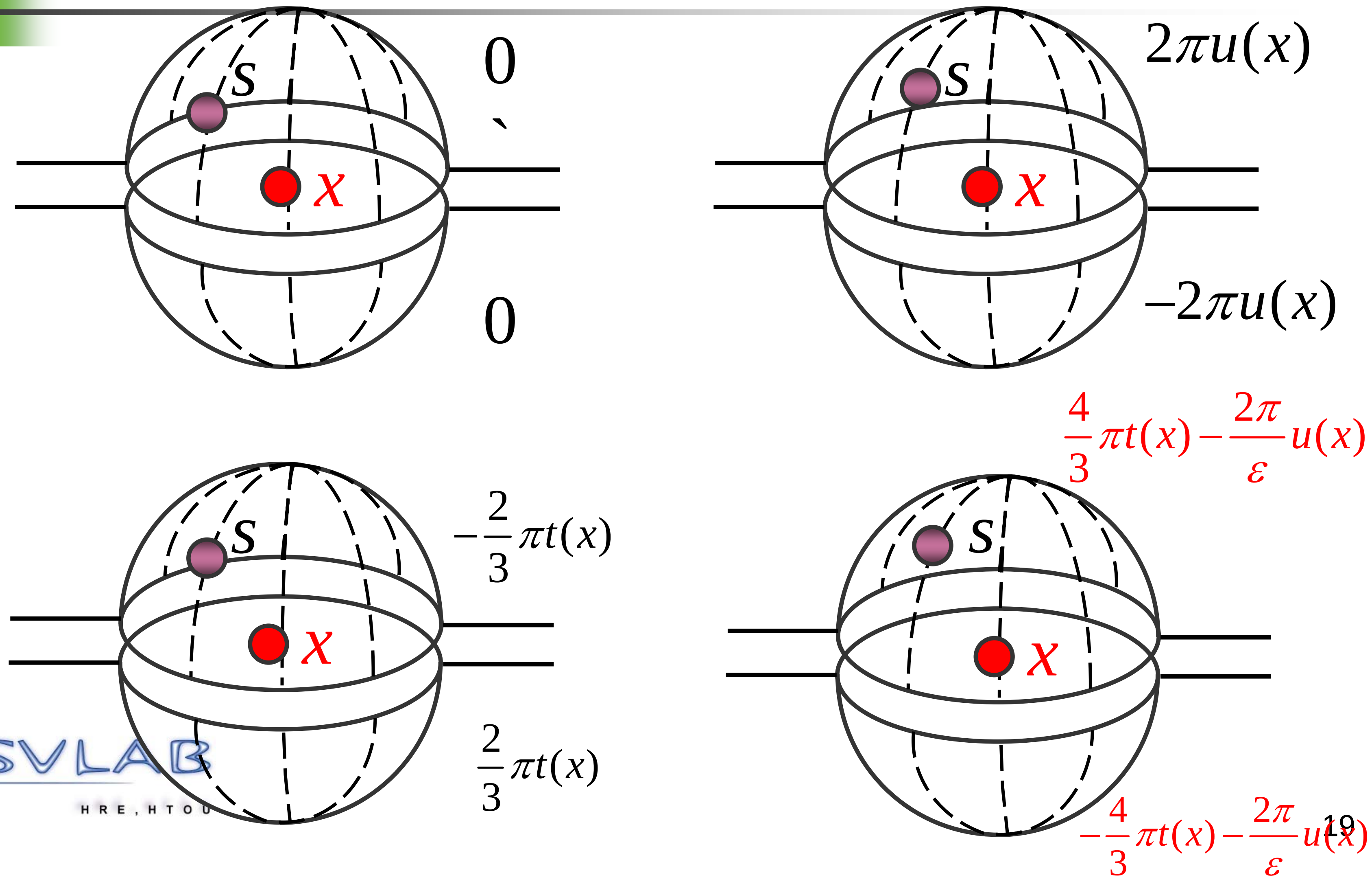
$$\frac{d}{dt}\left\{CPV\int_{-1}^{1}\frac{-1}{x-t}dx\right\}\Big|_{t=0}=-2$$

(Leibnitz rule should be considered)

Bump contribution (2-D)



Bump contribution (3-D)



Outlines (Direct problem)

- Motivation and literature review
- Mathematical formulation
  - Expansions of fundamental solution and boundary density
  - Adaptive observer system
  - Vector decomposition technique
  - Linear algebraic equation
- Numerical examples
- Degenerate scale
- Conclusions

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Gain of introducing the degenerate kernel

$$\varphi(\mathrm{x}) = \int_B \boxed{K(\mathrm{s},\mathrm{x})}\,\phi(\mathrm{s})\,dB(\mathrm{s})$$

Degenerate kernel →

$$K(\mathrm{s,x}) = \begin{cases} K^i(\mathrm{s,x}) = \sum_{j=0}^{\infty} a_j(\mathrm{s}) b_j(\mathrm{x}), & |\mathrm{x}| < |\mathrm{s}| \\ K^e(\mathrm{s,x}) = \sum_{j=0}^{\infty} a_j(\mathrm{x}) b_j(\mathrm{s}), & |\mathrm{x}| > |\mathrm{s}| \end{cases}$$

interior

exterior

→

$$\varphi(\mathrm{x}) = \int_B \sum_{j=0}^{\infty} a_j(\mathrm{x}) b_j(\mathrm{s}) \phi(\mathrm{s})\,dB(\mathrm{s})$$

→ No principal value?

Fundamental solution → CPV and HPV

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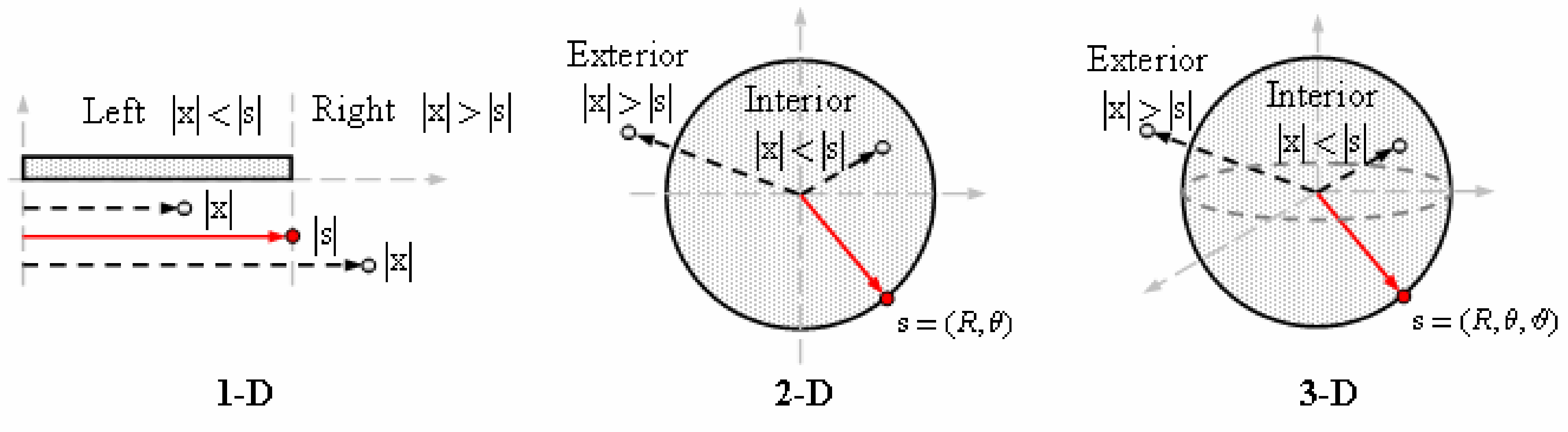
How to separate the region



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Expansions of fundamental solution and boundary density

- *Degenerate kernel - fundamental solution*

$$U(\mathrm{s},\mathrm{x})=\begin{cases} U^i(R,\theta;\rho,\phi)=\ln R-\sum_{m=1}^{\infty}\frac{1}{m}(\frac{\rho}{R})^m\cos m(\theta-\phi), & R\geq\rho \\ U^e(R,\theta;\rho,\phi)=\ln\rho-\sum_{m=1}^{\infty}\frac{1}{m}(\frac{R}{\rho})^m\cos m(\theta-\phi), & \rho>R \end{cases}$$

- *Fourier series expansions - boundary density*

$$u(\mathrm{s})=a_0+\sum_{n=1}^{M}(a_n\cos n\theta+b_n\sin n\theta),\ \ \mathrm{s}\in B$$

$$t(\mathrm{s})=p_0+\sum_{n=1}^{M}(p_n\cos n\theta+q_n\sin n\theta),\ \ \mathrm{s}\in B$$

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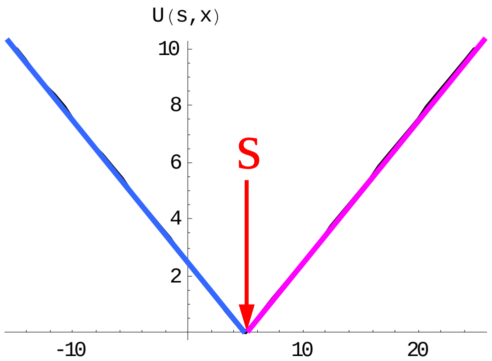
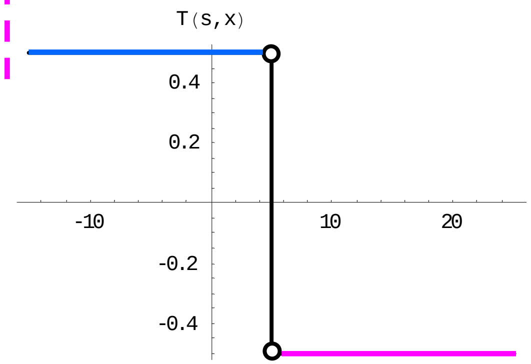
Separable form of fundamental solution (1D)

Separable property $U(s,x)=\begin{cases}\sum_{i=1}^{2} a_i(x)b_i(s), & s \ge x\\ \sum_{i=1}^{2} a_i(s)b_i(x), & x > s\end{cases}$



continuous



discontinuous

$$U(s,x)=\frac{1}{2}r=\begin{cases}\frac{1}{2}(s-x), & s \ge x\\ \frac{1}{2}(x-s), & x > s\end{cases}$$

$$T(s,x)=\begin{cases}\frac{1}{2}, & s > x\\ \frac{-1}{2}, & x > s\end{cases}$$

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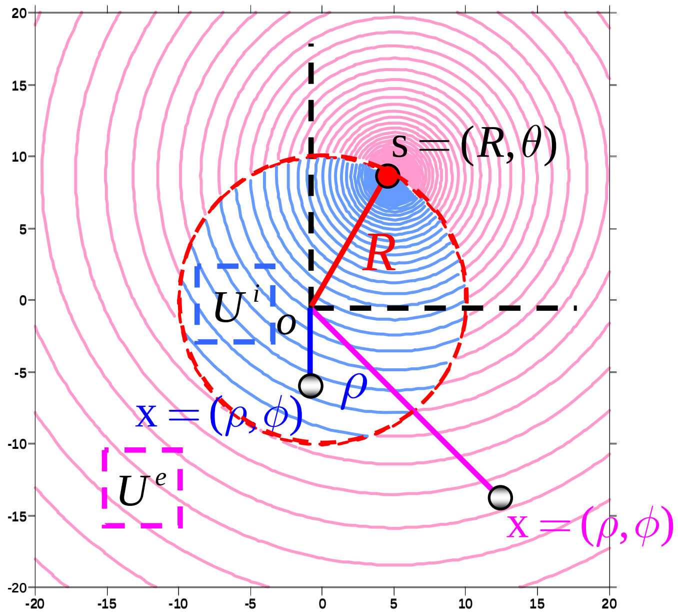
Separable form of fundamental solution (2D)

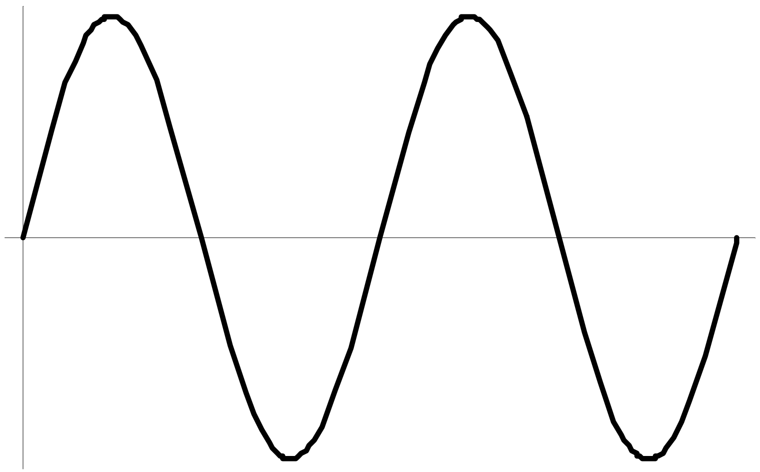
$$U(\mathrm{s},\mathrm{x}) = \begin{cases} U^{i}(R,\theta;\rho,\phi) = \ln R - \sum_{m=1}^{\infty} \frac{1}{m}\left(\frac{\rho}{R}\right)^{m} \cos m(\theta-\phi), & R \geq \rho \\ U^{e}(R,\theta;\rho,\phi) = \ln \rho - \sum_{m=1}^{\infty} \frac{1}{m}\left(\frac{R}{\rho}\right)^{m} \cos m(\theta-\phi), & \rho > R \end{cases}$$



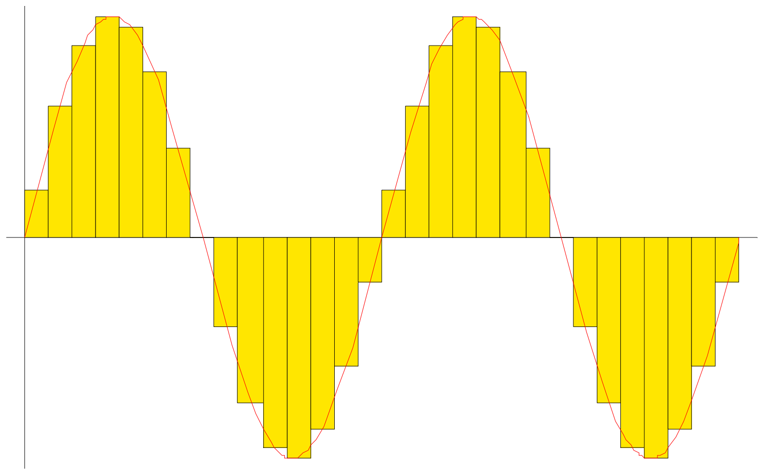
Boundary density discretization

Fourier series

Ex. constant element



Present method



Conventional BEM

Outlines

- Motivation and literature review
- Mathematical formulation
  - Expansions of fundamental solution and boundary density
  - Adaptive observer system
  - Vector decomposition technique
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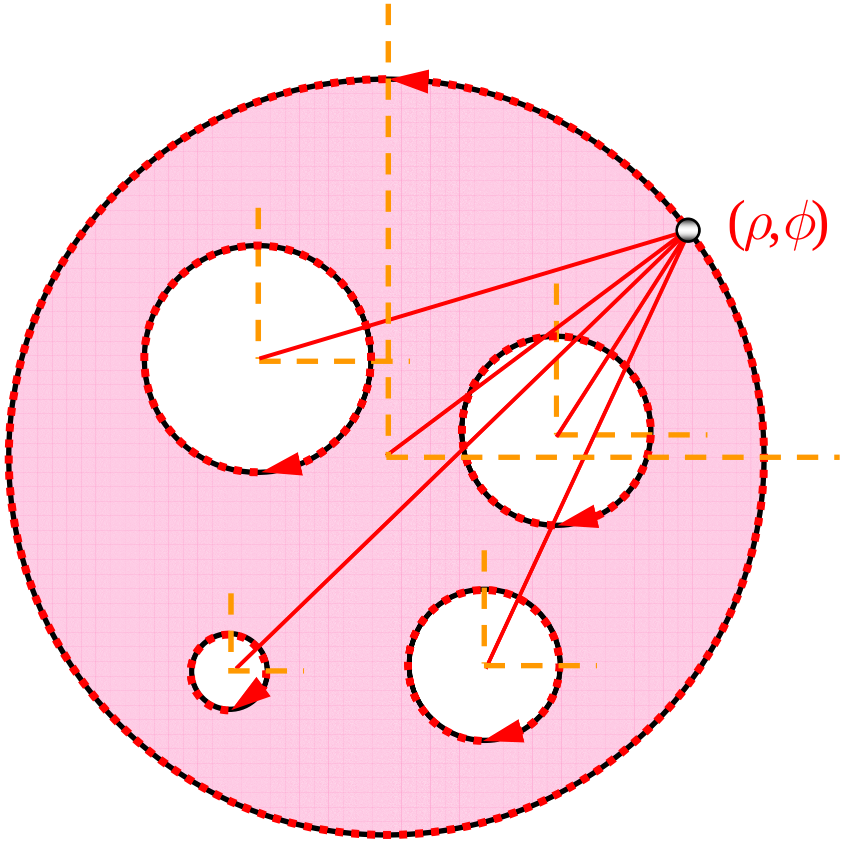
Adaptive observer system



○ *collocation point*

Outlines

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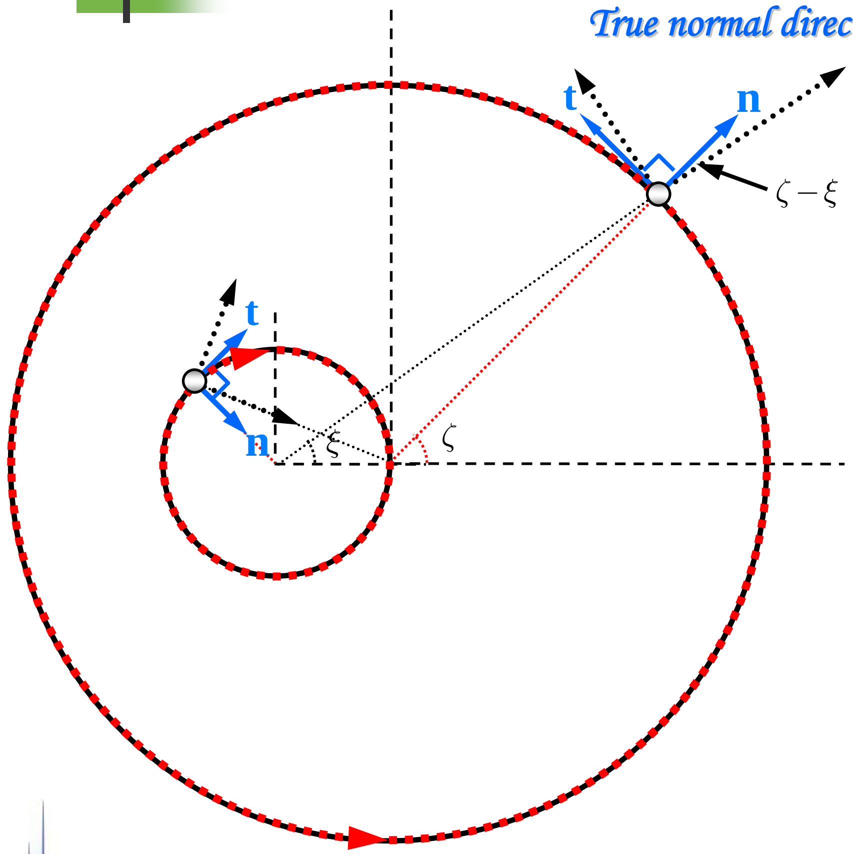
Vector decomposition technique for potential gradient



$$2\pi \frac{\partial u(\mathrm{x})}{\partial \mathbf{n}} = \int_B M_\rho(\mathrm{s},\mathrm{x})u(\mathrm{s})dB(\mathrm{s}) - \int_B L_\rho(\mathrm{s},\mathrm{x})t(\mathrm{s})dB(\mathrm{s}), \ \ \mathrm{x}\in D$$

$$2\pi \frac{\partial u(\mathrm{x})}{\partial \mathbf{t}} = \int_B M_\phi(\mathrm{s},\mathrm{x})u(\mathrm{s})dB(\mathrm{s}) - \int_B L_\phi(\mathrm{s},\mathrm{x})t(\mathrm{s})dB(\mathrm{s}), \ \ \mathrm{x}\in D$$

Non-concentric case:

$$L_\rho(\mathrm{s},\mathrm{x}) = \frac{\partial U(\mathrm{s},\mathrm{x})}{\partial \rho}\boxed{\cos(\zeta-\xi)} + \frac{1}{\rho}\frac{\partial U(\mathrm{s},\mathrm{x})}{\partial \phi}\boxed{\cos(\frac{\pi}{2}-\zeta+\xi)}$$

$$M_\rho(\mathrm{s},\mathrm{x}) = \frac{\partial T(\mathrm{s},\mathrm{x})}{\partial \rho}\boxed{\cos(\zeta-\xi)} + \frac{1}{\rho}\frac{\partial T(\mathrm{s},\mathrm{x})}{\partial \phi}\boxed{\cos(\frac{\pi}{2}-\zeta+\xi)}$$

Special case (concentric case): $\zeta=\xi$

$$L_\rho(\mathrm{s},\mathrm{x}) = \frac{\partial U(\mathrm{s},\mathrm{x})}{\partial \rho} \qquad M_\rho(\mathrm{s},\mathrm{x}) = \frac{\partial T(\mathrm{s},\mathrm{x})}{\partial \rho}$$

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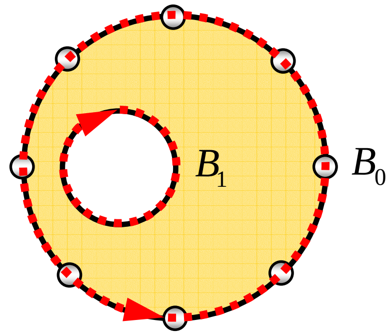
Linear algebraic equation



$$[\mathbf{U}]\{\mathbf{t}\} = [\mathbf{T}]\{\mathbf{u}\}$$

where

Index of collocation circle

$$[\mathbf{U}] = \begin{bmatrix} \mathbf{U}_{00} & \mathbf{U}_{01} & \cdots & \mathbf{U}_{0N} \\ \mathbf{U}_{10} & \mathbf{U}_{11} & \cdots & \mathbf{U}_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{U}_{N0} & \mathbf{U}_{N1} & \cdots & \mathbf{U}_{NN} \end{bmatrix}$$

Index of routing circle

$$\{\mathbf{t}\} = \begin{Bmatrix} \mathbf{t}_0 \\ \mathbf{t}_1 \\ \mathbf{t}_2 \\ \vdots \\ \mathbf{t}_N \end{Bmatrix}$$

Column vector of Fourier coefficients
(Nth routing circle)

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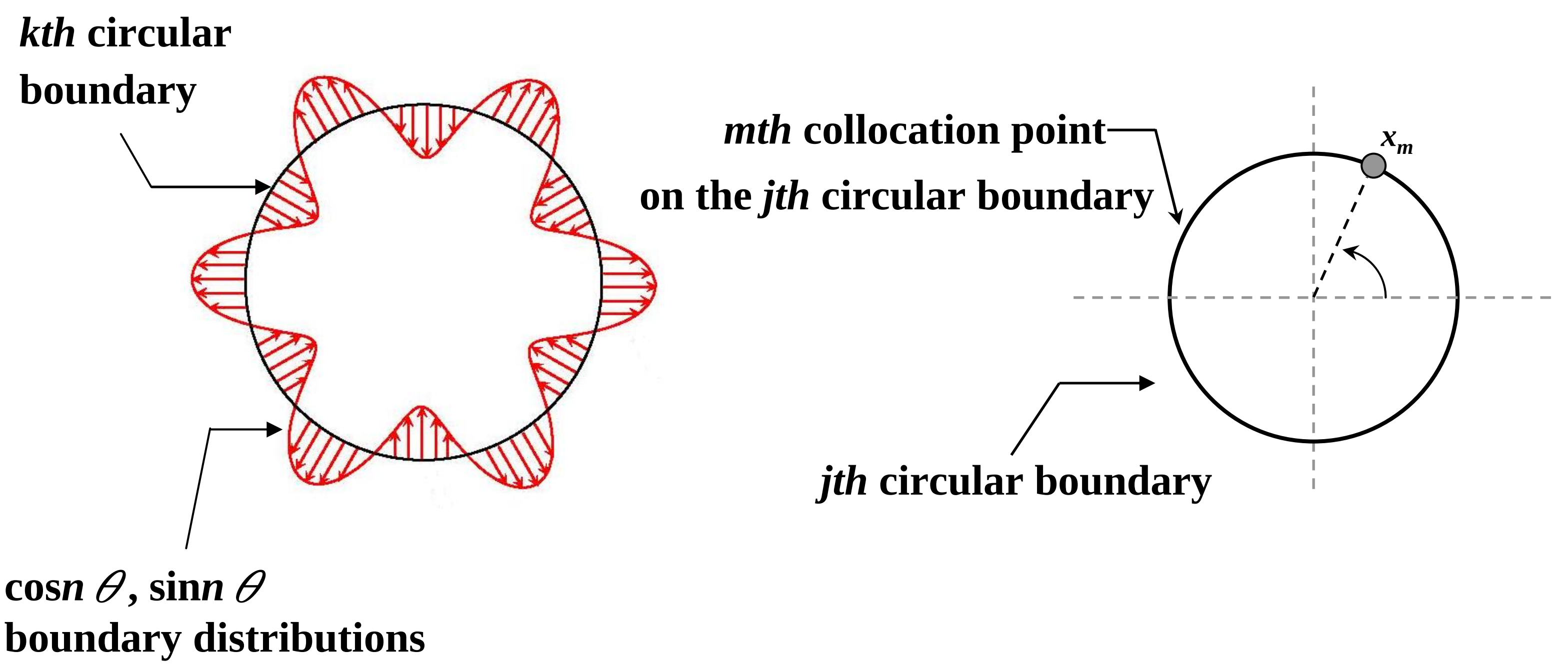
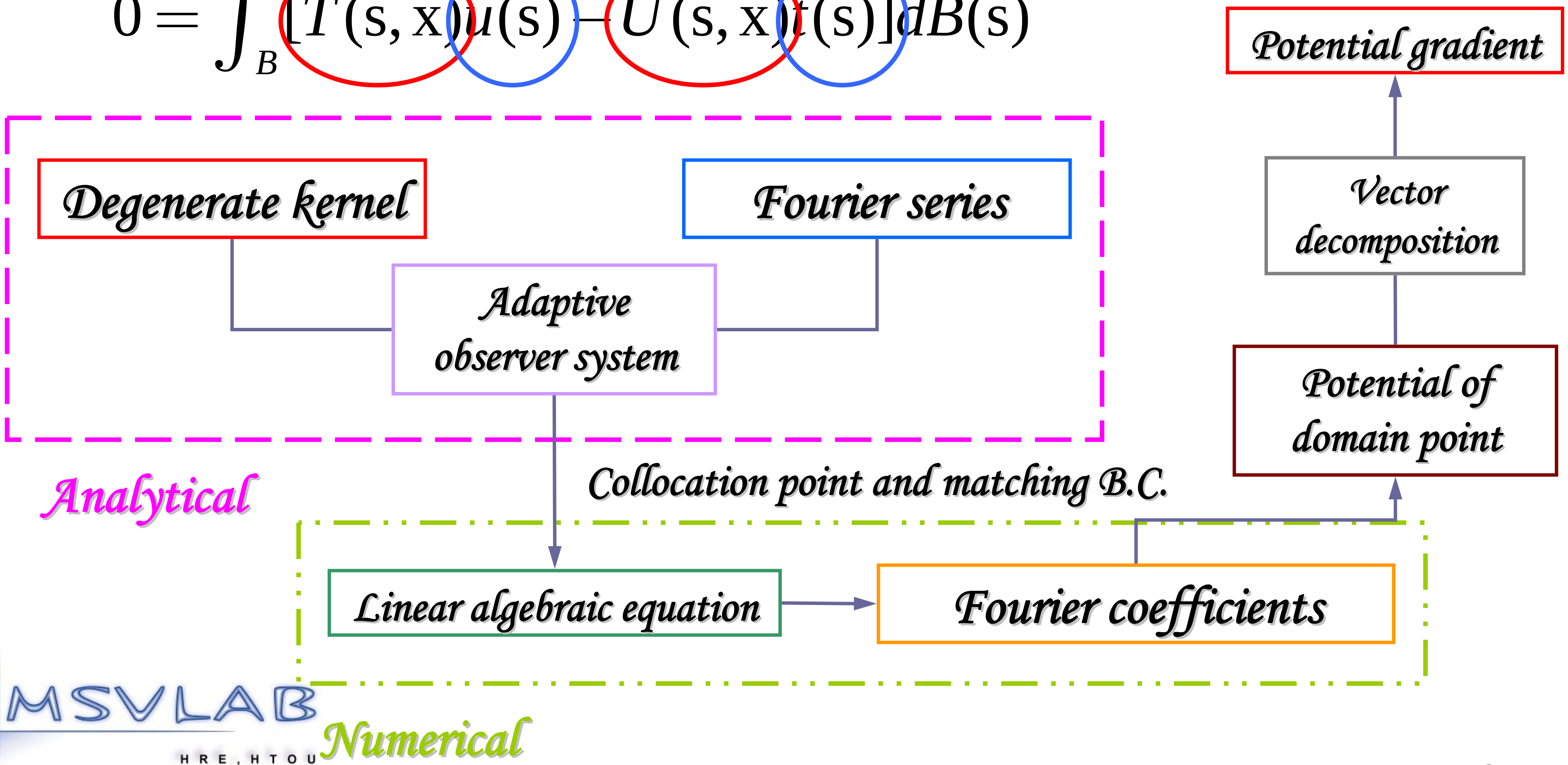
Physical meaning of influence coefficient



Physical meaning of the influence coefficient $U_{jk}^{nc}(\phi_m)$

Flowchart of present method

$$0 = \int_B [T(s,x)u(s) - U(s,x)t(s)]dB(s)$$



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Comparisons of conventional BEM and present method

	Boundary density discretization	Auxiliary system	Formulation	Observer system	Singularity	Convergence	Boundary layer effect
Conventional BEM	Constant, linear, quadratic… elements	Fundamental solution	Boundary integral equation	Fixed observer system	CPV, RPV and HPV	Linear	Appear
Present method	Fourier series expansion	Degenerate kernel	Null-field integral equation	Adaptive observer system	Disappear	Exponential	Eliminate

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Numerical examples

- *Laplace equation (EABE 2005, EABE 2007)*
  (CMES 2005, ASME 2007, JoM2007)
  (MRC 2007, NUMPDE revision)
- *Eigen problem (JCA revision)*
- *Exterior acoustics (CMAME, SDEE revision)*
- *Biharmonic equation (JAM, ASME 2006)*
- *Plate vibration (JSV revision)*

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Laplace equation

- *Steady state heat conduction problems*
- *Electrostatic potential of wires*
- *Flow of an ideal fluid pass cylinders*
- *A circular bar under torque*
- *An infinite medium under antiplane shear*
- *Half-plane problems*

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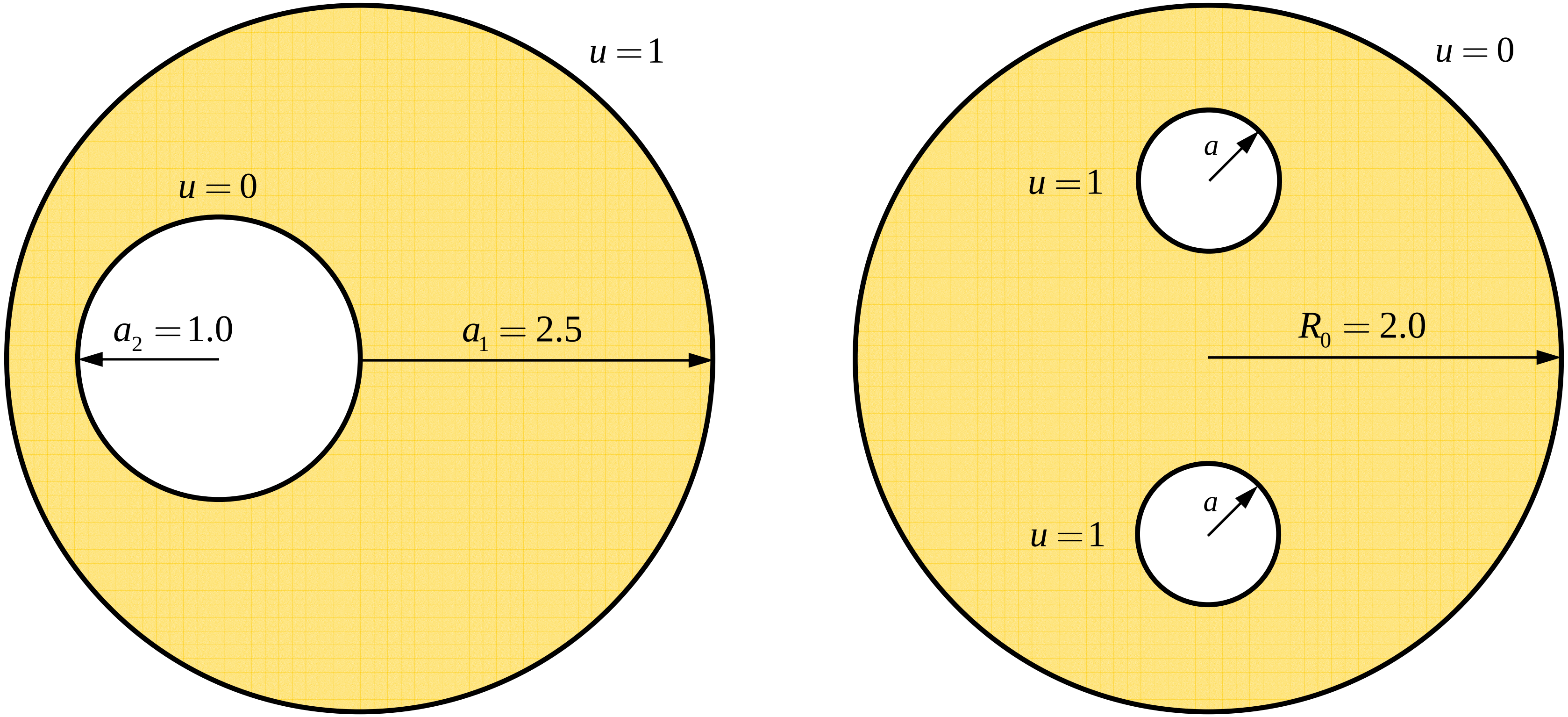
Steady state heat conduction problems



Case 1 *Case 2*

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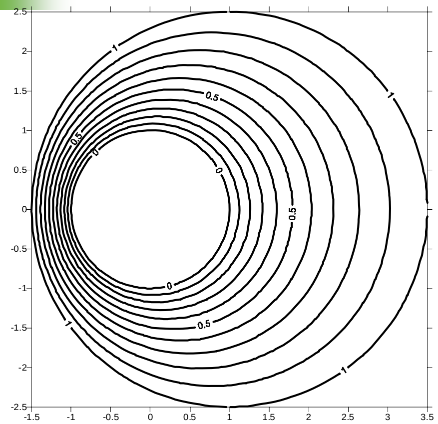
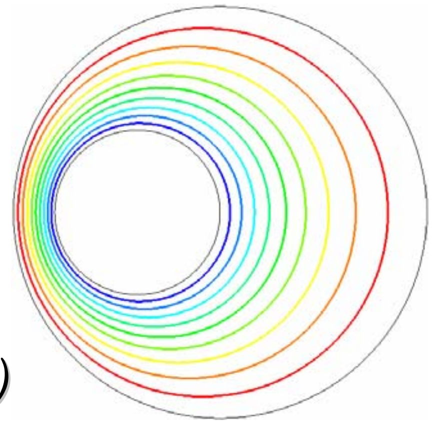
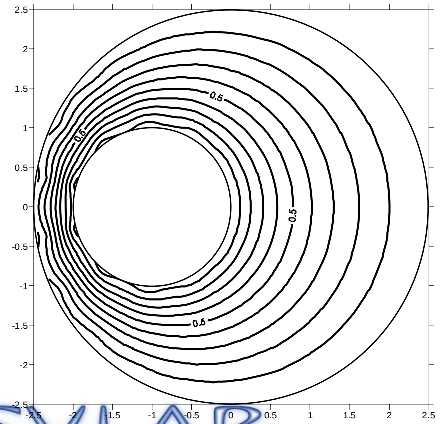
Case 1: Isothermal line



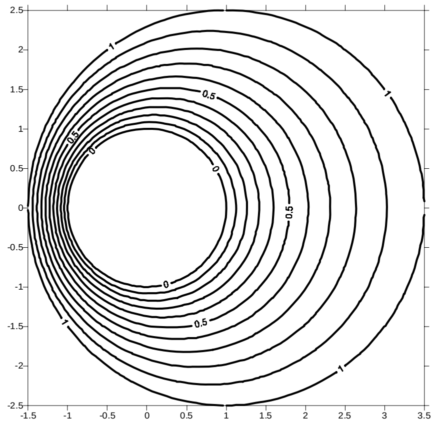
Exact solution (Carrier and Pearson)



FEM-ABAQUS (1854 elements)

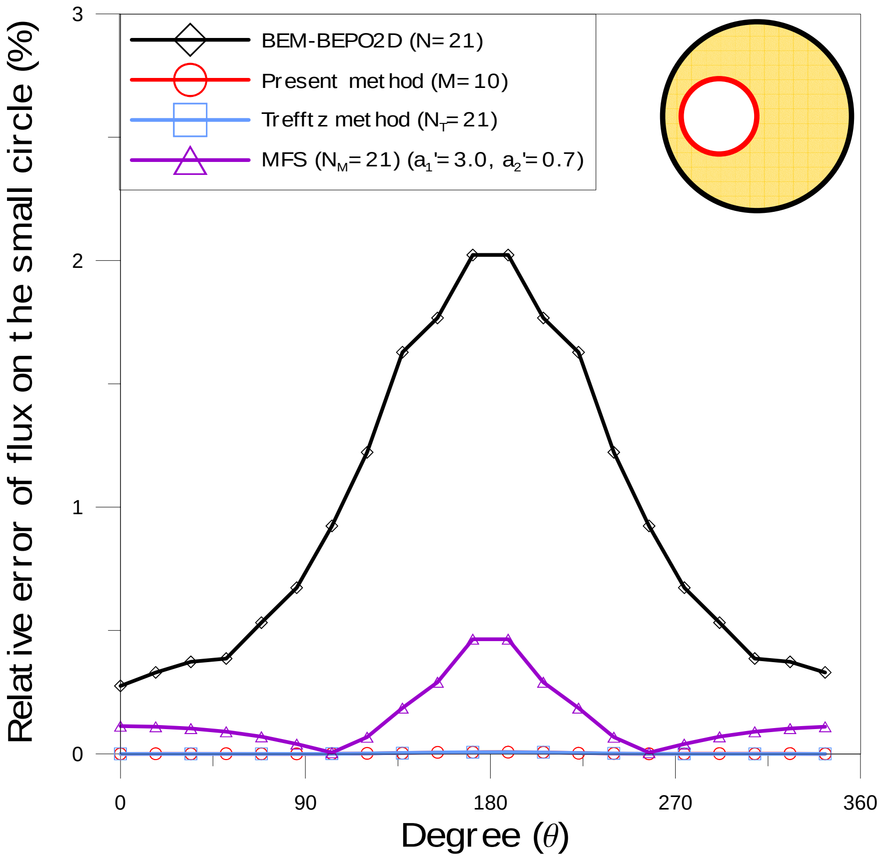


BEM-BEPO2D (N=21)



Present method (M=10)

Relative error of flux on the small circle



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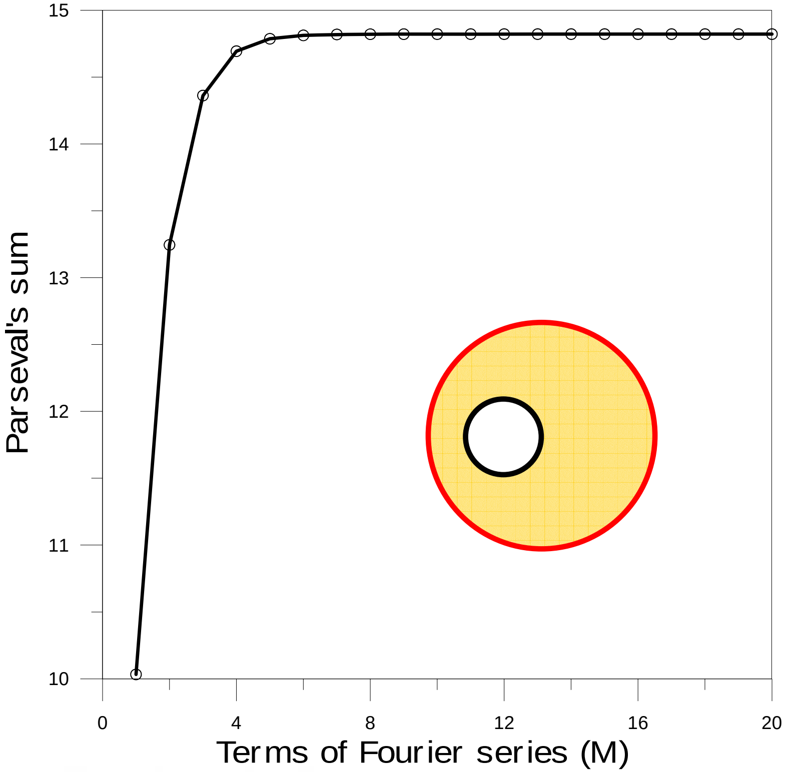
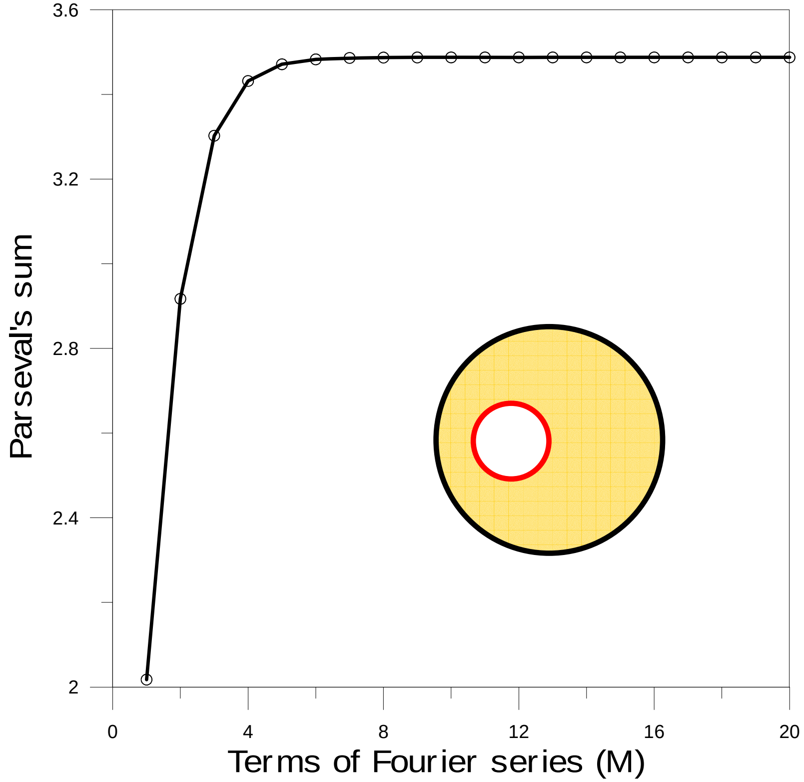
Convergence test - Parseval's sum for Fourier coefficients

Parseval's sum $\int_0^{2\pi} f^2(\theta)d\theta \doteq 2\pi a_0^{\ 2} + \pi\sum_{n=1}^{M}(a_n^{\ 2}+b_n^{\ 2})$





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Laplace equation

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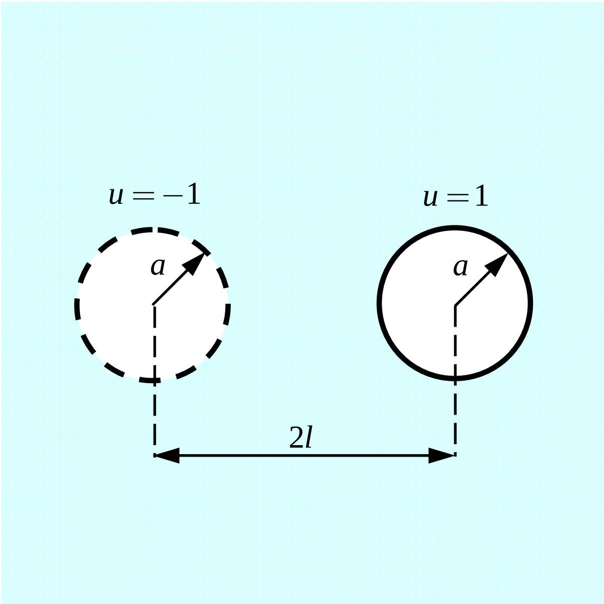
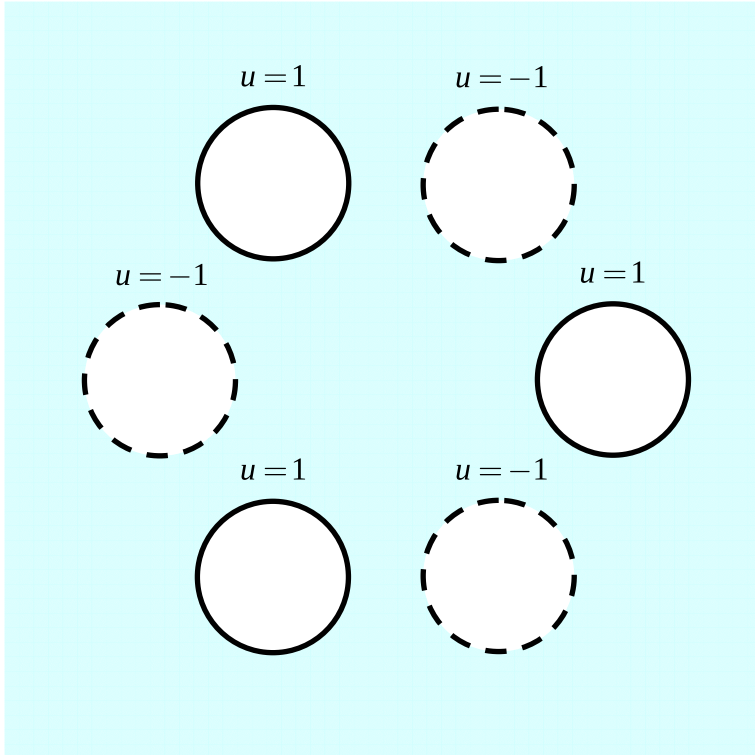
Electrostatic potential of wires



Two parallel cylinders held positive and negative potentials



Hexagonal electrostatic potential

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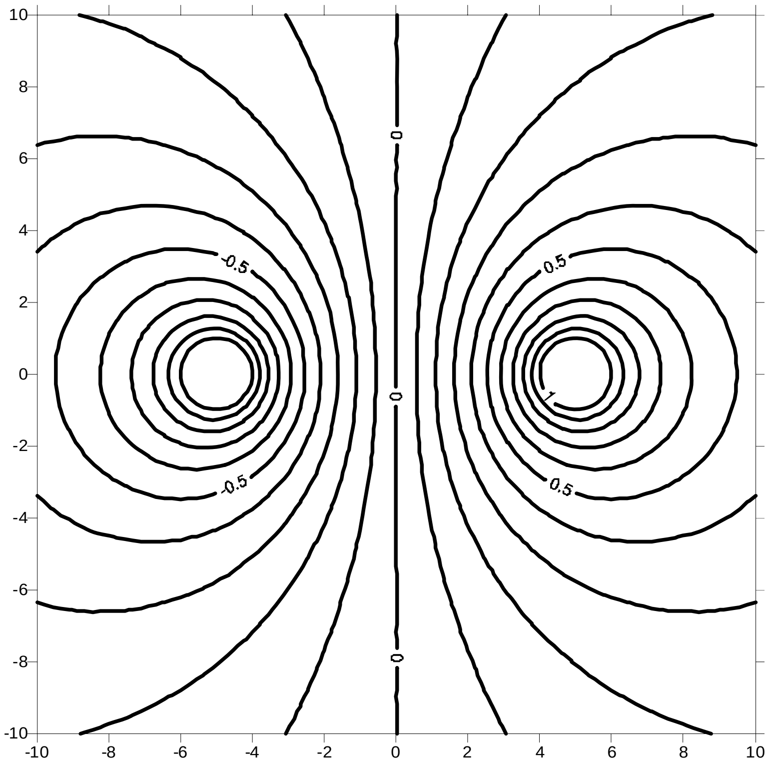
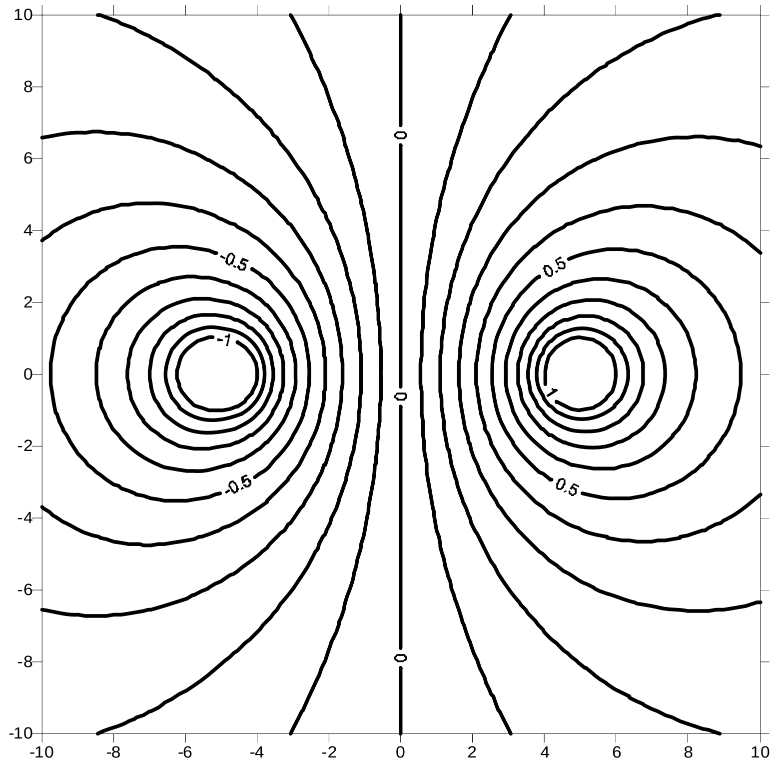
HRE, HTOU

Contour plot of potential



Exact solution (Lebedev et al.)



Present method (M=10)

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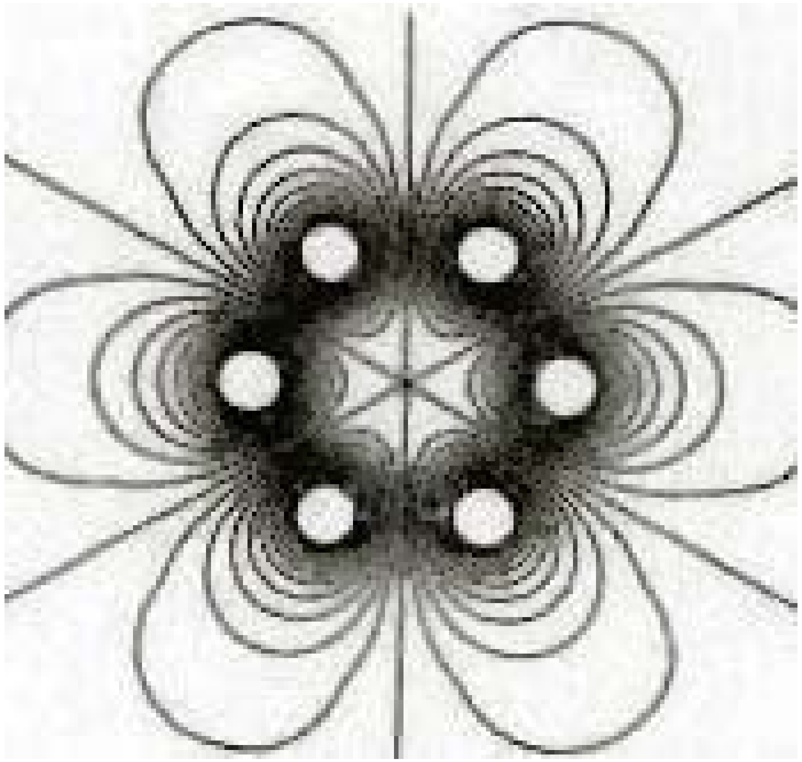
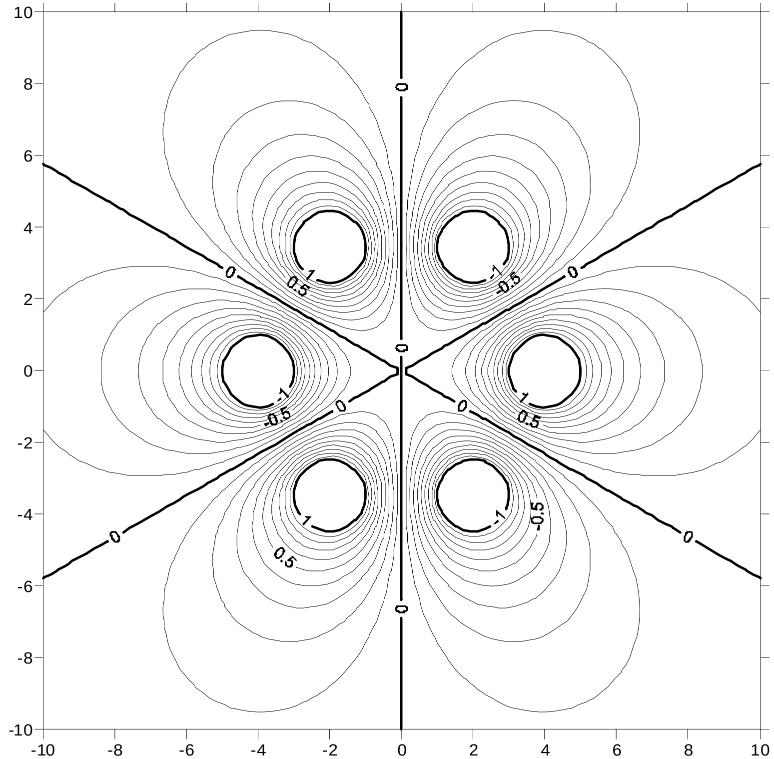
Contour plot of potential



Onishi's data (1991)

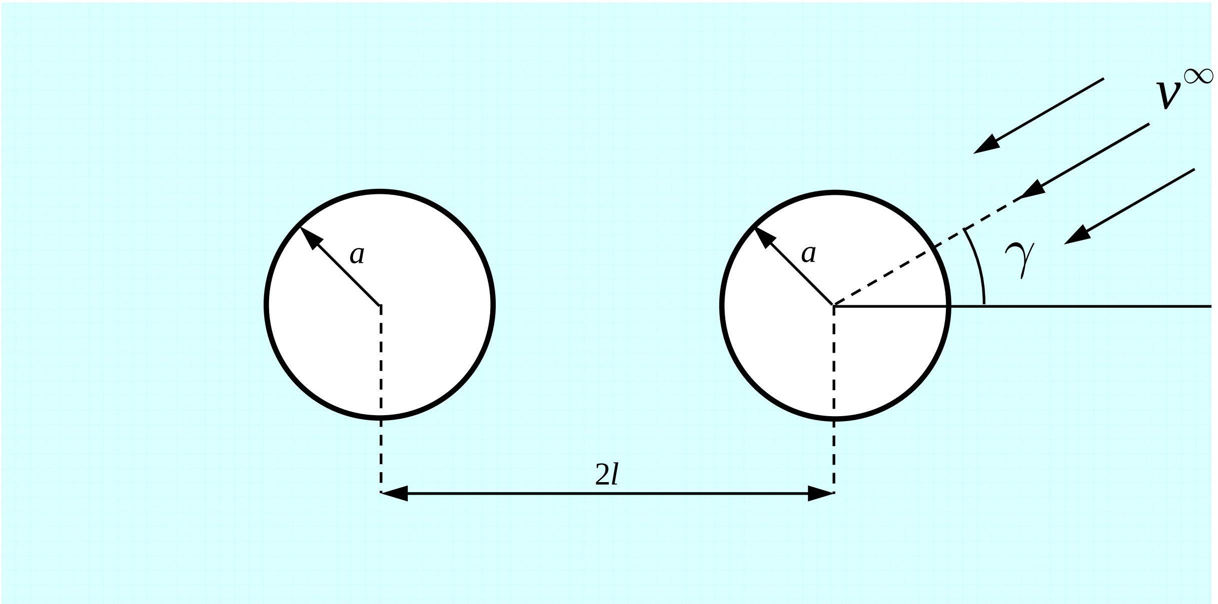


Present method (M=10)

Laplace equation

- *Steady state heat conduction problems*
- *Electrostatic potential of wires*
- *Flow of an ideal fluid pass cylinders*
- *A circular bar under torque*
- *An infinite medium under antiplane shear*
- *Half-plane problems*

Flow of an ideal fluid pass two parallel cylinders

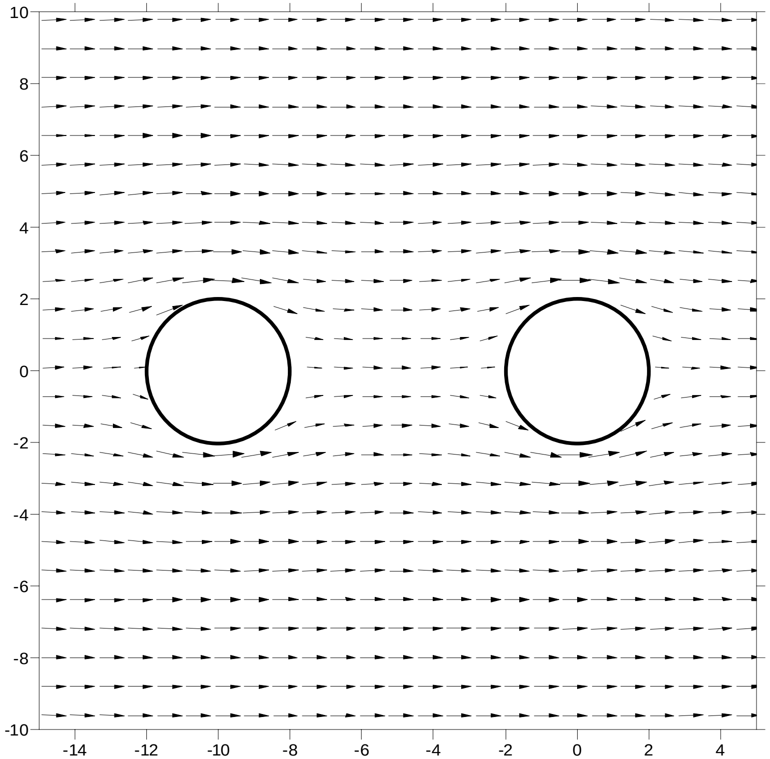


v^{∞} is the velocity of flow far from the cylinders

γ is the incident angle

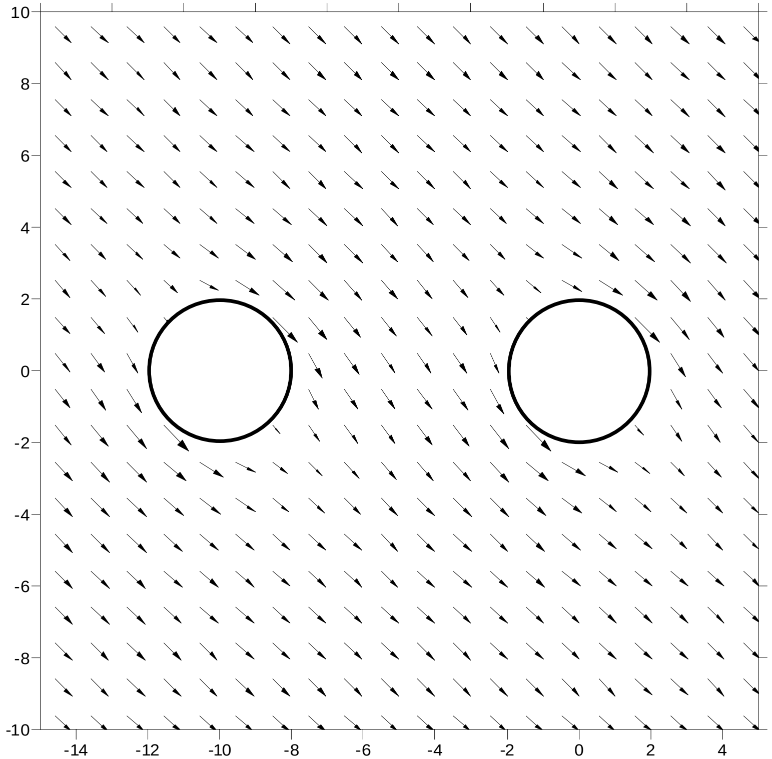
Velocity field in different incident angle

$\gamma = 180^\circ$



Present method (M=10)

$\gamma = 135^\circ$



Present method (M=10)

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Laplace equation

- *Steady state heat conduction problems*
- *Electrostatic potential of wires*
- *Flow of an ideal fluid pass cylinders*
- *A circular bar under torque*
- *An infinite medium under antiplane shear*
- *Half-plane problems*

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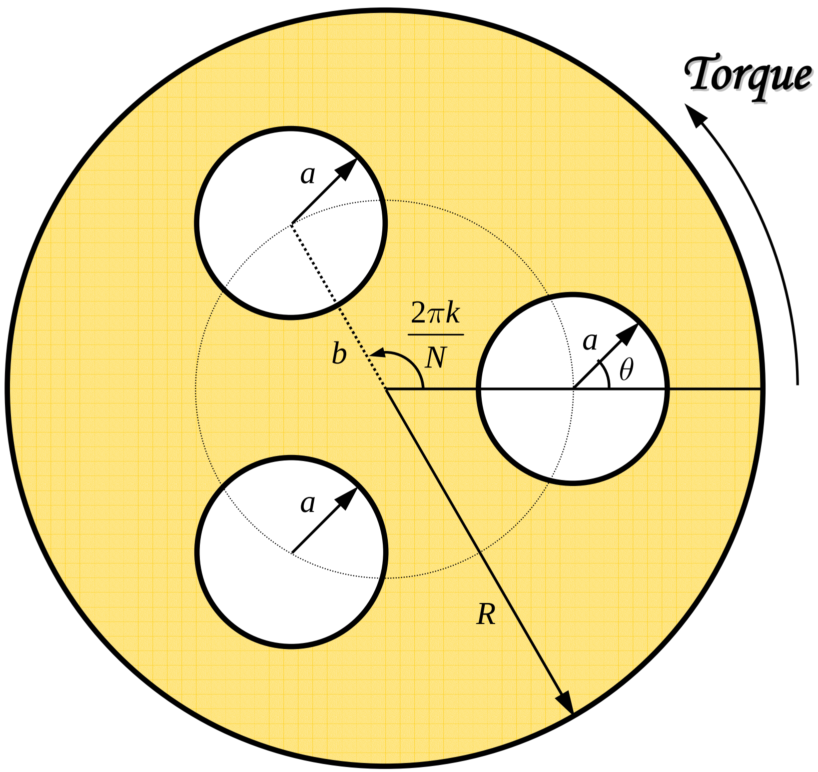
Torsion bar with circular holes removed



The warping function φ

$$\nabla^2 \varphi(x) = 0, \quad x \in D$$

Boundary condition

$$\frac{\partial \varphi}{\partial n} = x_k \sin\theta_k - y_k \cos\theta_k \quad \textit{on} \quad B_k$$

where

$$x_i = b\cos\frac{2\pi i}{N}, \quad y_i = b\sin\frac{2\pi i}{N}$$

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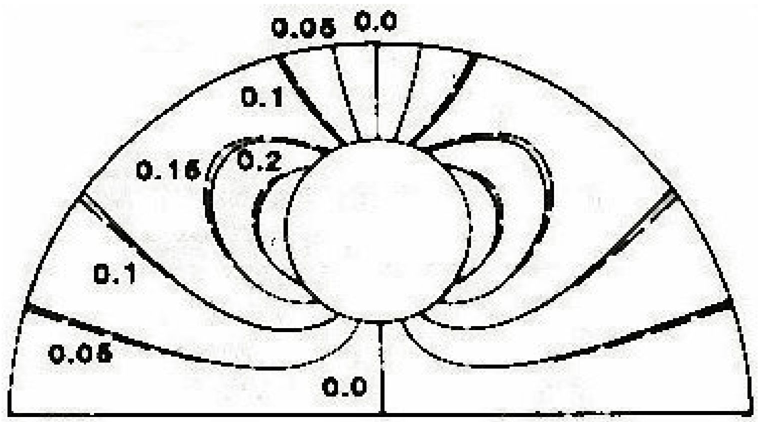
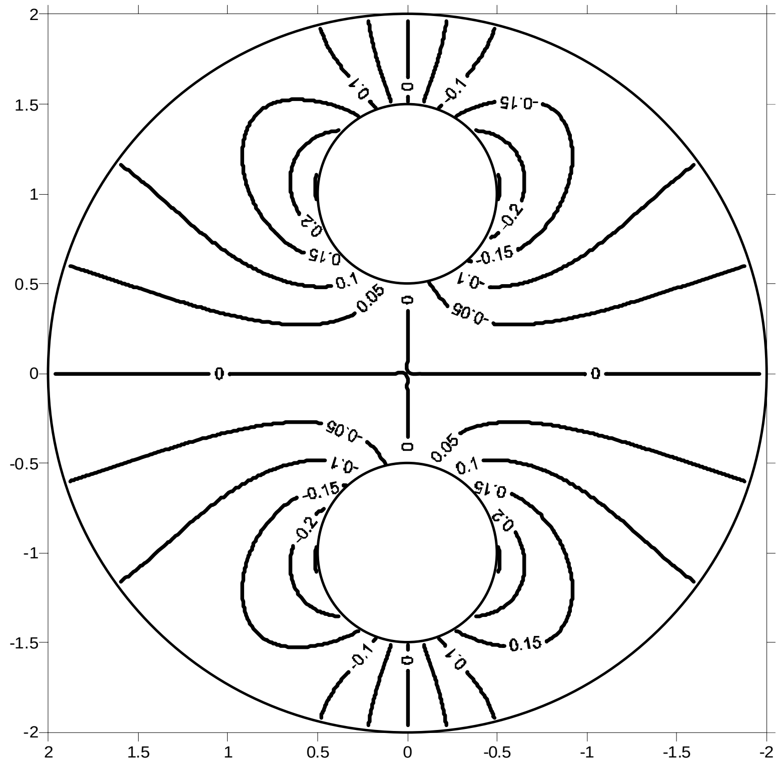
Axial displacement with two circular holes

Dashed line: exact solution
Solid line: first-order solution



Caulk's data (1983)
ASME Journal of Applied Mechanics



Present method (M=10)

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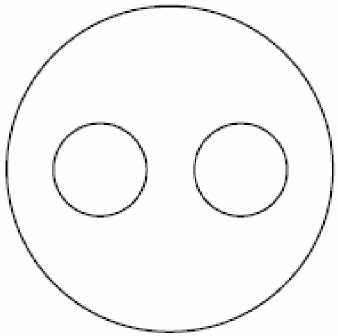
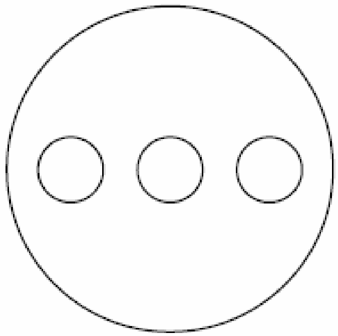
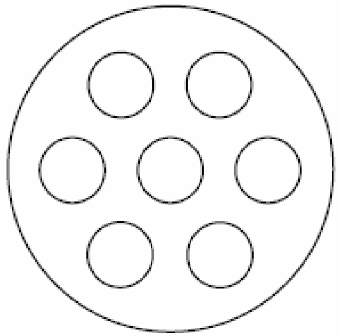
Torsional rigidity

	Case			
		$N=2, c/R=0$ $a/R=2/7, b/R=3/7$	$N=2, c/R=1/5$ $a/R=1/5, b/R=3/5$	$N=6, c/R=1/5$ $a/R=1/5, b/R=3/5$
$\frac{2G}{(\mu\pi R^4)}$	Caulk(First-order approximate)	0.8739	0.8741	0.7261
	Exact BIE formulation	0.8713	0.8732	0.7261
	Ling's results	0.8809	0.8093 ?	0.7305
	The present method	0.8712	0.8732	0.7245

Laplace equation

- *Steady state heat conduction problems*
- *Electrostatic potential of wires*
- *Flow of an ideal fluid pass cylinders*
- *A circular bar under torque*
- *An infinite medium under antiplane shear*
- *Half-plane problems*

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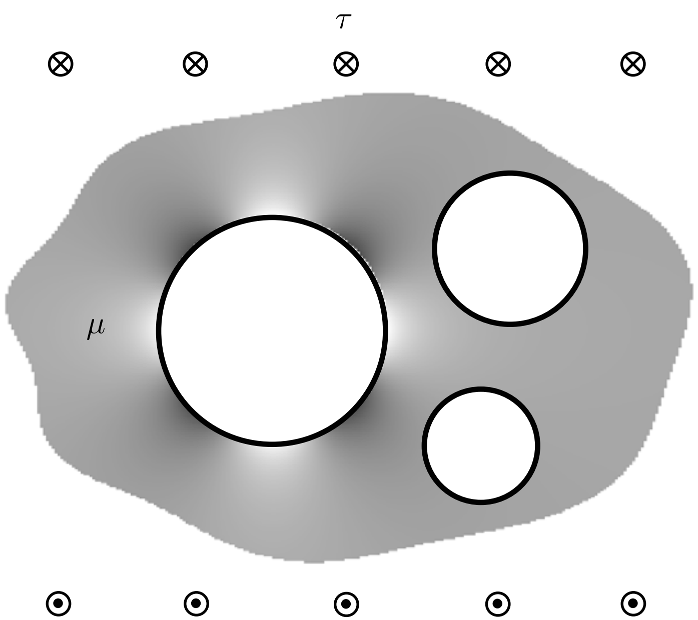
Infinite medium under antiplane shear



The displacement w^s

$$\nabla^2 w^s(x) = 0, \quad x \in D$$

Boundary condition

$$\frac{\partial w^s(x)}{\partial n} = \frac{\tau}{\mu}\sin\theta \quad on \quad B_k$$

Total displacement

$$w = w^s + w^\infty$$

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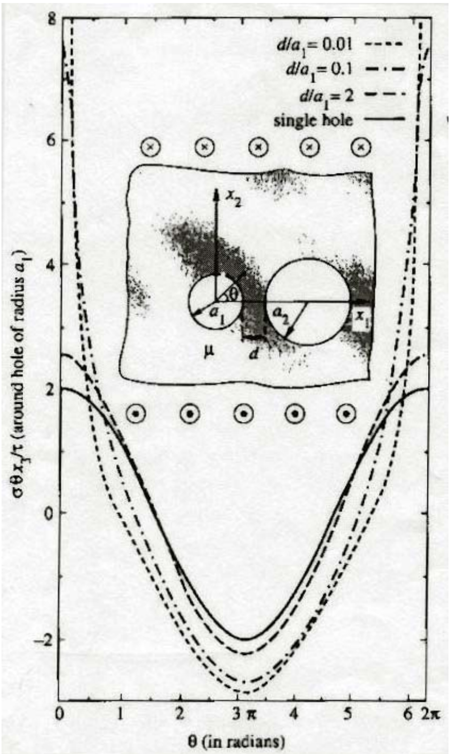
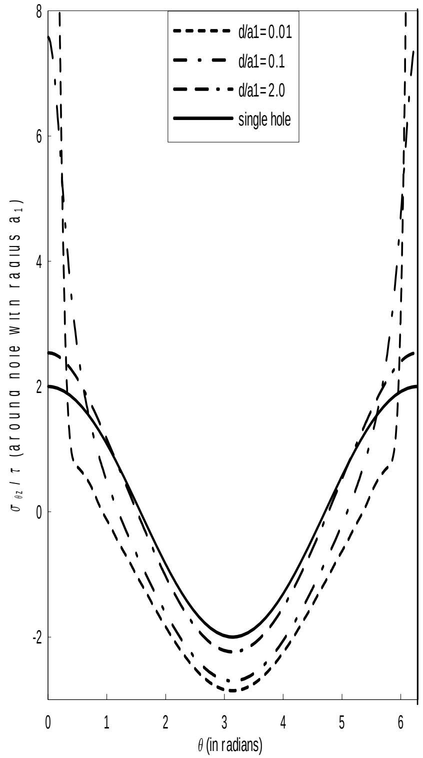
Shear stress $\sigma_{z\theta}$ around the hole of radius a_1 (x axis)



Honein's data (1992)

Quarterly of Applied Mathematics



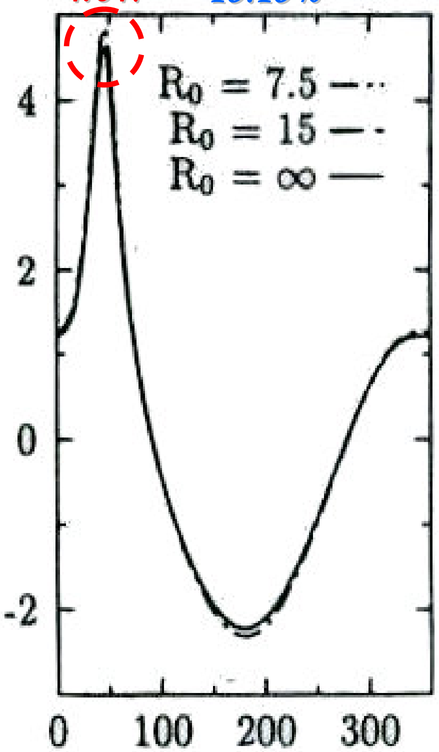
Present method (M=20)

Shear stress $\sigma_{z\theta}$ around the hole of radius a_1

Stress approach | **Analytical** | **Displacement approach**

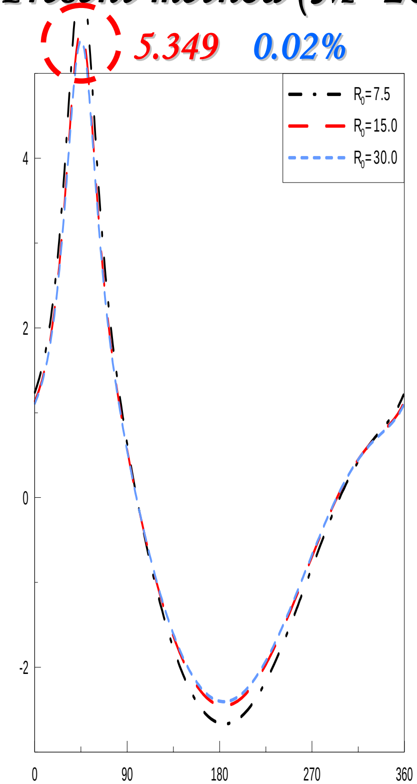
Steele's data (1992)

4.647 13.13%



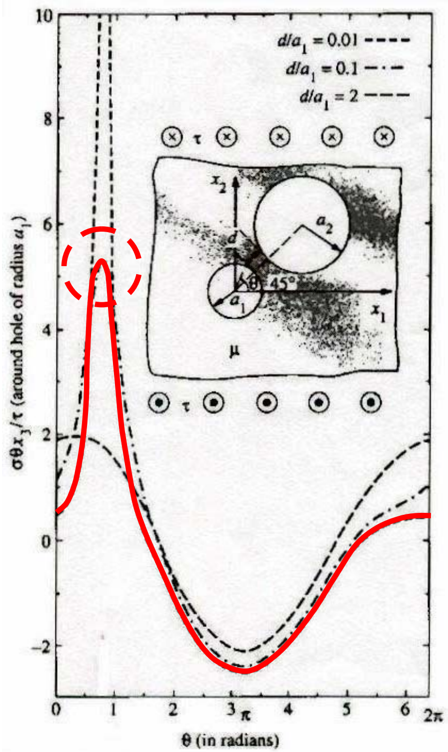
Present method (M=20)

5.349 0.02%



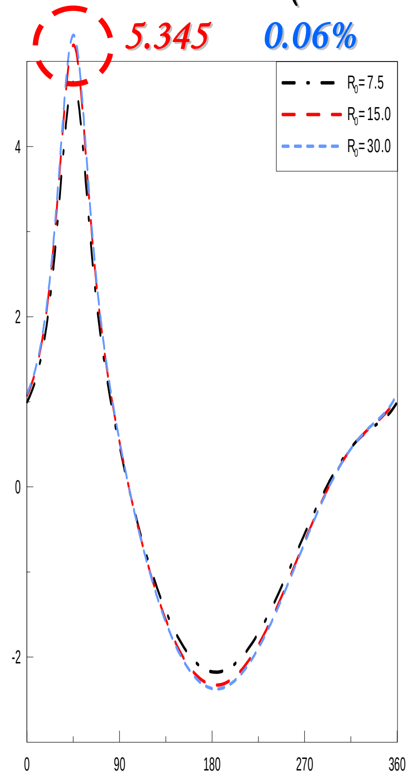
Honein's data (1992)

5.348



Present method (M=20)

5.345 0.06%



Extension to inclusion

- Anti-plane piezoelectricity problems
- In-plane electrostatics problems
- Anti-plane elasticity problems

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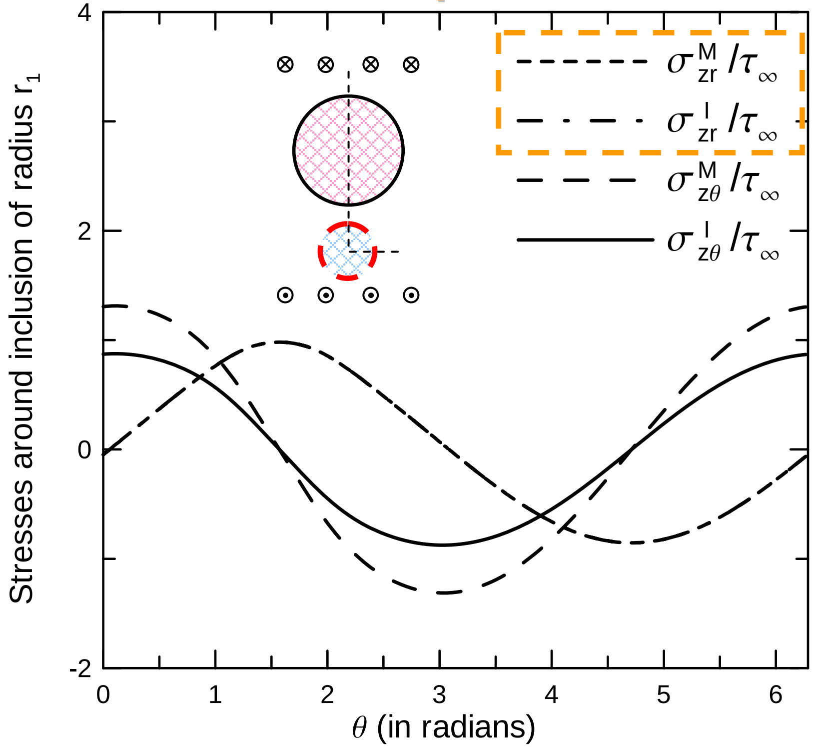
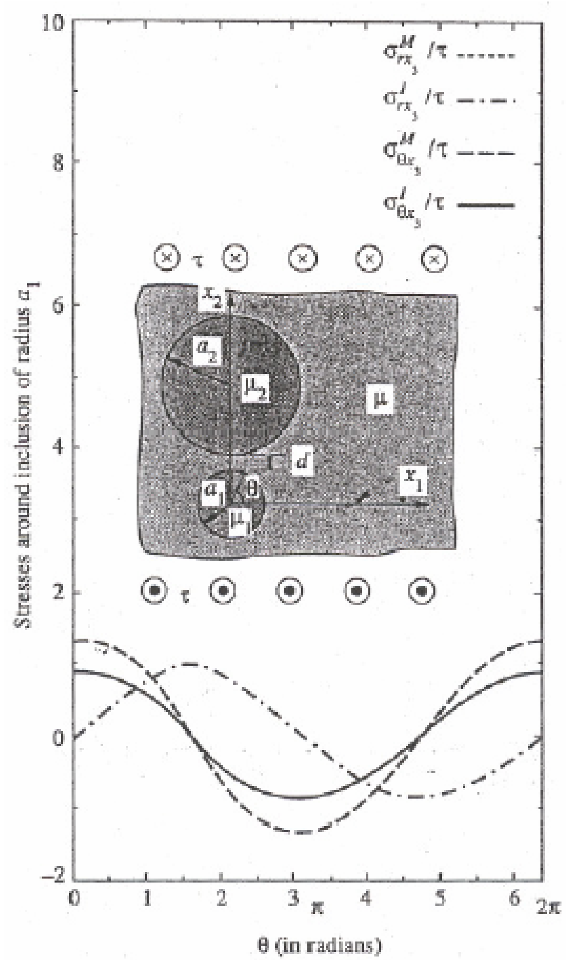
Two circular inclusions with centers on the *y* axis

Equilibrium of traction



Present method (L=20)



Honein *et al.*'sdata (1992)

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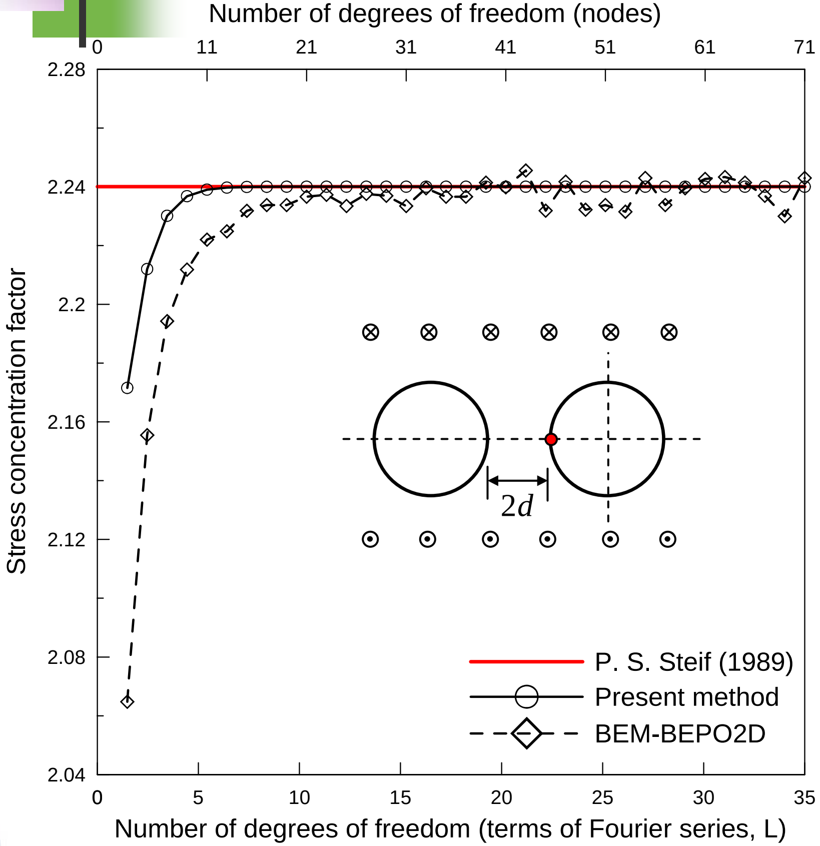
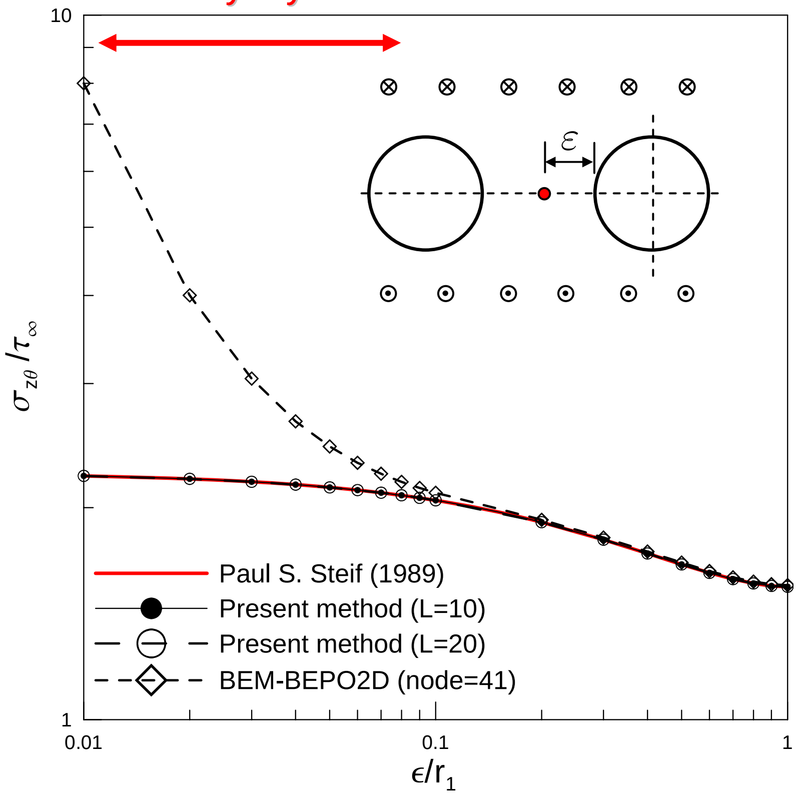
Convergence test and boundary-layer effect analysis

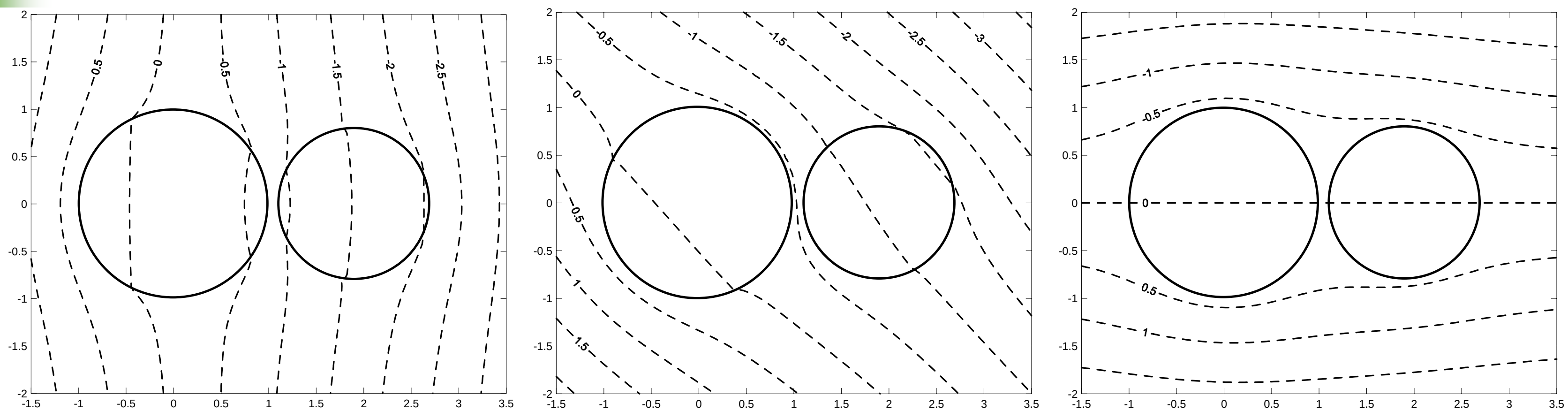


boundary-layer effect



Patterns of the electric field for ϵ_0=2, ϵ_1=9 and ϵ_2=5

Present method (L=20)

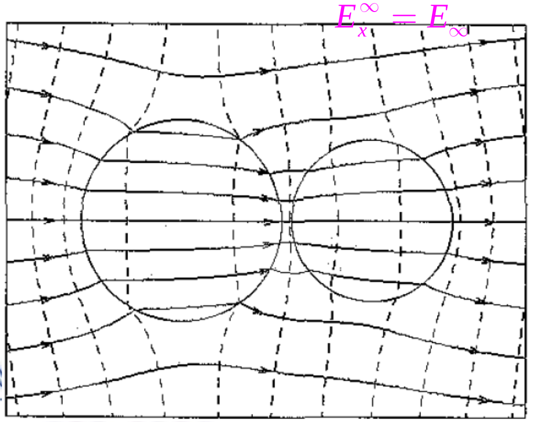


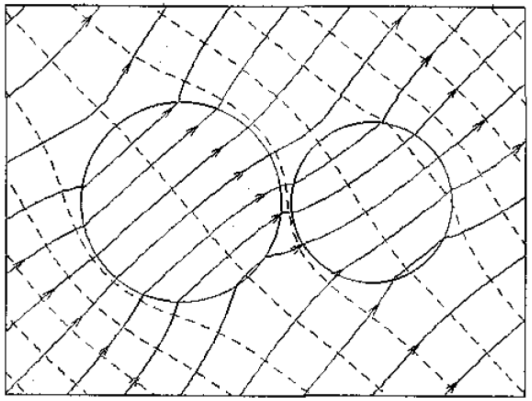
$E_x^\infty = E_\infty$

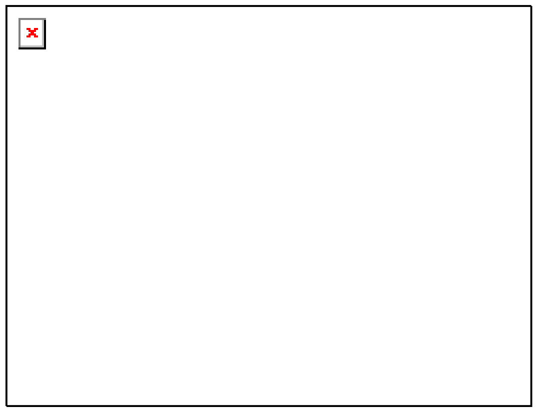
$E_x^\infty = E_\infty \cos 45°, E_y^\infty = E_\infty \sin 45°$

$E_y^\infty = E_\infty$

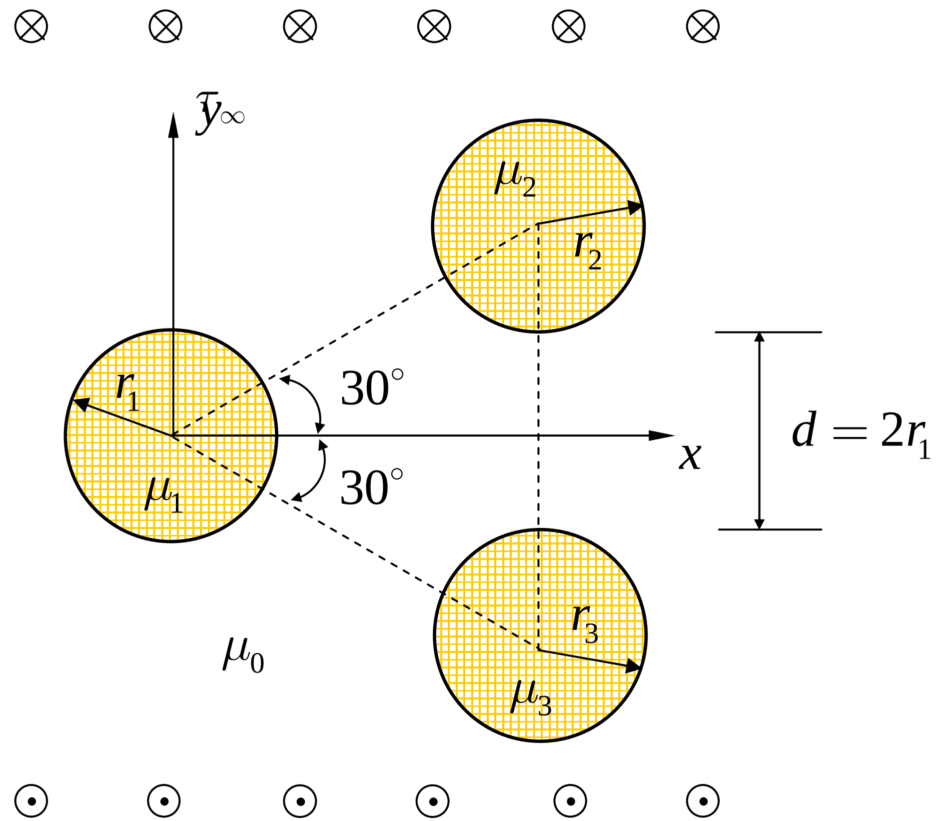
Emets & Onofrichuk (1996)







Three identical inclusions forming an equilateral triangle



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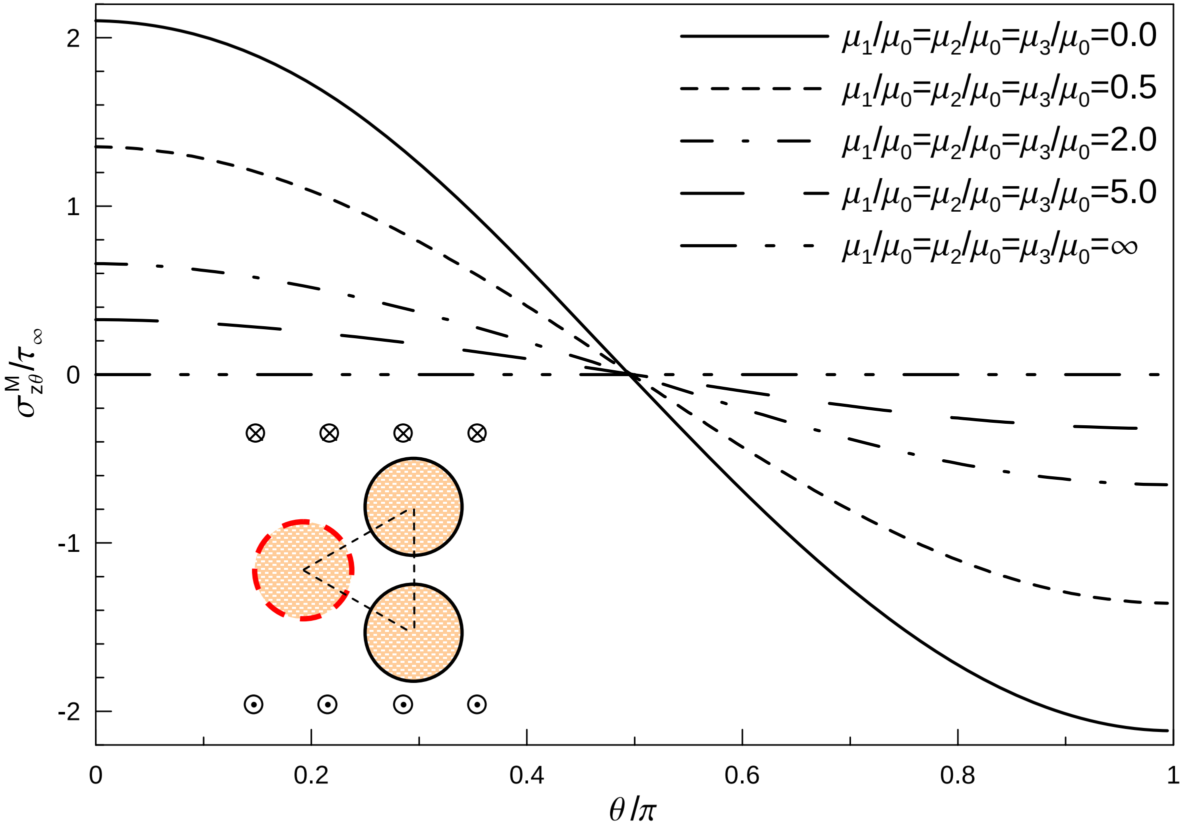
Tangential stress distribution around the inclusion located at the origin

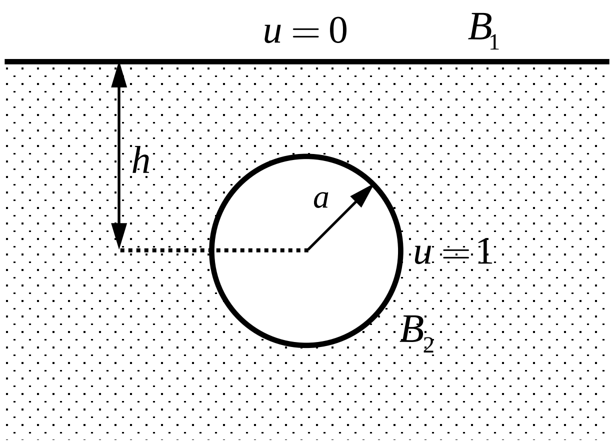


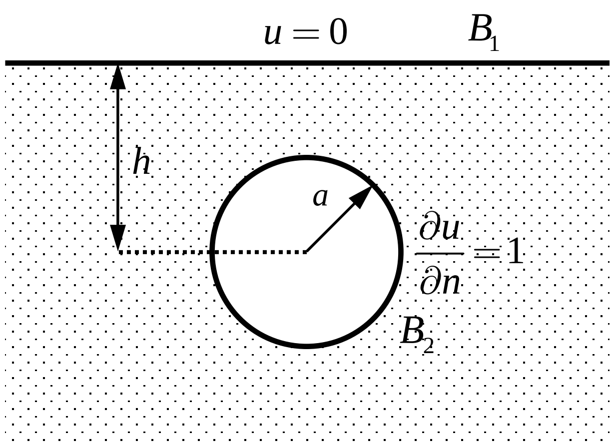
Present method (L=20),
agrees well with Gong's data (1995)

Laplace equation

- *Steady state heat conduction problems*
- *Electrostatic potential of wires*
- *Flow of an ideal fluid pass cylinders*
- *A circular bar under torque*
- *An infinite medium under antiplane shear*
- *Half-plane problems*

Half-plane problems





Dirichlet boundary condition
(Lebedev et al.)

Mixed-type boundary condition
(Lebedev et al.)

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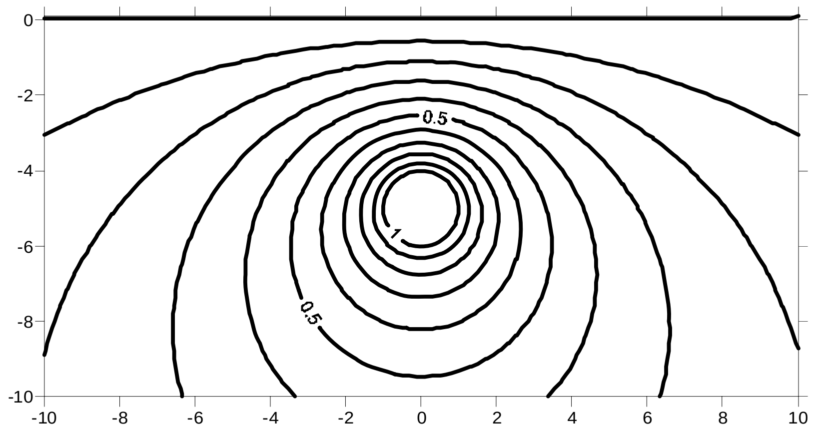
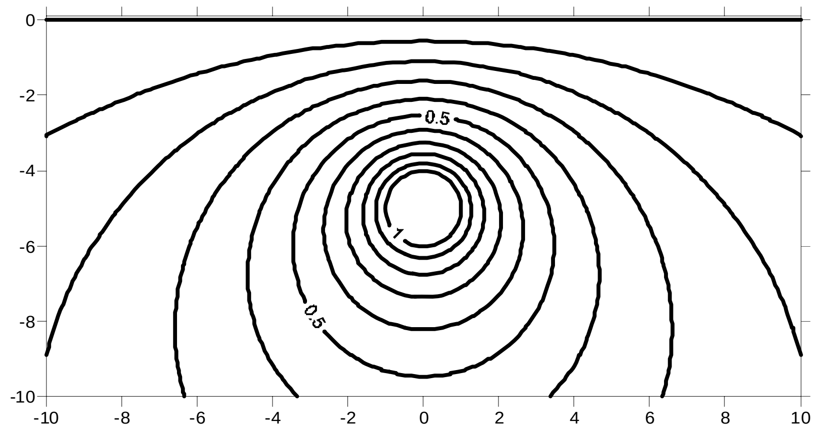
Dirichlet problem

Isothermal line



Exact solution (Lebedev et al.)



Present method (M=10)

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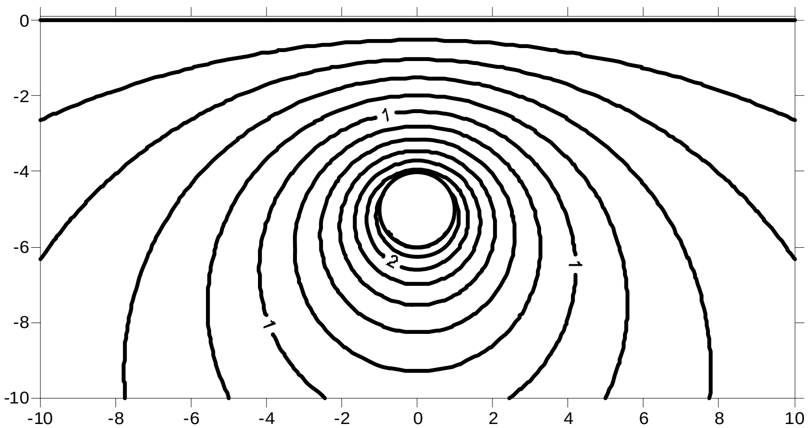
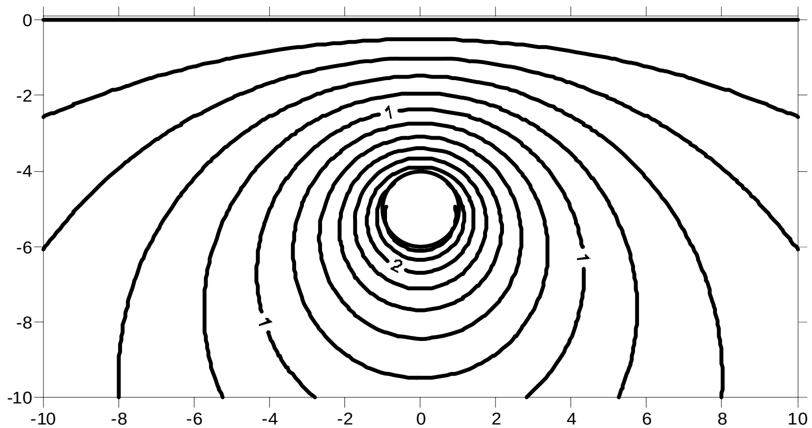
Mixed-type problem

Isothermal line



Exact solution (Lebedev et al.)



Present method (M=10)

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Numerical examples

- *Laplace equation*
- *Eigen problem*
- *Exterior acoustics*
- *Biharmonic equation*

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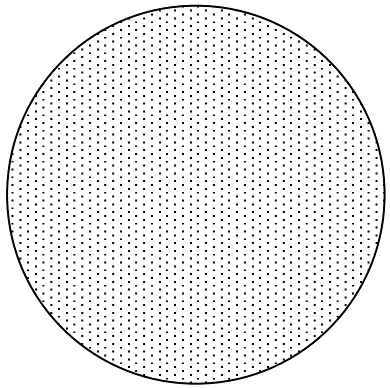
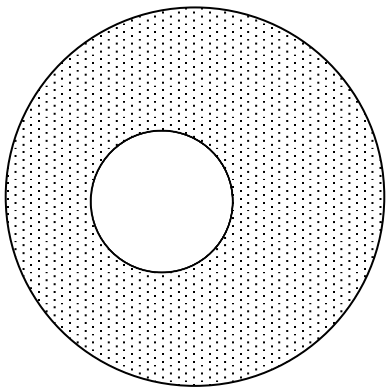
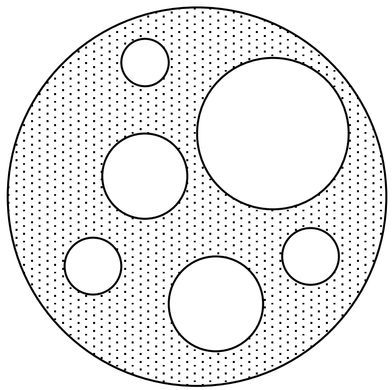
Problem statement



Simply-connected domain



Doubly-connected domain



Multiply-connected domain

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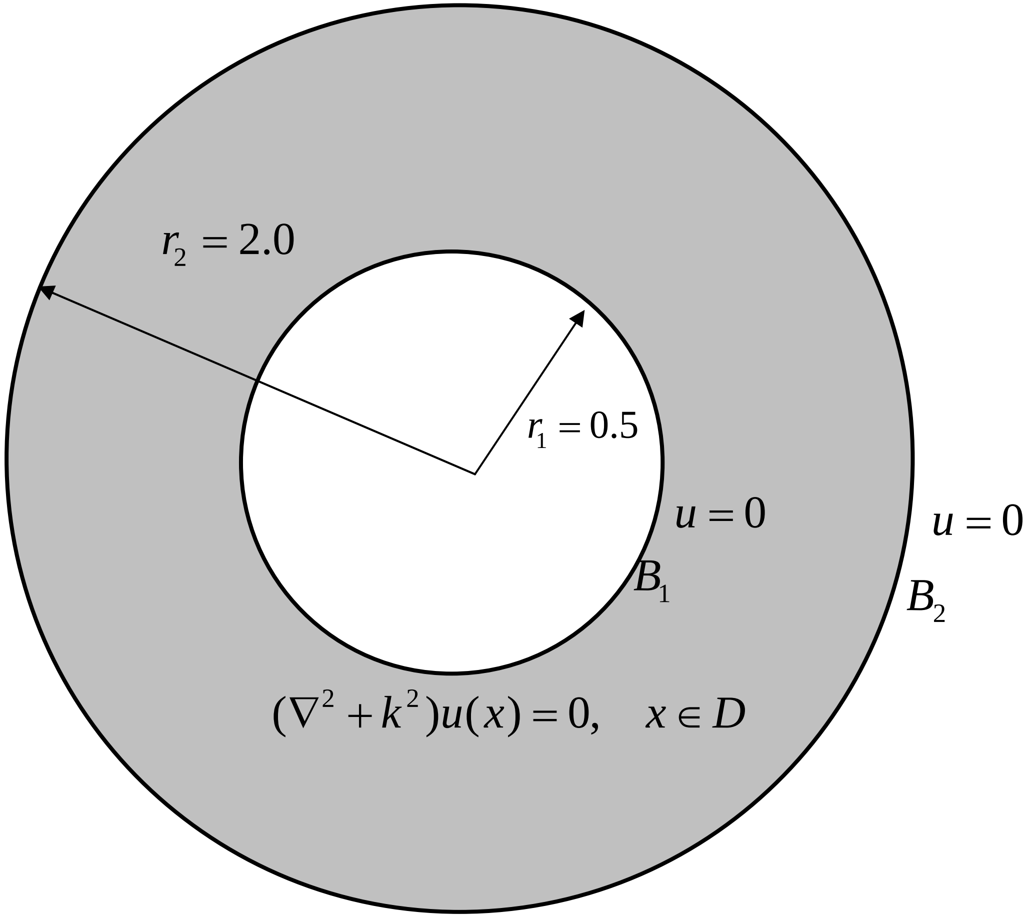
Example 1



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The former five true eigenvalues by using different approaches

	k_1	k_2	k_3	k_4	k_5
FEM (ABAQUS)	2.03	2.20	2.62	3.15	3.71
BEM (Burton & Miller)	2.06	2.23	2.67	3.22	3.81
BEM (CHIEF)	2.05	2.23	2.67	3.22	3.81
BEM (null-field)	2.04	2.20	2.65	3.21	3.80
BEM (fictitious)	2.04	2.21	2.66	3.21	3.80
Present method	2.05	2.22	2.66	3.21	3.80
Analytical solution[19]	2.05	2.23	2.66	3.21	3.80

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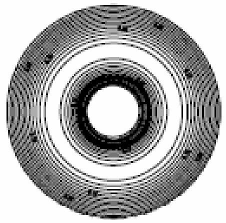
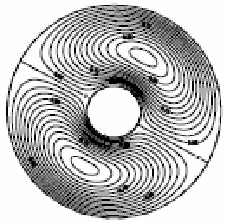
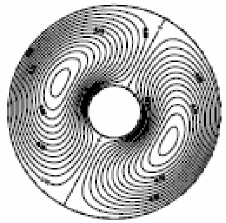
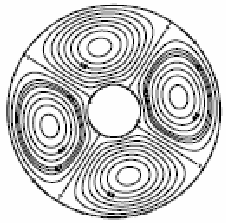
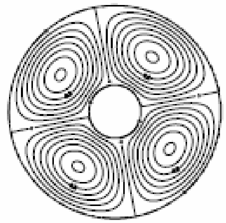
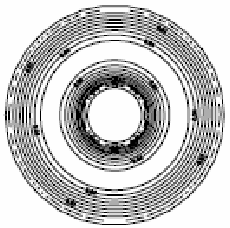
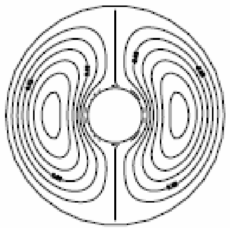
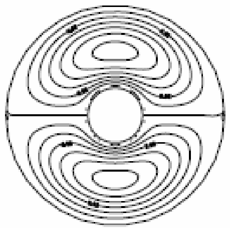
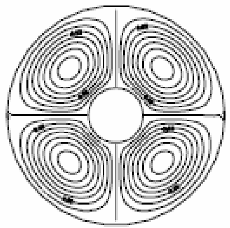
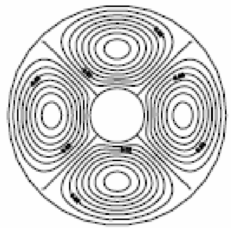
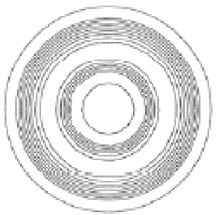
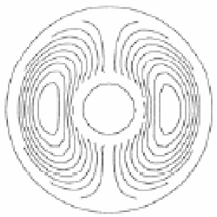
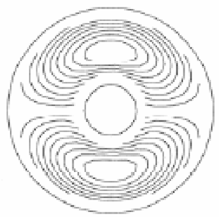
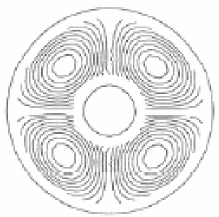
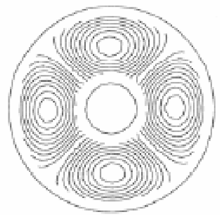
The former five eigenmodes by using present method, FEM and BEM

Method \ Mode	1	2	3	4	5
Present method					
	$k = 2.05$	$k = 2.22$	$k = 2.22$	$k = 2.66$	$k = 2.66$
BEM					
	$k = 2.06$	$k = 2.23$	$k = 2.23$	$k = 2.67$	$k = 2.67$
FEM					
	$k = 2.03$	$k = 2.20$	$k = 2.20$	$k = 2.62$	$k = 2.62$

Numerical examples

- *Laplace equation*
- *Eigen problem*
- *Exterior acoustics*
- *Biharmonic equation*

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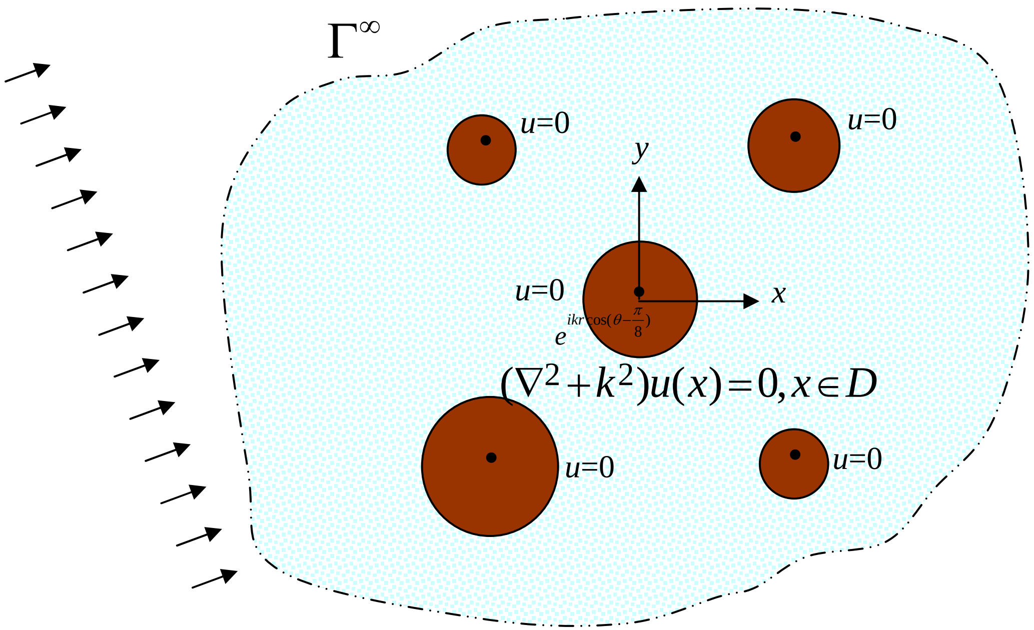
Sketch of the scattering problem (Dirichlet condition) for five cylinders



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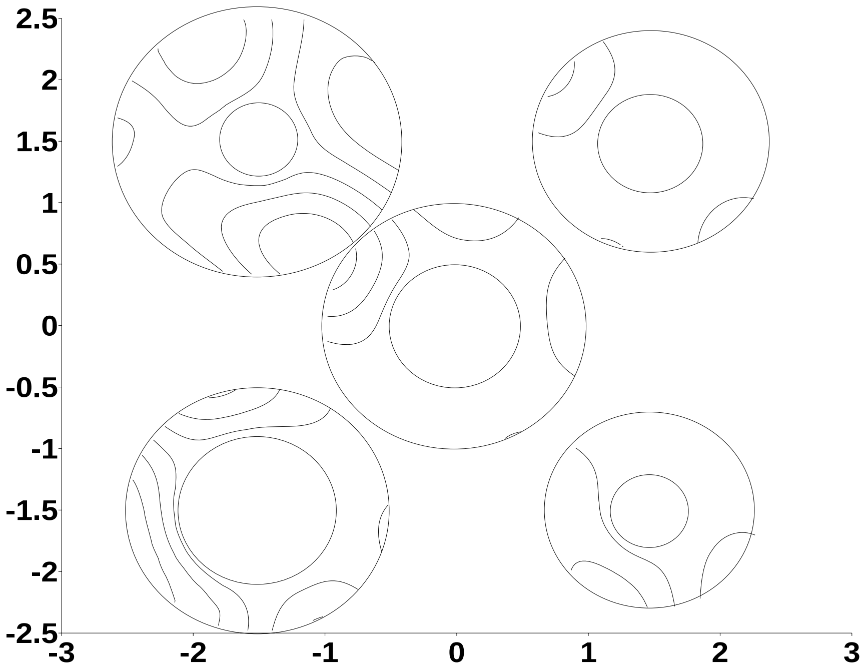
The contour plot of the real-part solutions of total field for $k = \pi$



(a) Present method (M=20)

(b) Multiple DtN method (N=50)

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The contour plot of the real-part solutions of total field for $k = 8\pi$

(a) Present method (M=20)

(b) Multiple DtN method (N=50)

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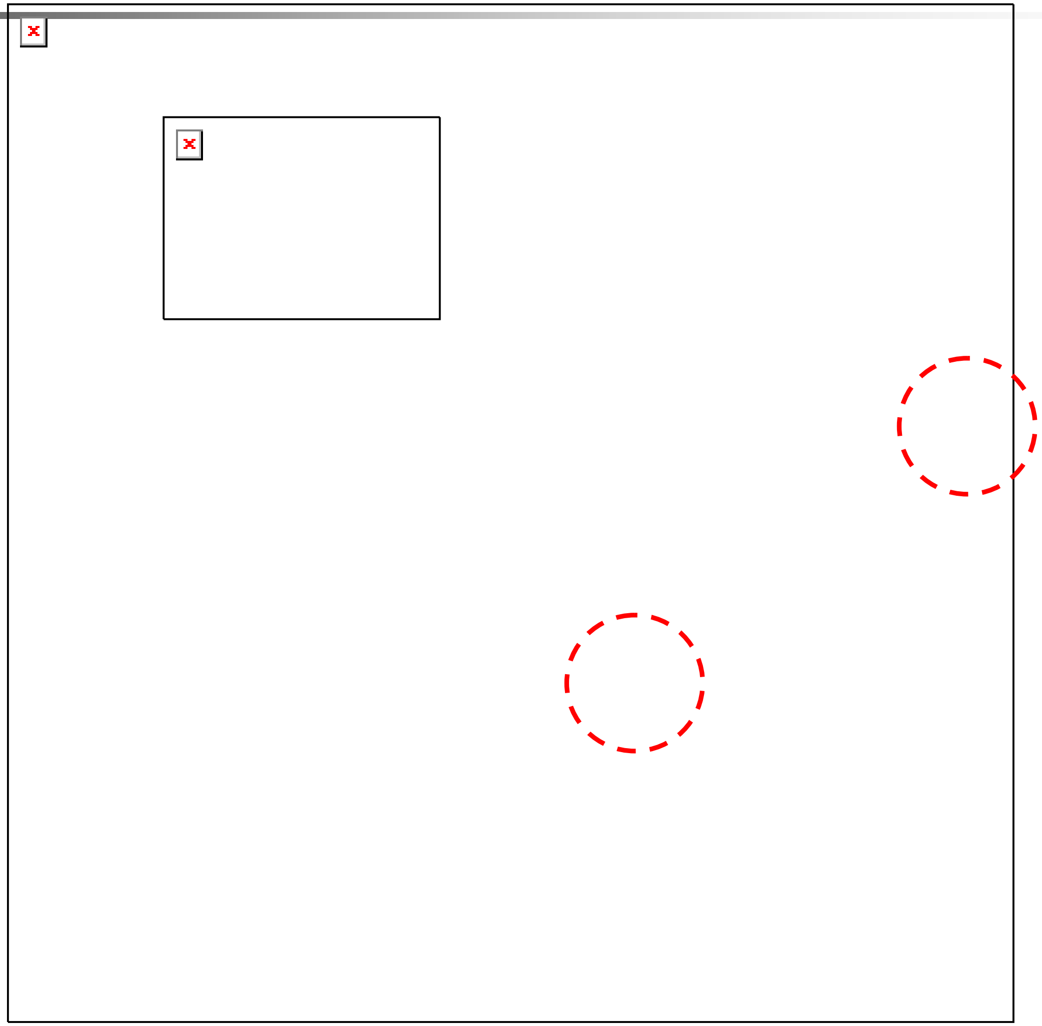
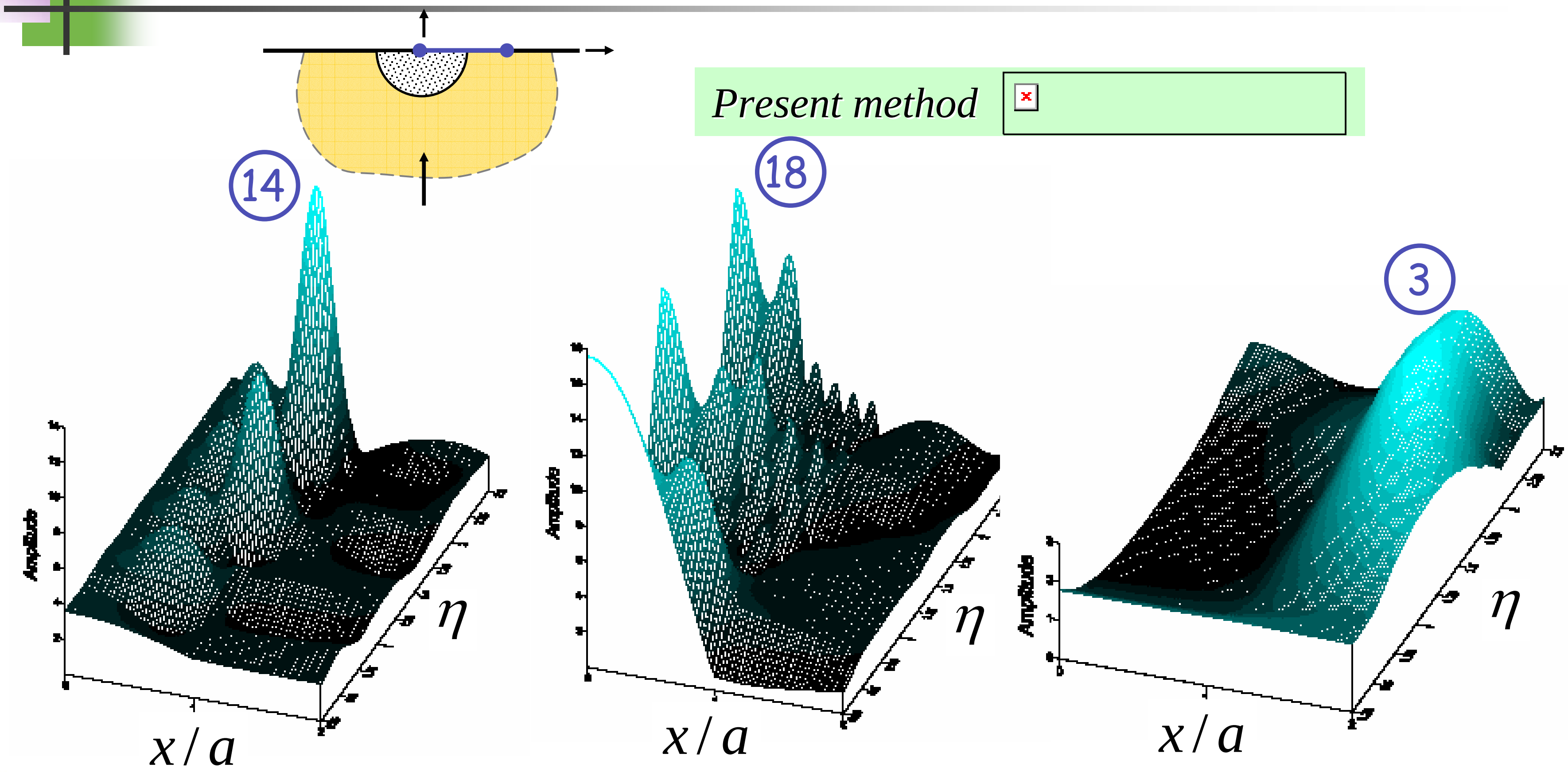
Fictitious frequencies



Soft-basin effect



Numerical examples

- *Laplace equation*
- *Eigen problem*
- *Exterior acoustics*
- *Biharmonic equation*

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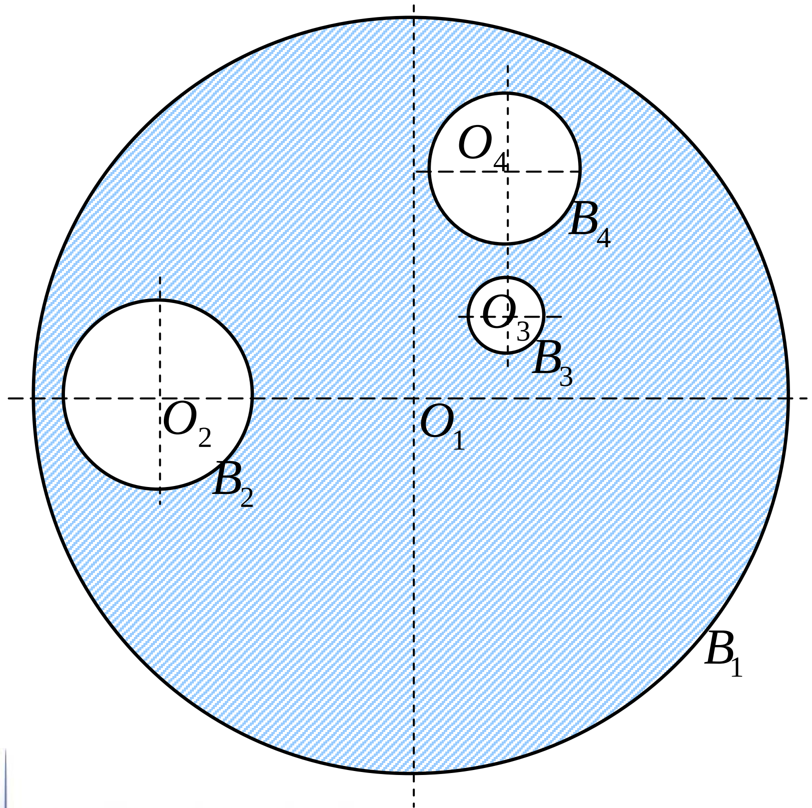
Plate problems



(Bird & Steele, 1991)

Geometric data:

Essential boundary conditions:

and on B_1

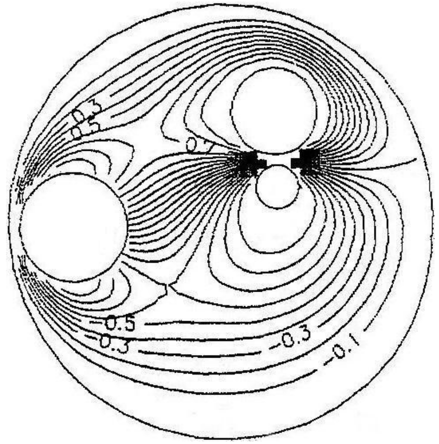
and on B_2

and on B_3

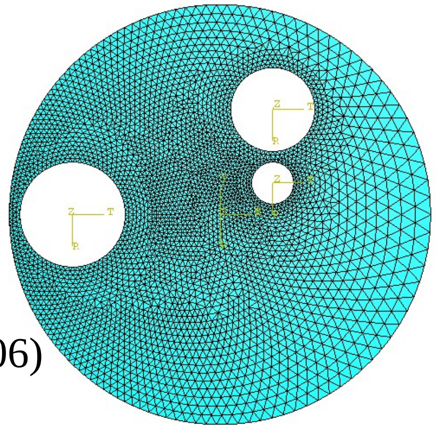
and on B_4

Contour plot of displacement

Present method (N=101)



Bird and Steele (1991)

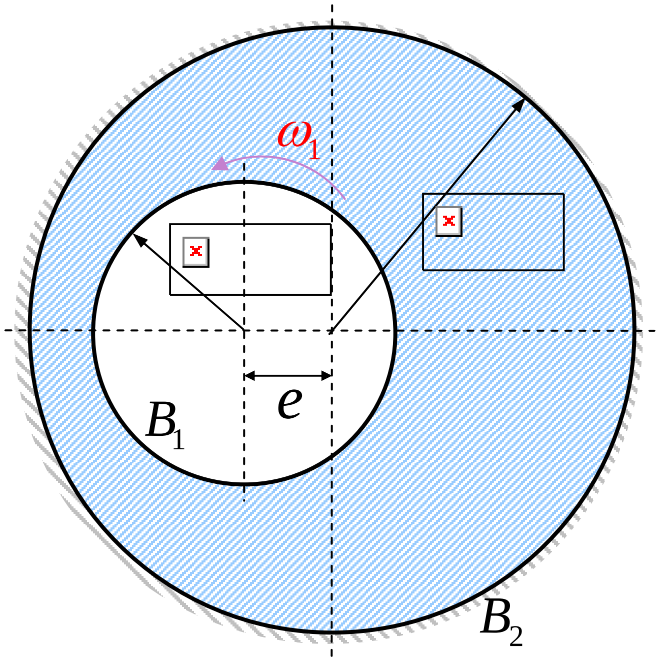


(No. of nodes=3,462,
No. of elements=6,606)

FEM mesh

FEM (ABAQUS)

Stokes flow problem



Governing equation:

Angular velocity:

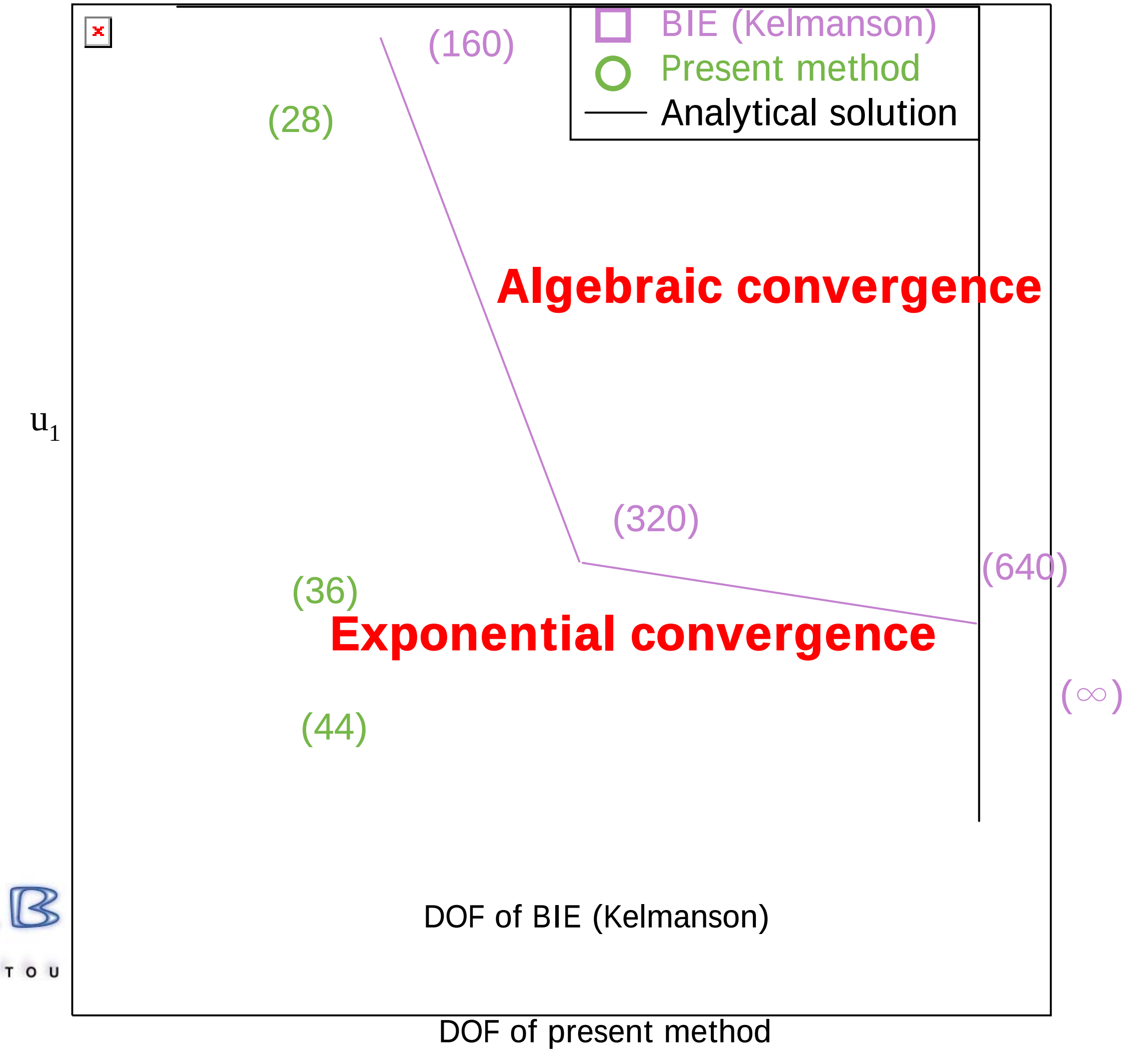
Boundary conditions:

and on B_1

and on B_2 (Stationary)

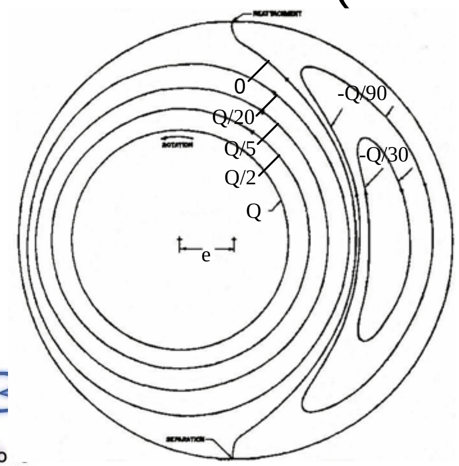
Eccentricity:

Comparison for

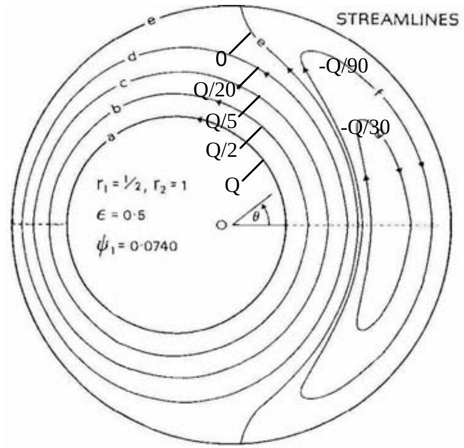


Contour plot of Streamline for

Present method (N=81)



Kamal (Q=0.0738)

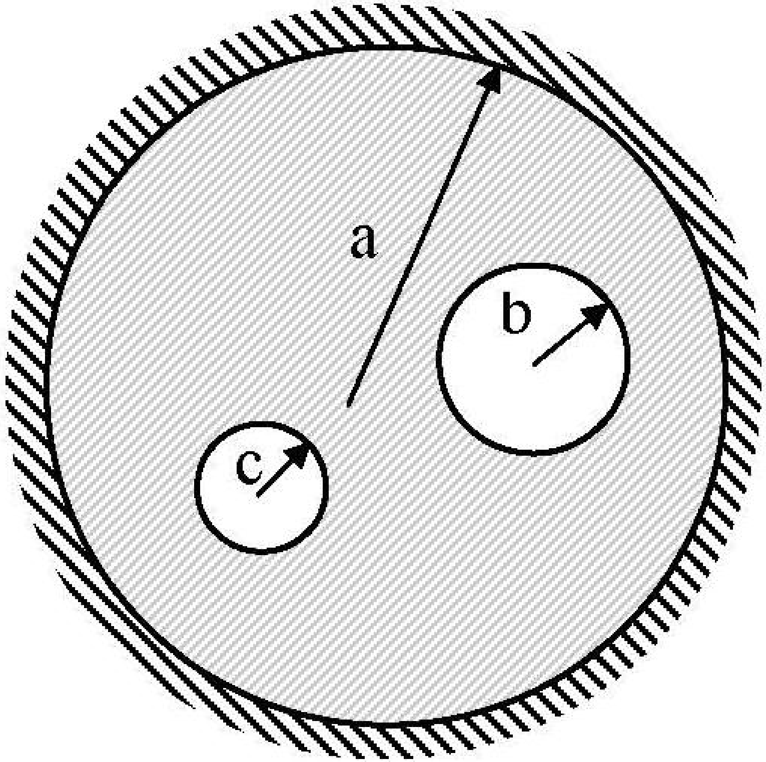


Kelmanson (Q=0.0740, n=160)

Numerical examples

- *Laplace equation*
- *Eigen problem*
- *Exterior acoustics*
- *Biharmonic equation*
- *Plate vibration*

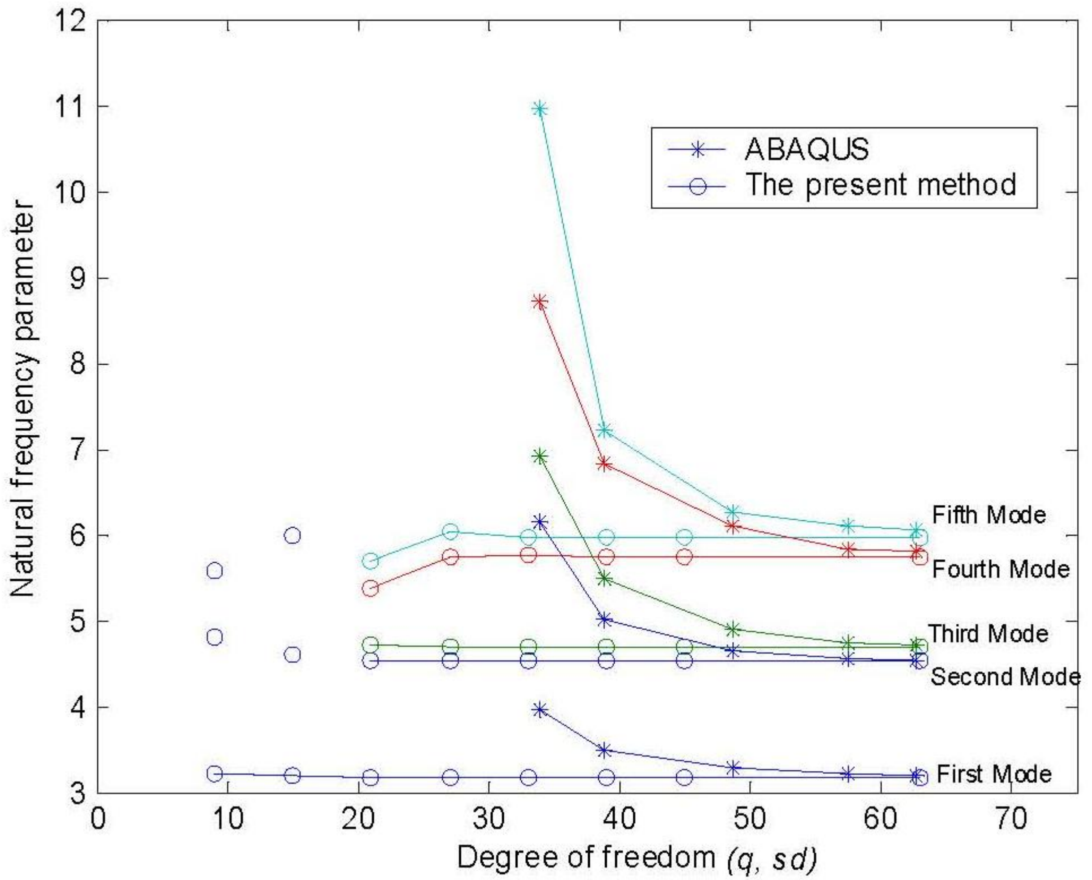
Free vibration of plate



Case3:

Geometric data:
a=1m
b=0.25m
c=0.125m
Boundary condition:
Inner circle : free
Outer circle: clamped

Comparisons with FEM



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BEM trap ? Why engineers should learn mathematics ?

- Well-posed ?
- Existence ?
- Unique ?

- Mathematics versus Computation

- Some examples

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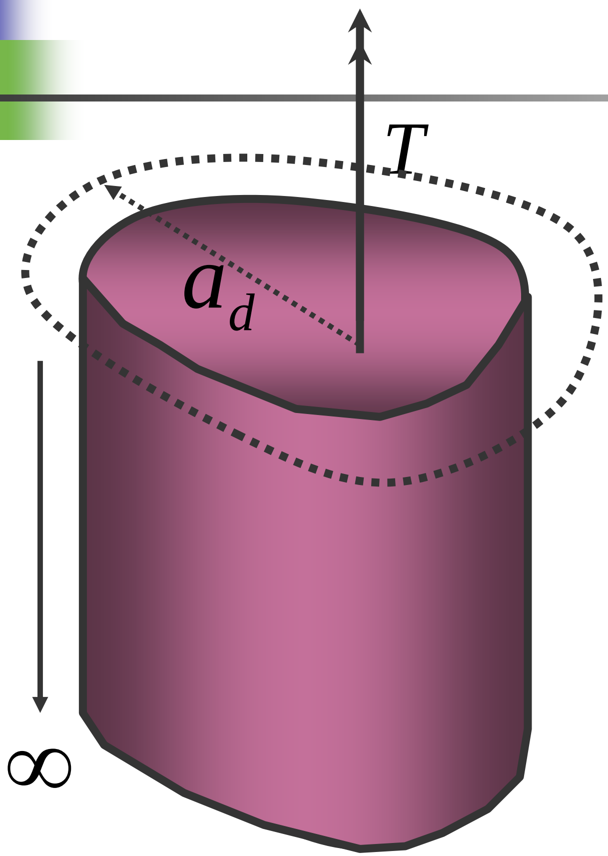
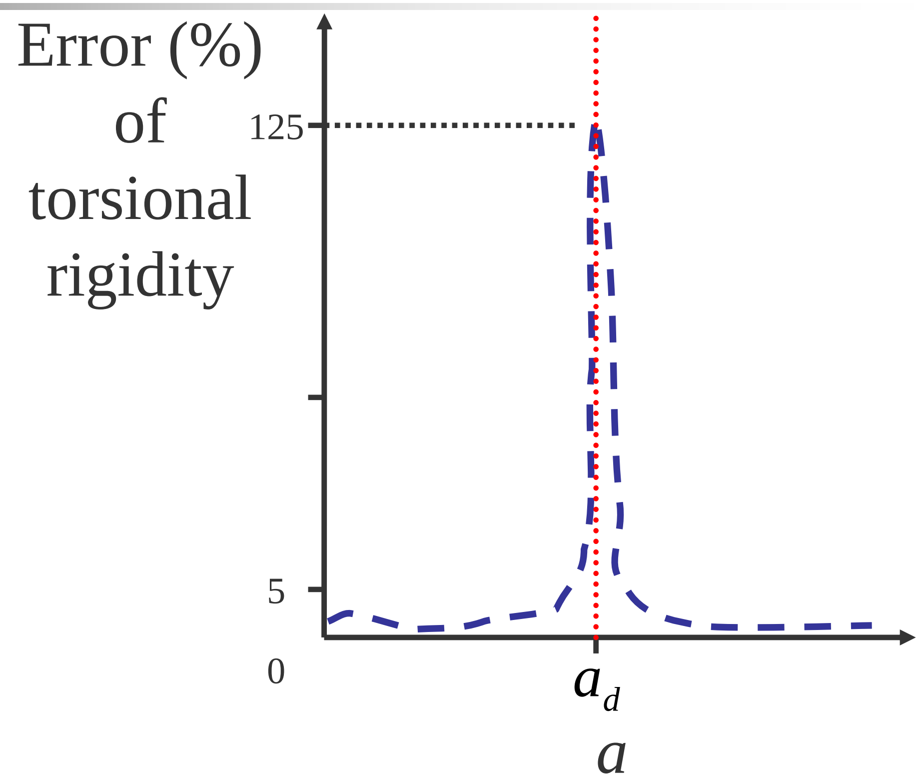
Numerical phenomena

(Degenerate scale)



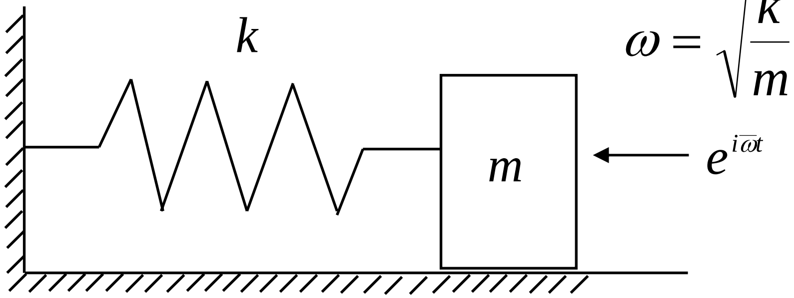
Commercial ode output ?



Previous approach : Try and error on a
Present approach : Only one trial

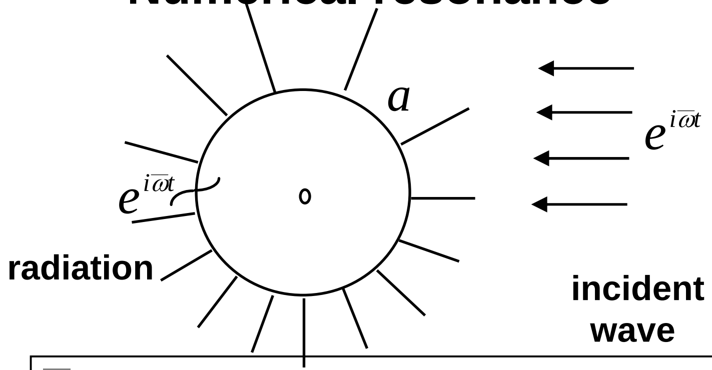
Numerical and physical resonance

Physical resonance



$$\omega = \sqrt{\frac{k}{m}}$$

Numerical resonance



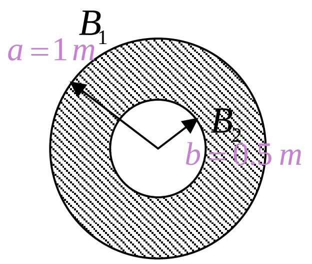
$\varpi \to \omega$

Numerical phenomena
(Fictitious frequency)

$t(a,0)$

A story of NTU Ph.D. students

Numerical phenomena
(Spurious eigensolution)



Outlines

- Motivation and literature review
- Mathematical formulation
  - Expansions of fundamental solution and boundary density
  - Adaptive observer system
  - Vector decomposition technique
  - Linear algebraic equation
- Numerical examples
- Conclusions

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Conclusions

- *A systematic approach using degenerate kernels, Fourier series and null-field integral equation has been successfully proposed to solve Laplace Helmholtz and Biharminic problems with circular boundaries.*
- *Numerical results agree well with available exact solutions, Caulk's data, Onishi's data and FEM (ABAQUS) for only few terms of Fourier series.*

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Conclusions

- *Free of boundary-layer effect*
- *Free of singular integrals*
- *Well posed*
- *Exponetial convergence*
- *Mesh-free approach*

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The End

Thanks for your kind attentions.

Your comments will be highly appreciated.

URL: http://msvlab.hre.ntou.edu.tw/

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- 最新消息
- 指導教授
- 成員介紹
- 邊界元素
- 程式集錦
- 研究成果
- 計劃專題
- 課程相關
- 好站介紹
- 線上期刊
- 相關文獻
- 生活剪影
- 工作月曆
- 訪客留言

Papers

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如有任何意見請 E-mail 告訴我們，本網頁負責人：陳正宗，李應德，廖奐禎，柯佳男，方衡尉，周克勳

網頁內容部分檔案格式為*.PDF，請安裝Acrobat Reader來瀏覽

NEW	06/11/08	ICOME 2006 邀陳正宗 特聘教授 Plenary lecture
NEW	06/11/07	邊界元素法入門一書，近期推出，欲購者請洽本研究室或留言告知
NEW	06/11/06	恭賀吳國綸學弟甄試高中台大土木工程學系研究所-水利組、結構組碩士班
NEW	06/11/02	下星期一(11/6)晚上7點~9點工數(一)期中考，在河二館404與405教室，請攜帶學生證以備查驗!
NEW	06/11/01	本研究室研究成果已突破410篇論文引用
NEW	06/11/01	恭賀柯永彥博士進入國家地震中心服務
NEW	06/10/30	恭賀吳國綸,蔡明宏,林裕桀獲國立臺灣海洋大學94學年度第2學期書卷獎
NEW	06/10/27	恭賀吳安傑,陳柏源,高政宏,蕭嘉俊學長獲國立臺灣海洋大學95年度「大學部及碩士班在校生論文發表於SC
NEW	06/10/25	恭賀河工系陳正宗、周宗仁、陳俶季、黃然、蕭葆羲與簡連貴等教授在校服務期間得免辦理評估
NEW	06/10/24	恭賀 吳安傑學長 入圍第三十屆力學會議學生論文競賽
NEW	06/10/13	新加坡南洋科技大學提供博士生獎學金一名(從事邊界元素法研究)
NEW	06/10/13	恭賀林盛益學長普考土木科通過
NEW	06/10/11	感謝中興工程科技研究基金會再度贊助李應德學長博士班獎助學金
NEW	06/09/27	對偶邊界元素法世界最具影響力學者前三名: Aliabadi M.H. , Chen J.T. , Power H.
NEW	06/09/27	邊界元素法世界最具影響力學者前四名:Aliabadi M.H. ,Mukherjee.S , Chen J.T. , Tanaka M.