ECCOMAS

Thematic Conference on Meshless Methods

True and spurious eigensolutions for membrane and plate problems by using method of fundamental solutions

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Outlines

1. Introduction
2. Problem statements
3. Mathematical analysis
4. Treatment methods
5. Numerical examples
6. Conclusions

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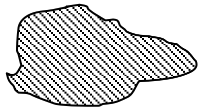
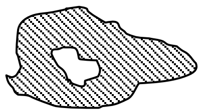
Spurious eignesolutions in BEM

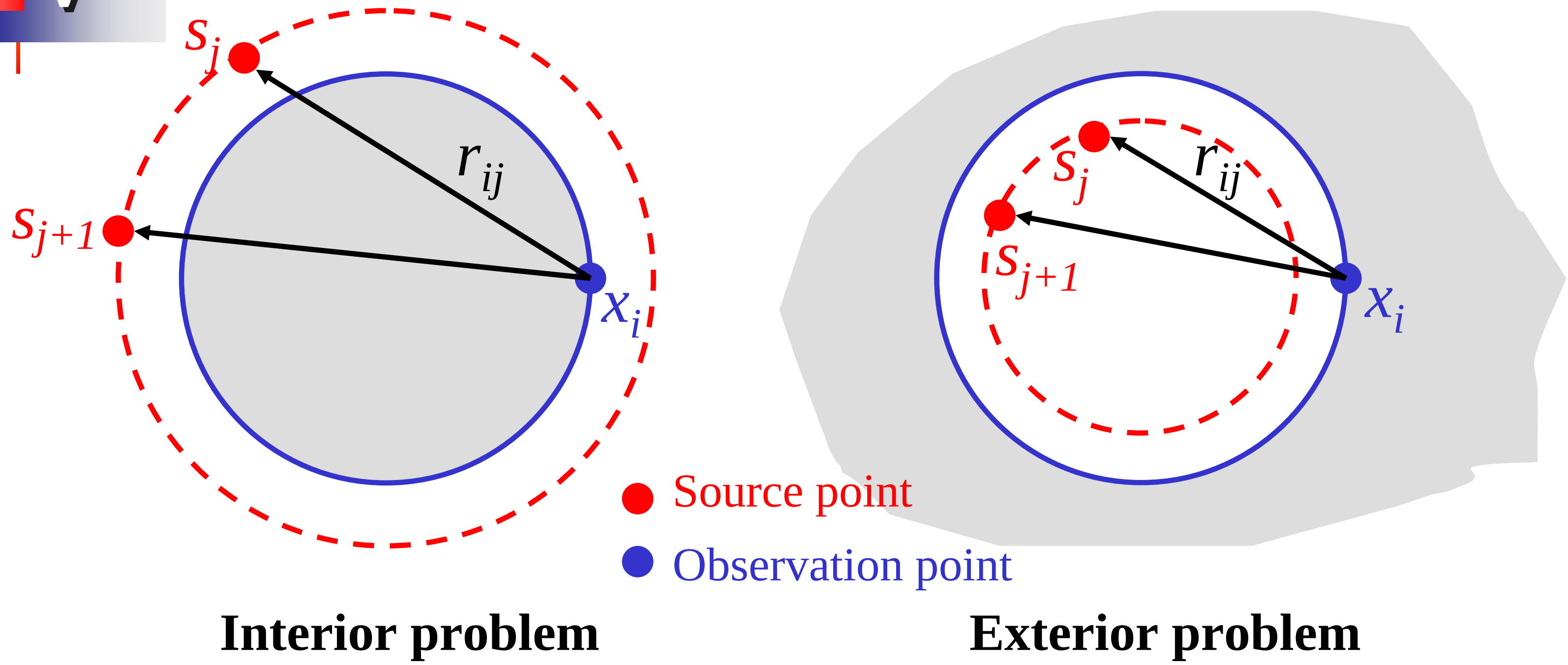
Simply-connected problem  (Membrane and plate)

	Real/MRM	Imaginary	Complex
Saving CPU time	Yes	Yes	No
Avoid singular integral	No	Yes	No
Spurious eigenvalues	Appear	Appear	No

Multiply-connected problem  (Membrane and plate)

	Complex
Spurious eigenvalues	Appear

What happens in MFS ?



Field representation: $u(x_i) = \sum_j c_j \psi(s_j, x_i)$

$$\psi(s_j, x_i) = \psi(r_{ij}), \quad r_{ij} \equiv |s_j - x_i|$$

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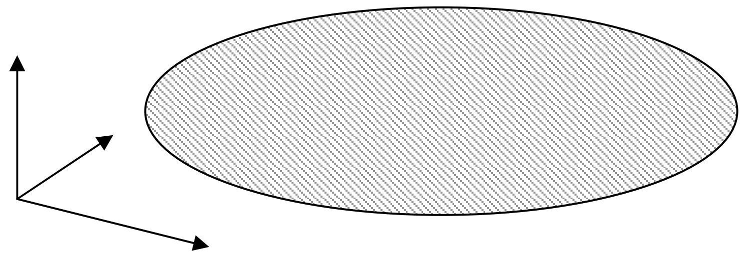
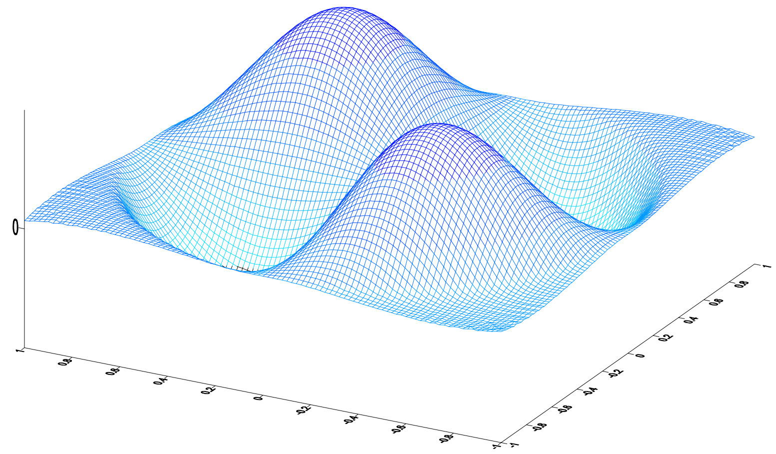
Governing equation





Governing equation

Membrane vibration

$$(\nabla^2 + k^2)u(x) = 0$$

Plate vibration

$$(\nabla^4 - \lambda^4)u(x) = 0$$

Fundamental solution

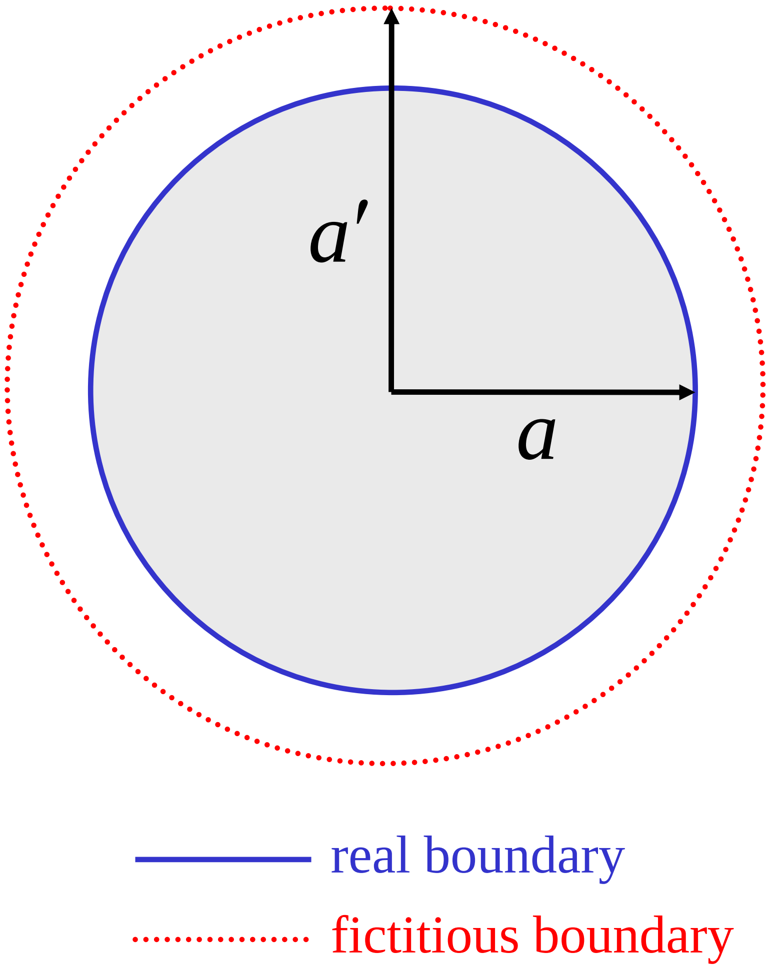
Membrane vibration

$$U(s,x) = iJ_0(kr) - Y_0(kr)$$

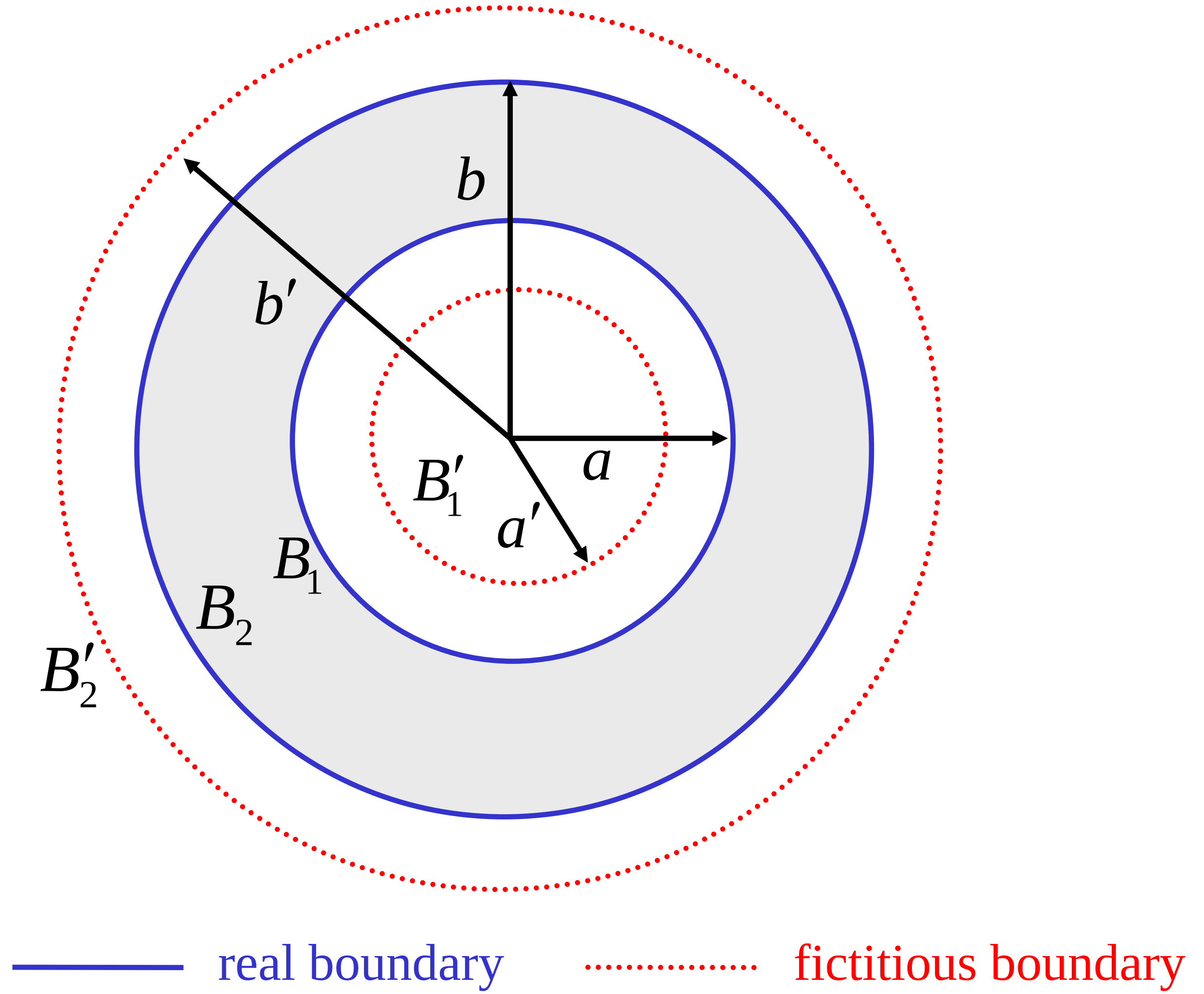
Plate vibration

$$U(s,x) = \frac{1}{8\lambda^2}\{[Y_0(\lambda r) - iJ_0(\lambda r)] + \frac{2}{\pi}[K_0(\lambda r) - iI_0(\lambda r)]\}$$

Simply-connected problem



Multiply-connected problem



Two MFS for membrane problem

Single-layer potential approach

$$u(x_i) = \sum_j U(s_j, x_i)\phi_j$$

$$t(x_i) = \sum_j L(s_j, x_i)\phi_j$$

Double-layer potential approach

$$u(x_i) = \sum_j T(s_j, x_i)\psi_j$$

$$t(x_i) = \sum_j M(s_j, x_i)\psi_j$$

$$U(s,x) \xrightarrow{\frac{\partial}{\partial n_s}} T(s,x)$$

$$U(s,x) \xrightarrow{\frac{\partial}{\partial n_x}} L(s,x), \qquad T(s,x) \xrightarrow{\frac{\partial}{\partial n_x}} M(s,x)$$

$$L(s,x) \xrightarrow{\frac{\partial}{\partial n_s}} M(s,x)$$

ASME, App. Mech. Rev.

(Chen and Hong, 1999)

Kernel functions of plate vibration

$U(s,x) = $ Fundamental solution

Slope operator

$$\Theta(s,x) = K_{\theta,s}(U(s,x)) = \frac{\partial U(s,x)}{\partial n_s}$$

Moment operator

$$M(s,x) = K_{m,s}(U(s,x))$$

$$= \nu \nabla_s^2 U(s,x) + (1-\nu)\frac{\partial^2 U(s,x)}{\partial n_s^2}$$

Shear operator

$$V(s,x) = K_{v,s}(U(s,x))$$

$$= \frac{\partial \nabla_s^2 U(s,x)}{\partial n_s} + (1-\nu)\frac{\partial}{\partial t_s}\left(\frac{\partial^2 U(s,x)}{\partial n_s \partial t_s}\right)$$

Plate problem

Selected two from four (U, Θ, M and V) (C_2^4 options)

Displacement

$$u(x) = \sum_{j=1}^{2N} P(s_j, x)\phi(s_j) + \sum_{j=1}^{2N} Q(s_j, x)\psi(s_j)$$

Slope $\theta(x) = K_{\theta,x}(u(x))$

Moment $m(x) = K_{m,x}(u(x))$

Shear $v(x) = K_{v,x}(u(x))$

Matrix form

Membrane

$$\{u\} = [U]\{\phi\}$$ Dirichlet problem

$$\{t\} = [L]\{\phi\}$$ Neumann problem

Plate

Clamped:

$$\{u\} = [P]\{\phi\} + [Q]\{\psi\}$$

$$\{\theta\} = [P_\theta]\{\phi\} + [Q_\theta]\{\psi\}$$

Free:

$$\{m\} = [P_m]\{\phi\} + [Q_m]\{\psi\}$$

$$\{v\} = [P_v]\{\phi\} + [Q_v]\{\psi\}$$

Simply-supported: $\{u\}$ and $\{m\}$

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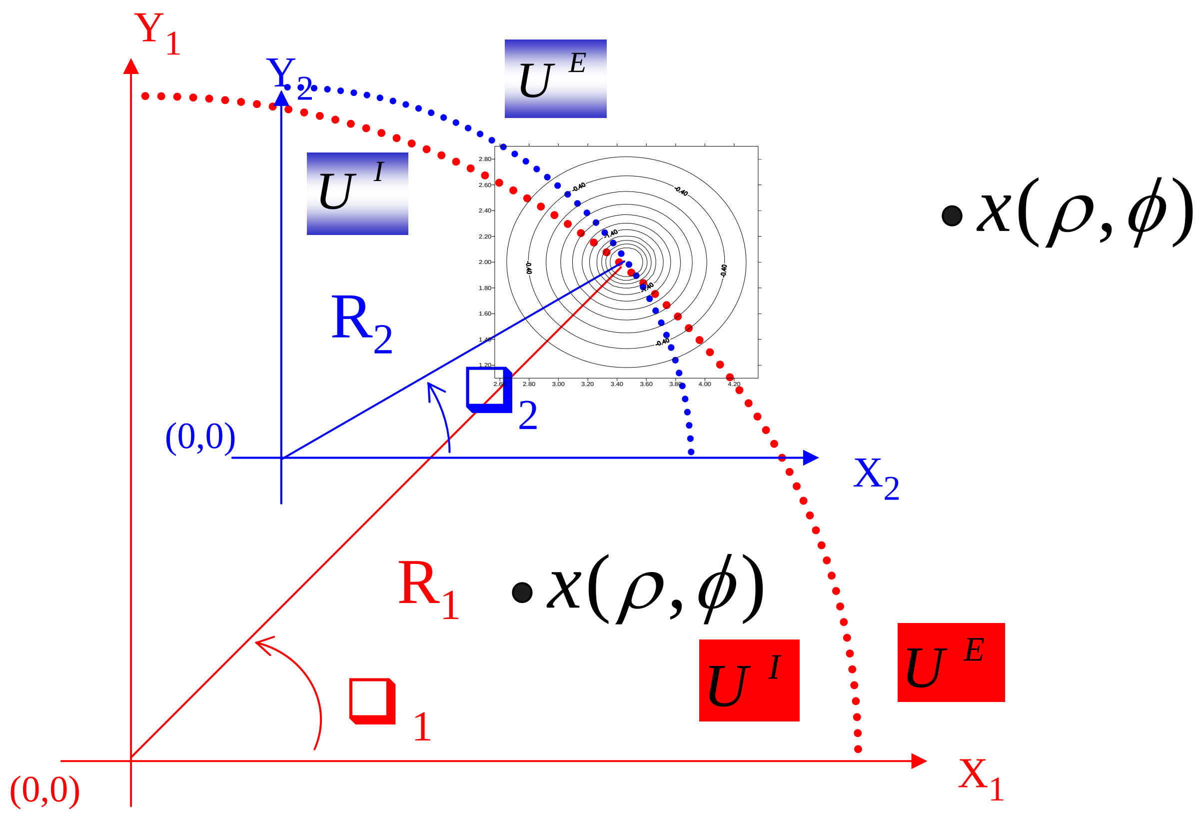
Degenerate kernel



Circulant

Uniform discritization into *2N* nodes on the circular boundary

$$[U]=\begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_{2N-2} & a_{2N-1} \\ a_{2N-1} & a_0 & a_1 & \cdots & a_{2N-3} & a_{2N-2} \\ a_{2N-2} & a_{2N-1} & a_0 & \cdots & a_{2N-4} & a_{2N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_2 & a_3 & a_4 & \cdots & a_0 & a_1 \\ a_1 & a_2 & a_3 & \cdots & a_{2N-1} & a_0 \end{bmatrix}$$

$$[U]=a_0 I + a_1 C_{2N} + a_2 (C_{2N})^2 + \cdots + a_{2N-1}(C_{2N})^{2N-1}$$

Circulant

$$C_{2N} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{2N \times 2N}$$

$$\alpha_\ell = e^{i\frac{2\pi\ell}{2N}} = \cos\left(\frac{2\pi\ell}{2N}\right) + i\sin\left(\frac{2\pi\ell}{2N}\right)$$ ：eigenvalue of C_{2N}

Membrane: $\lambda_\ell = 2NJ_\ell(ka)[iJ_\ell(ka') - Y_\ell(ka')]$

Plate: $\lambda_\ell^{[U]} = \dfrac{N}{4\lambda^2}\{J_m(\lambda\rho)[Y_m(\lambda R) - iJ_m(\lambda R)] + \dfrac{2}{\pi}(-1)^m I_m(\lambda\rho)[(-1)^m K_m(\lambda R) - iI_m(\lambda R)]\}$

Singular Value Decomposition

$$[U] = \Phi \Sigma_{[U]} \Phi^H$$

$$= \Phi \begin{bmatrix} \lambda_0^{[U]} & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \lambda_1^{[U]} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \lambda_{-1}^{[U]} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_{(N-1)}^{[U]} & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \lambda_{-(N-1)}^{[U]} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \lambda_N^{[U]} \end{bmatrix}_{2N \times 2N} \Phi^H$$

H is transpose and conjugate

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Methods of extracting true and spurious eigenvalues

SVD updating document	$[C_R] = [U_R \mid T_R]^T$	**Extraction true**
Burton & Miller method	$\left[[U_R] + i[T_R]\right]\{\varphi\} = \{0\}$	**Extraction true**
SVD updating term	$[P_R] = \begin{bmatrix} U_R \\ L_R \end{bmatrix}$	**Extraction spurious**

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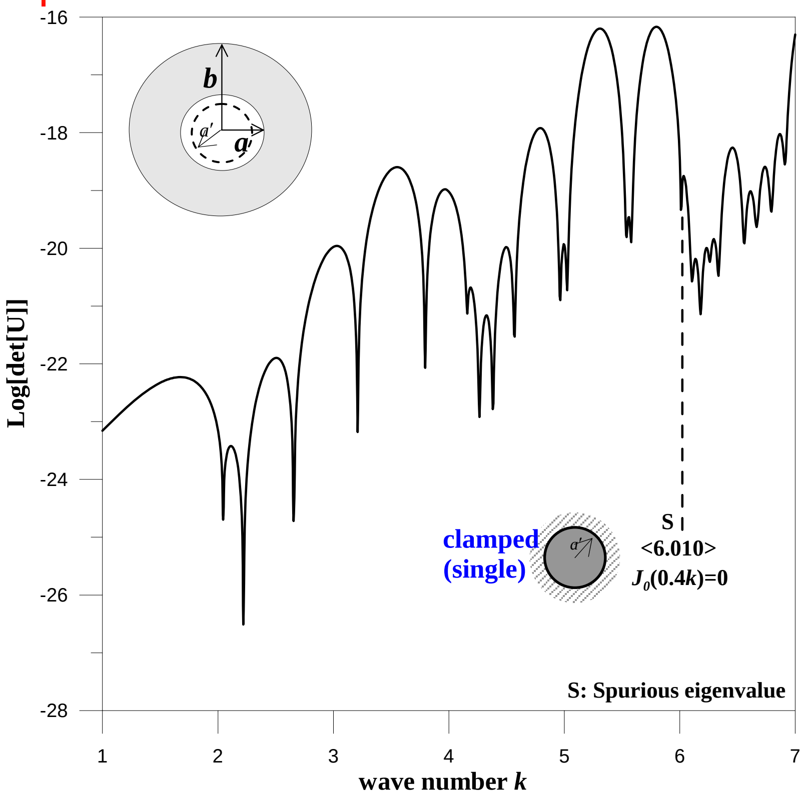
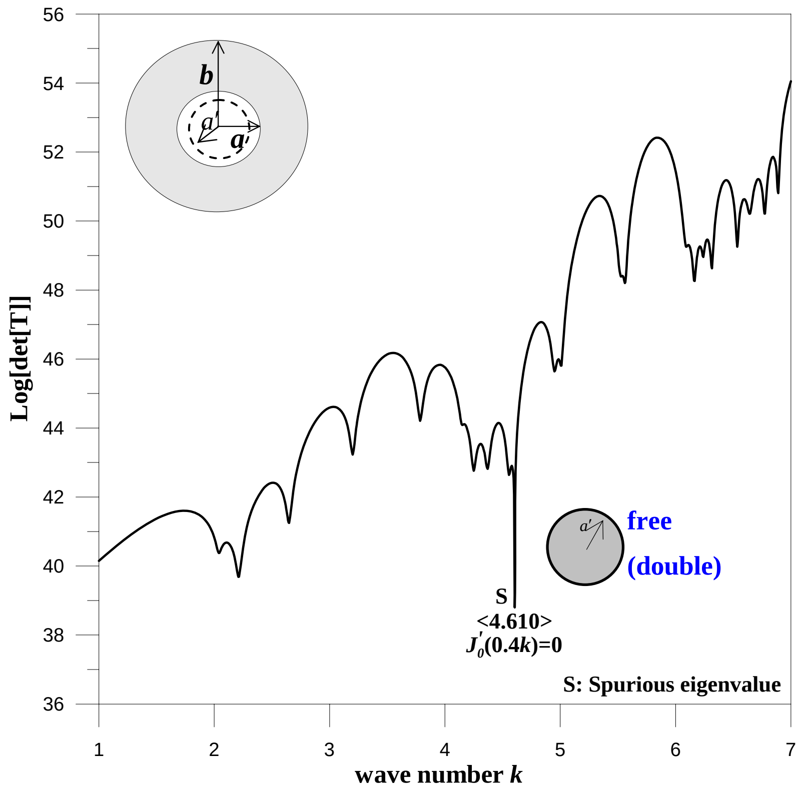
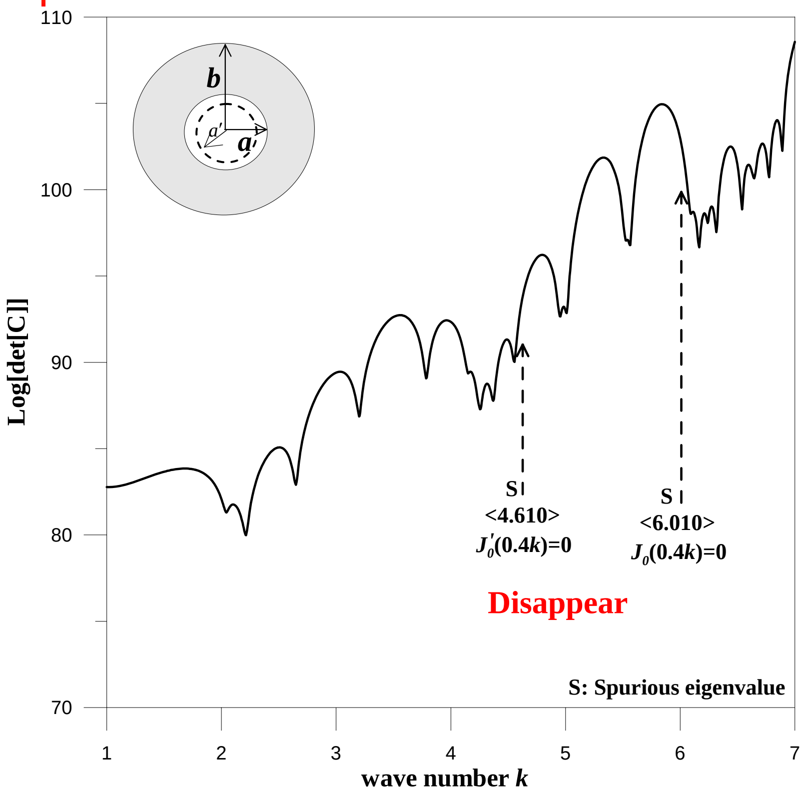
Clamped-clamped problem (membrane)

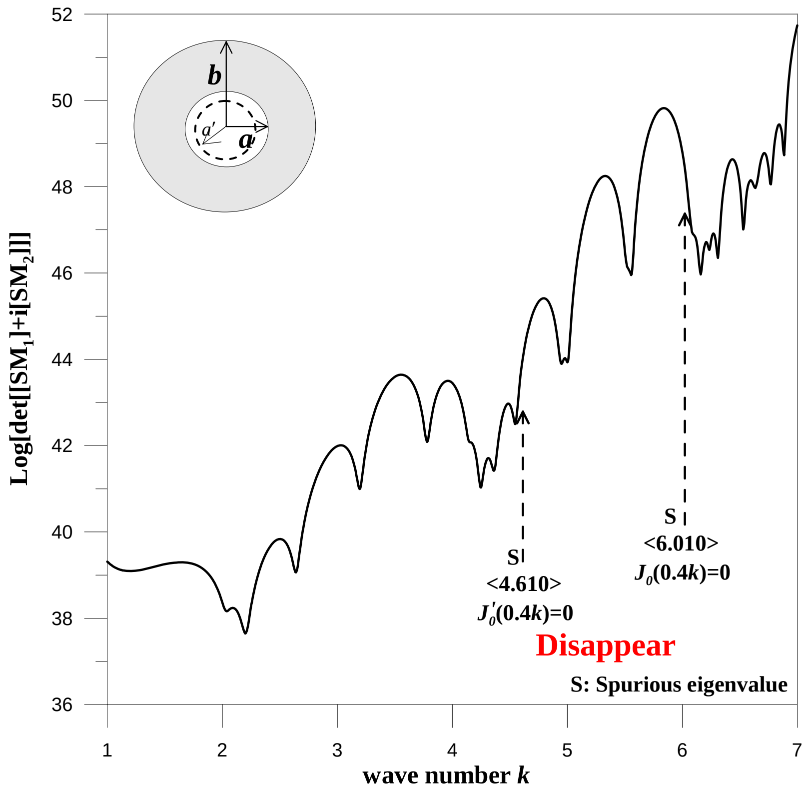




Single-layer potential approach **Double-layer potential approach**

Clamped-clamped problem (membrane)

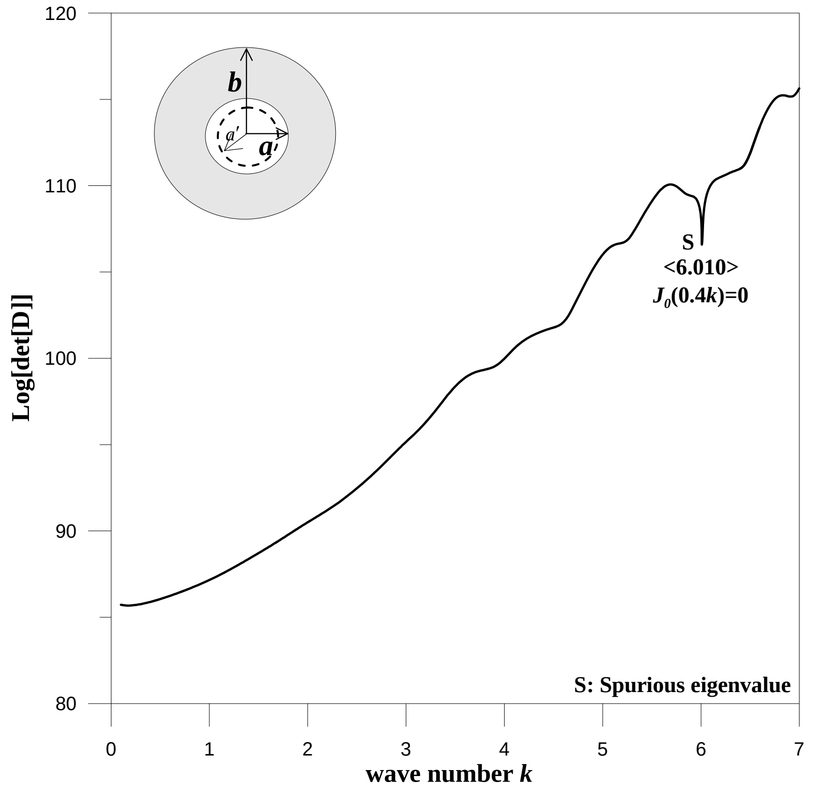




Single-layer potential approach + SVD updating document

Single-layer potential approach + Burton & Miller method

Clamped-clamped problem (membrane)



Single-layer potential approach

+ SVD updating term

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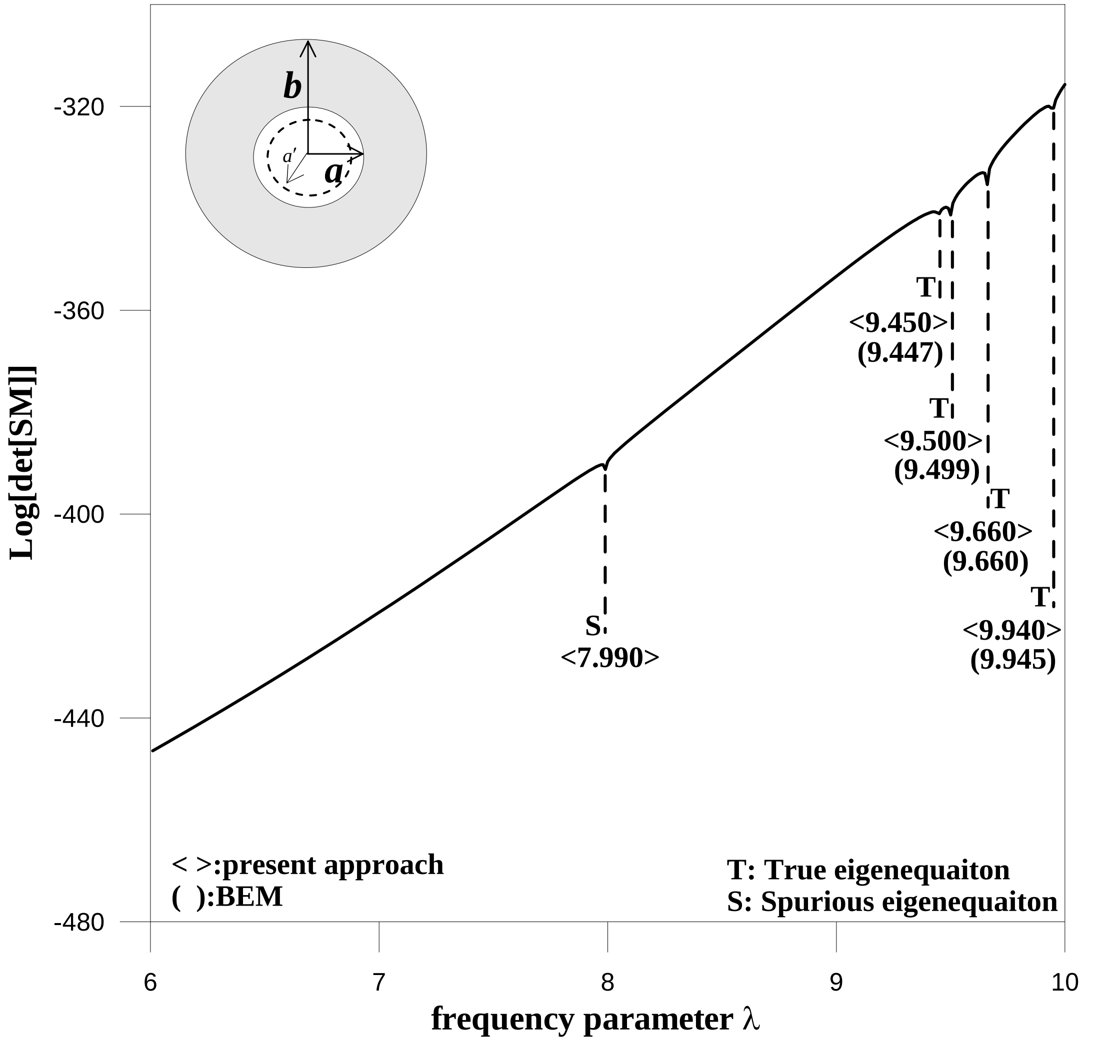
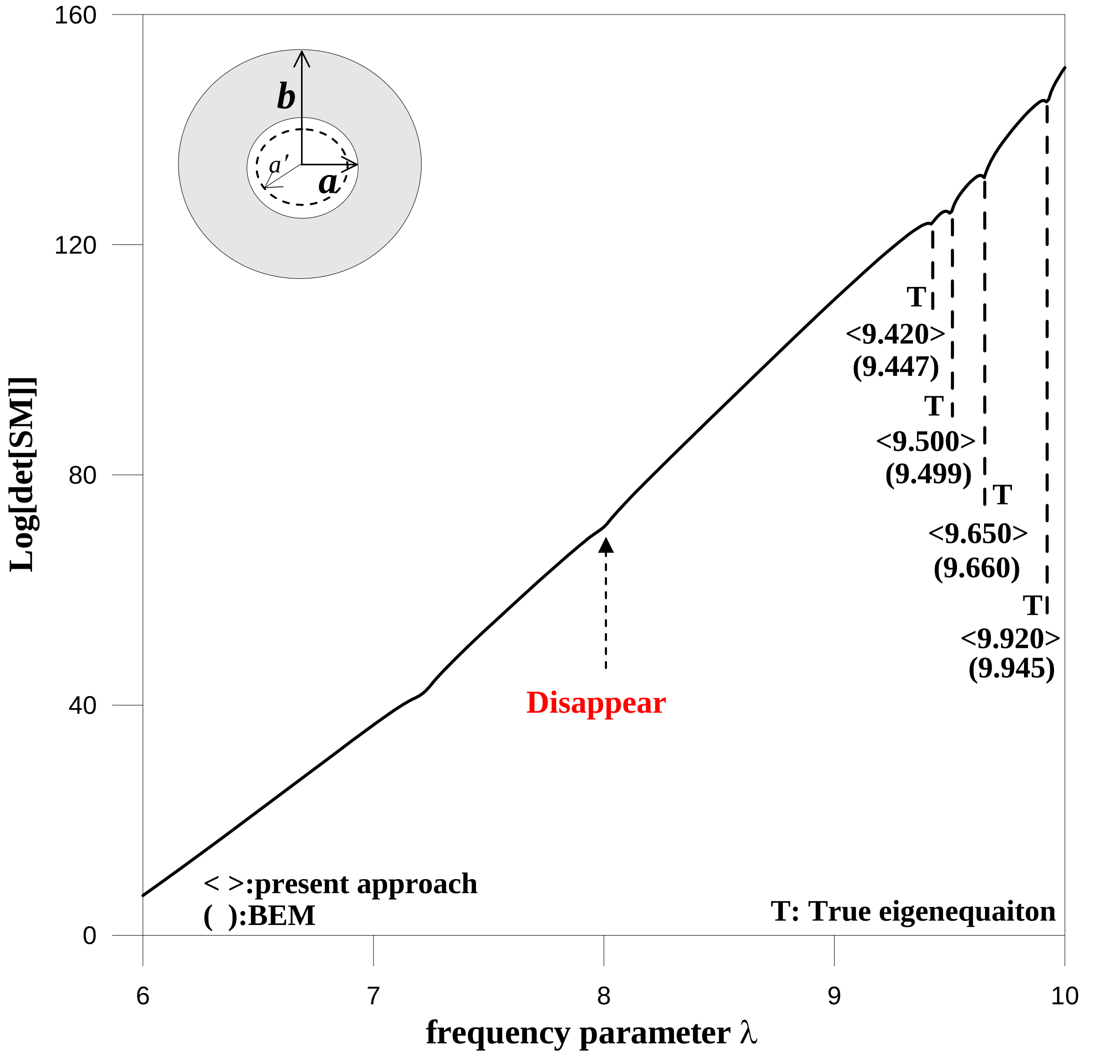
Clamped-clamped plate (U- Θ formulation)



Burton & Miller method



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Conclusions

1. We have verified that the true eigenequation depends on the boundary condition while spurious eigenequation is embedded in each formulation.

2. The spurious eigenvalues occurring in the multiply-connected eigenproblem for membrane and plate are the true eigenvalues of the associated problem bounded by the inner fictitious boundary where the sources are distributed.

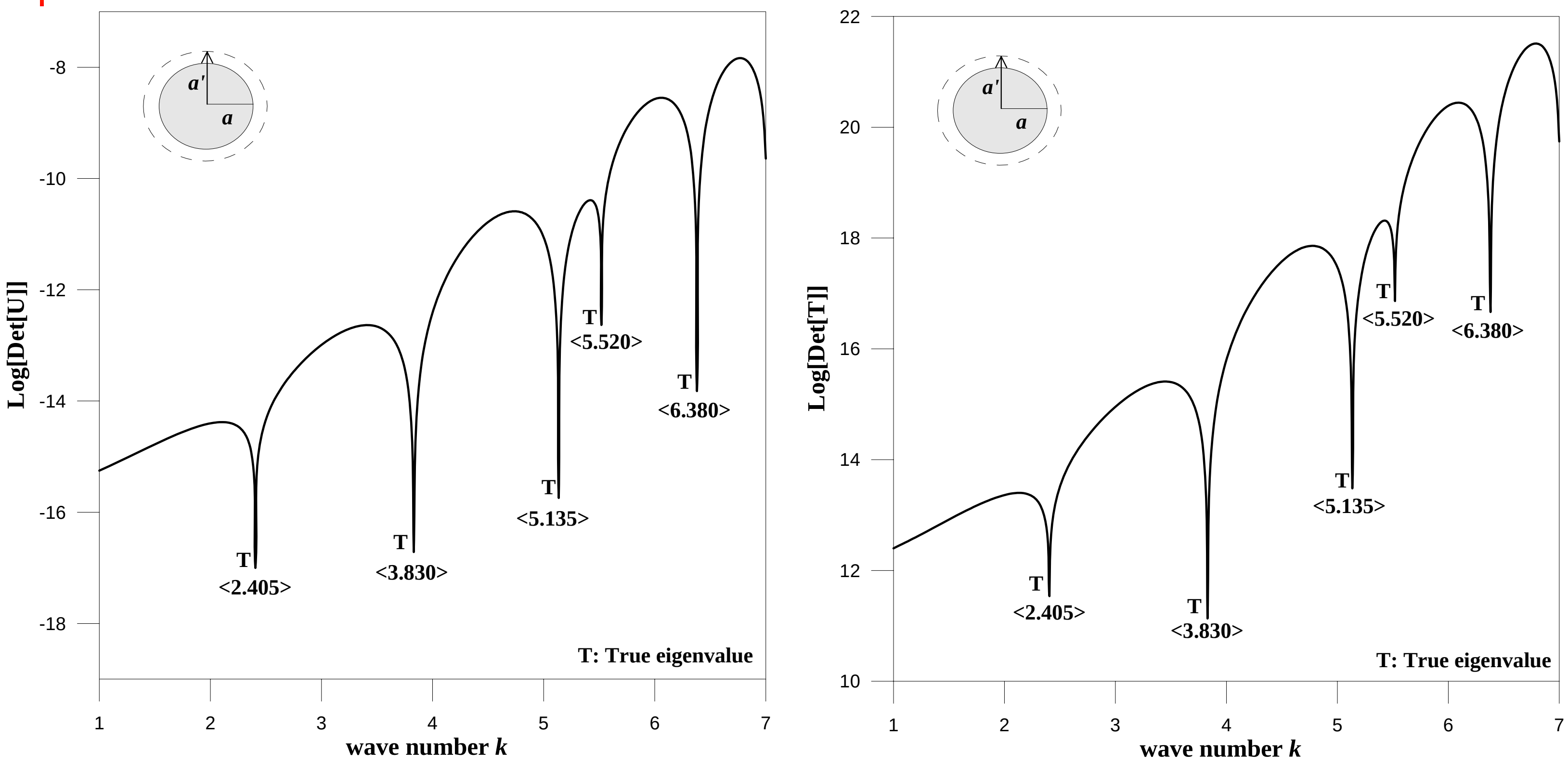
3. Three remedies, the SVD updating document, the SVD updating term and the Burton & Miller method,were successfully employed to suppress the appearance of the spurious eigenvalues

The End

Thanks for your kind attention

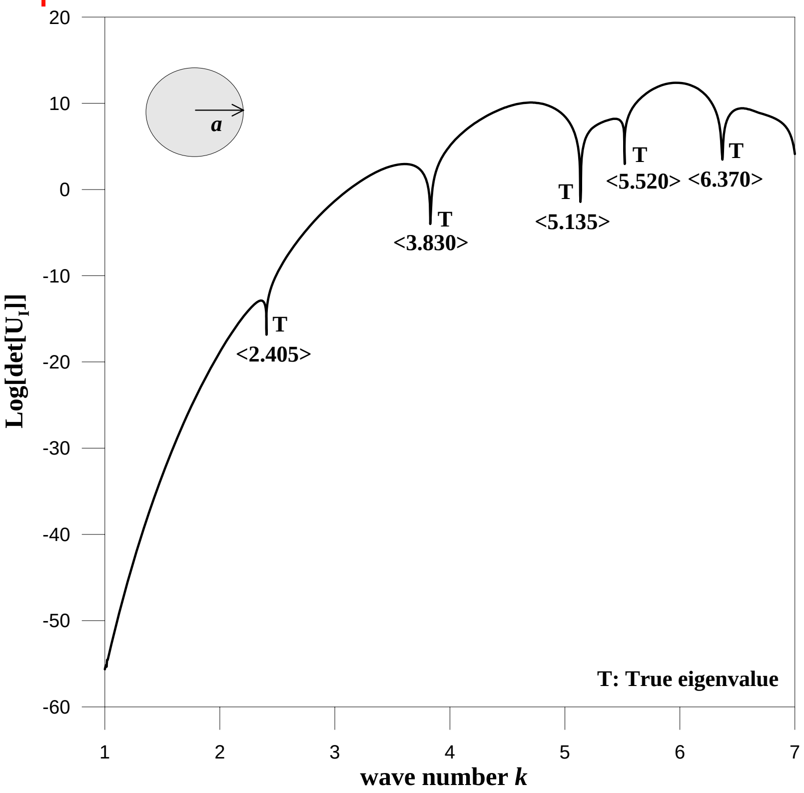
http://ind.ntou.edu.tw/~msvlab/

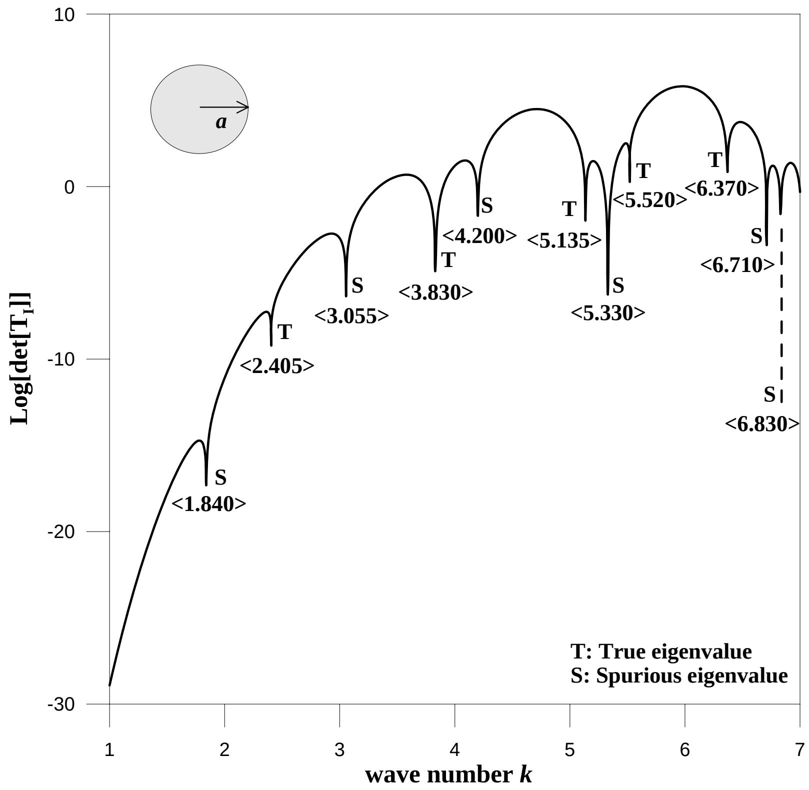
Dirichlet problem (Complex-valued MFS)



Single-layer potential approach **Double-layer potential approach**

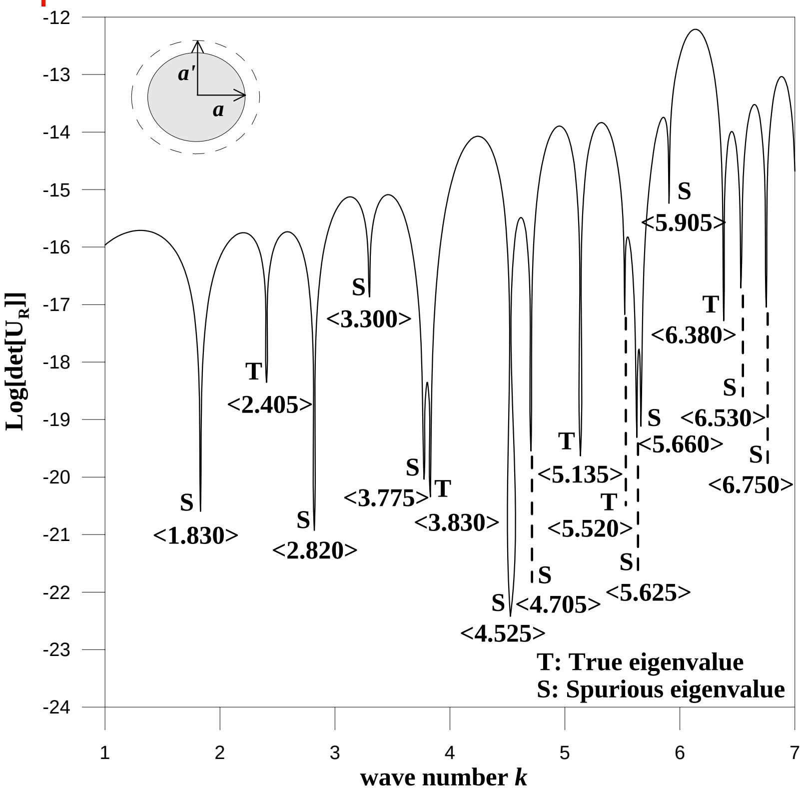
Dirichlet problem (Imaginary-part MFS)

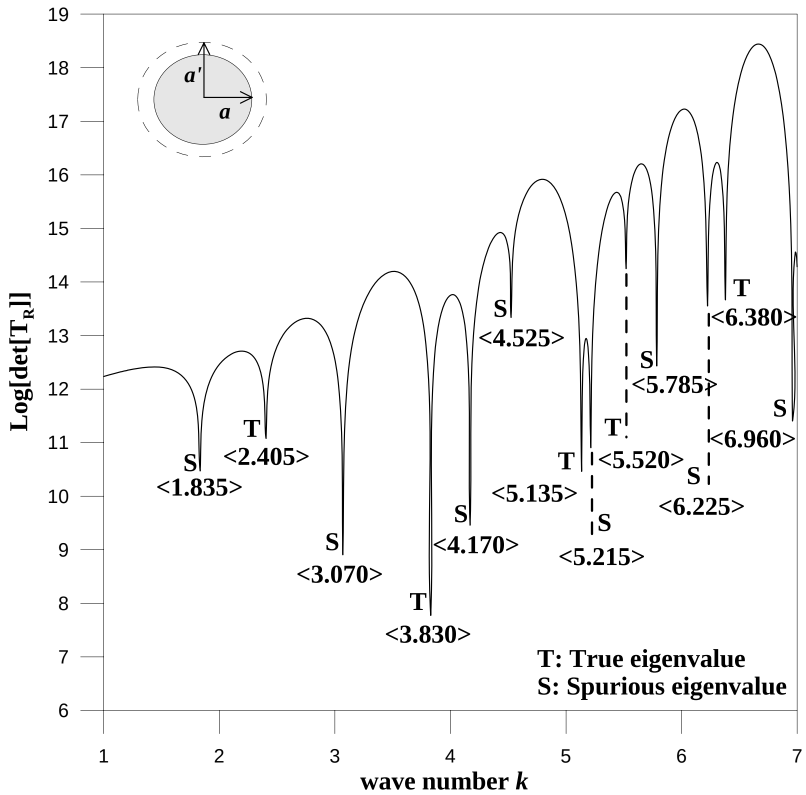




Single-layer potential approach **Double-layer potential approach**

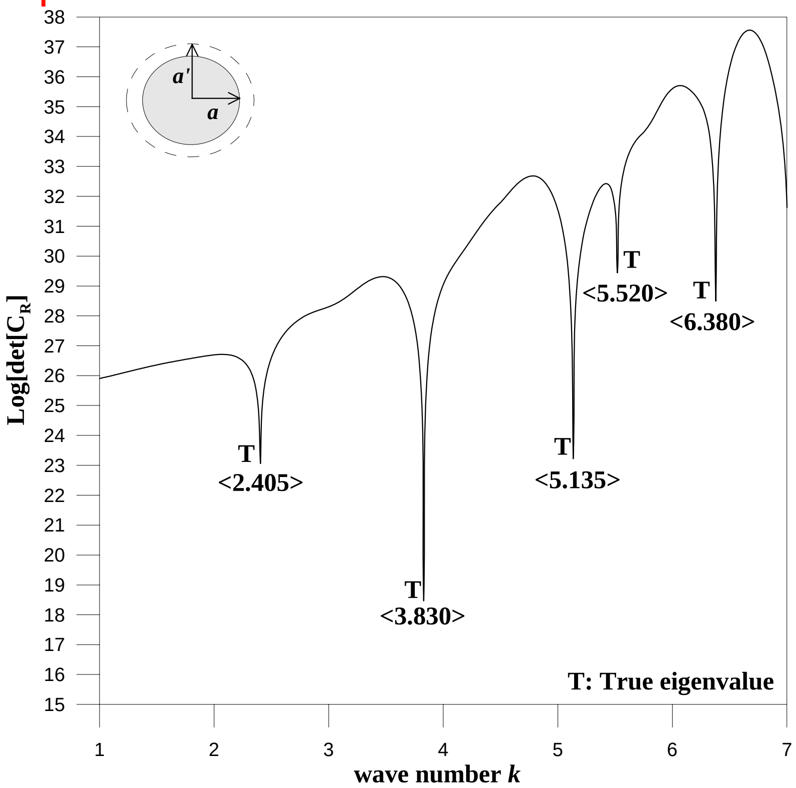
Dirichlet problem (Real-part MFS)

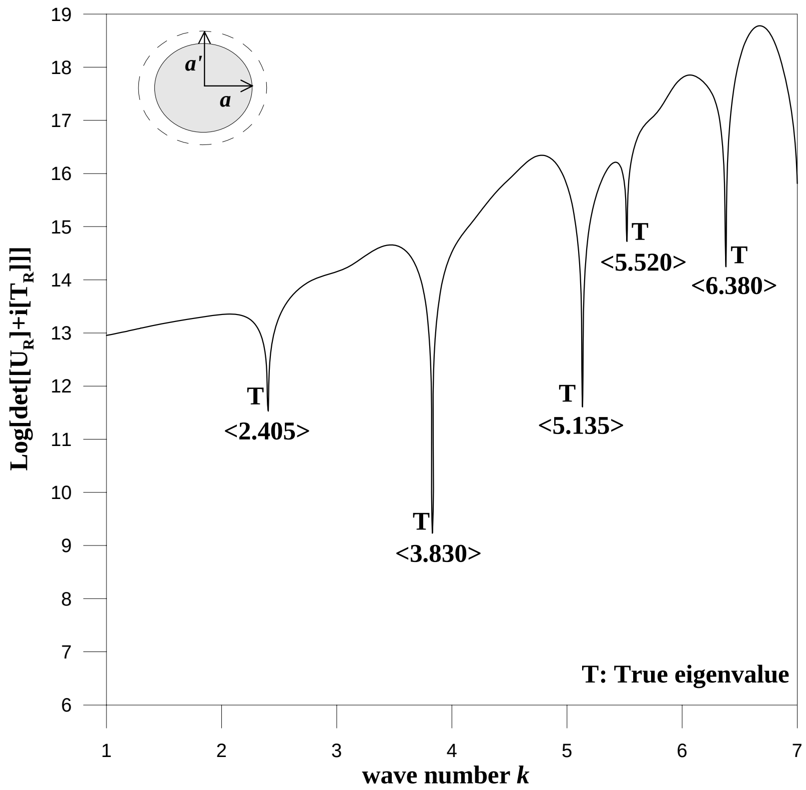




Single-layer potential approach **Double-layer potential approach**

Dirichlet problem (Real-part MFS)

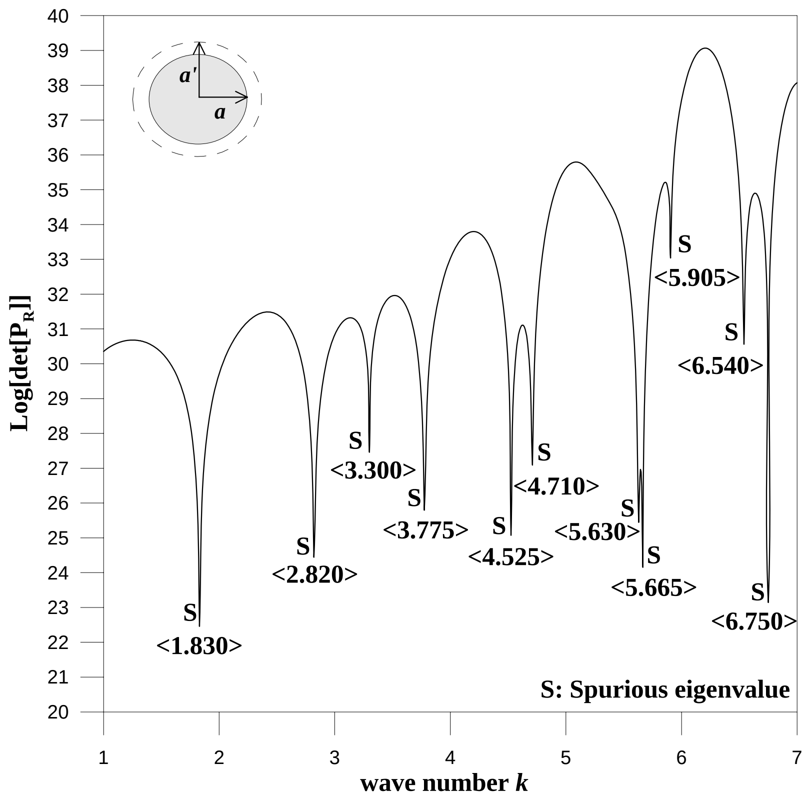




Single-layer potential approach +SVD updating document

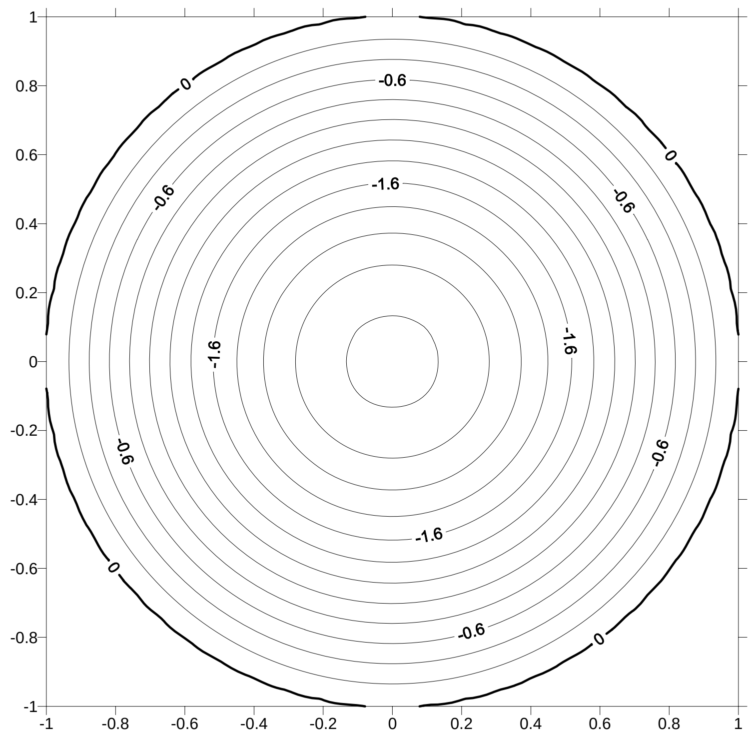
Single-layer potential approach +Burton & Miller method

Dirichlet problem (Real-part MFS)

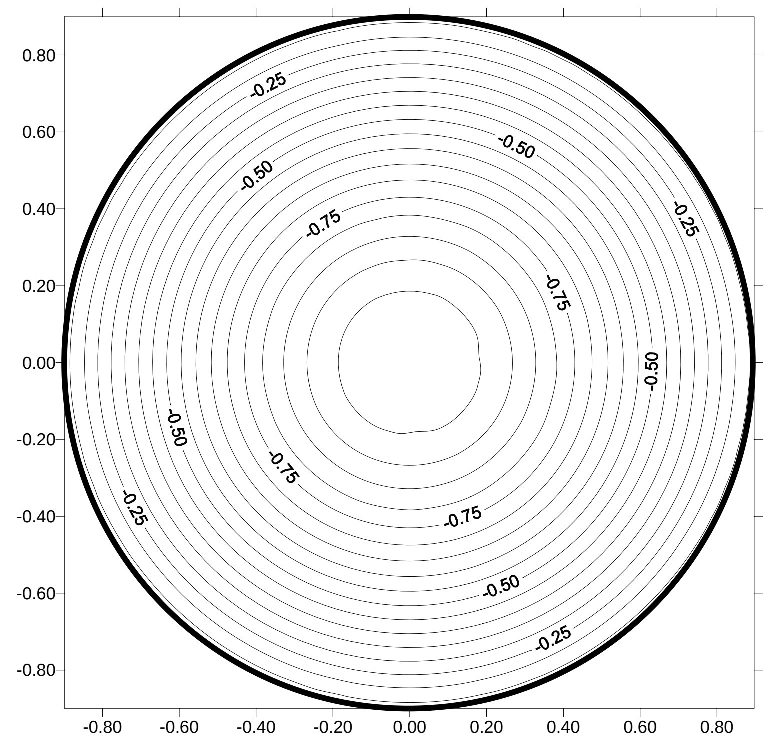


Single-layer potential approach
+SVD updating term

Mode 1 (k_1=2.045)

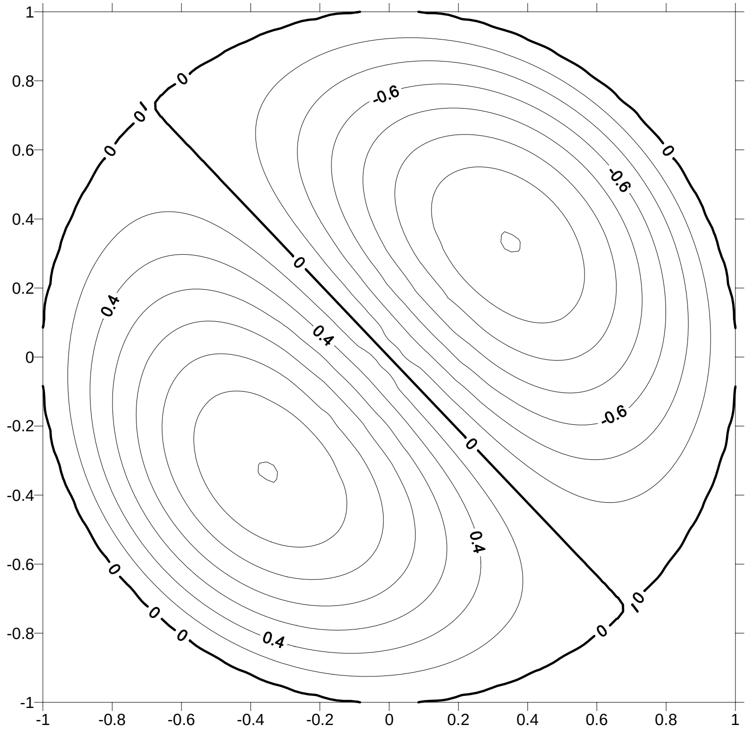


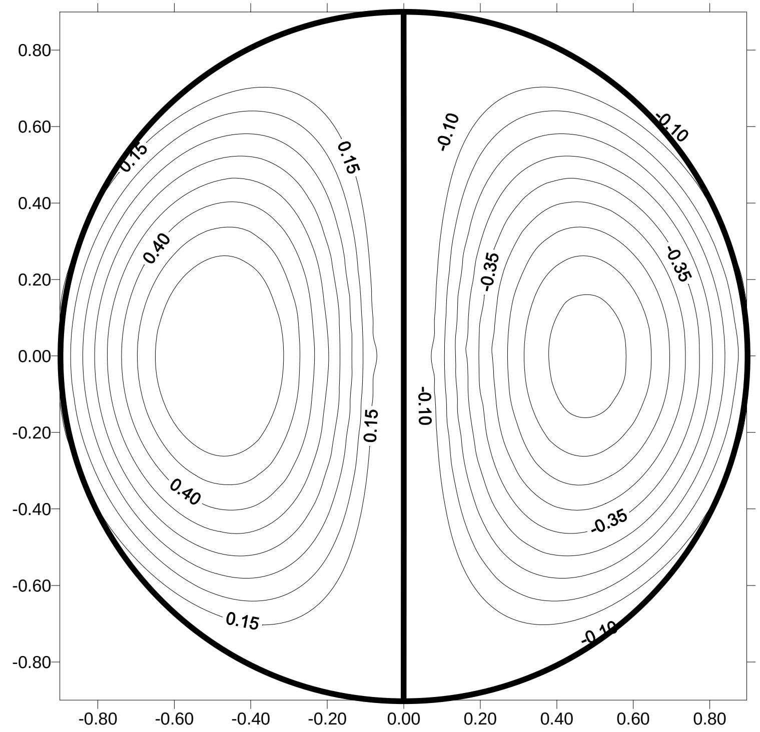
Present method



Analytical solution

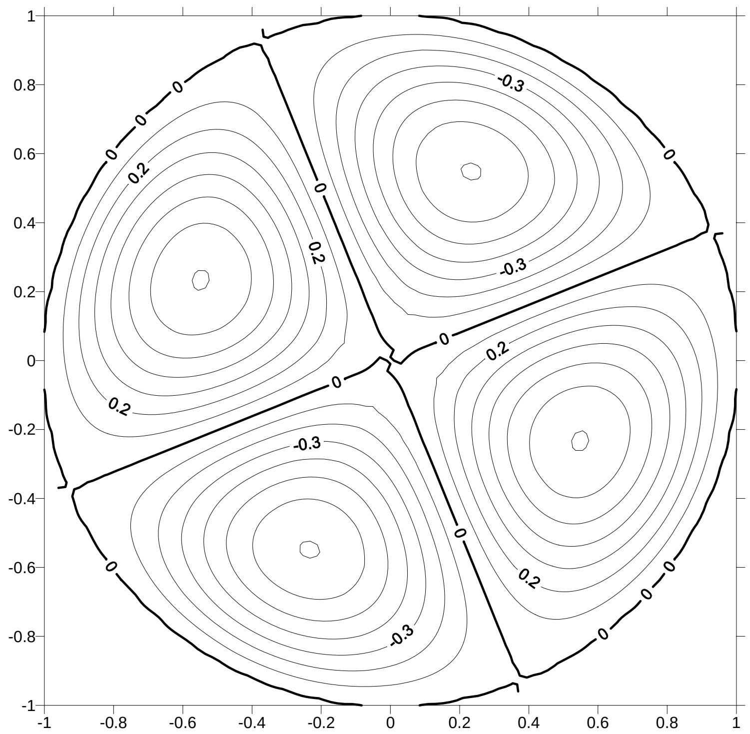
Mode 2 (k_2=3.083)



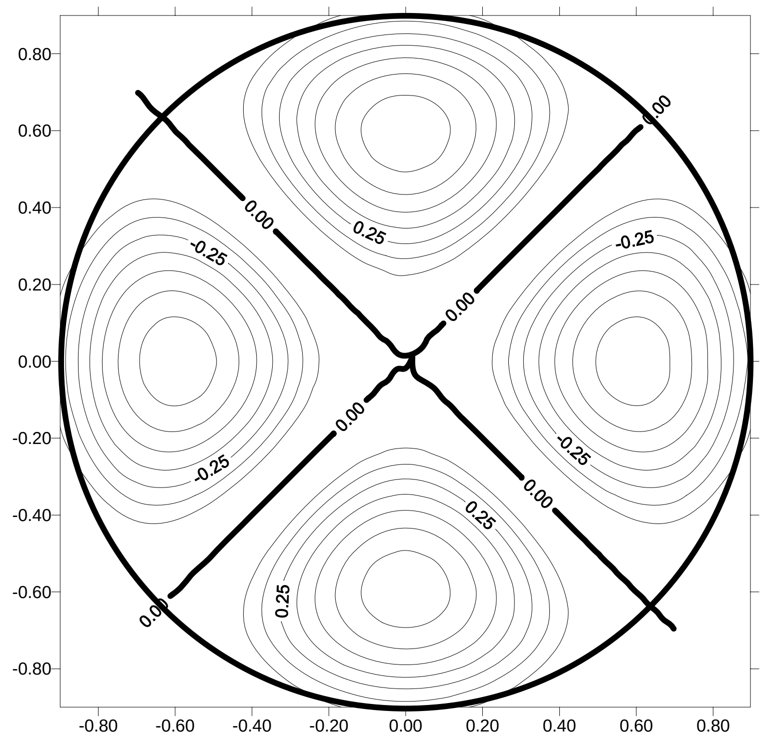


Present method **Analytical solution**

Mode 3 (k_3=5.135)



Present method



Analytical solution

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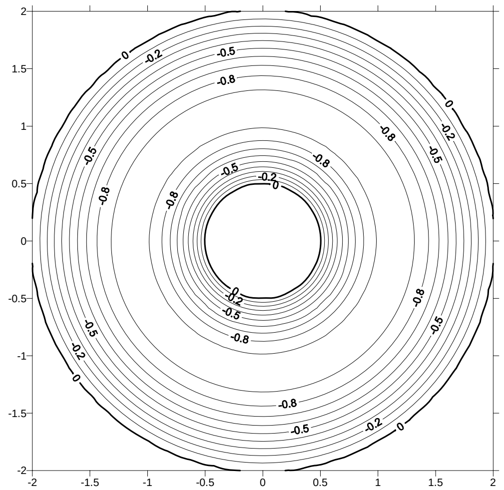
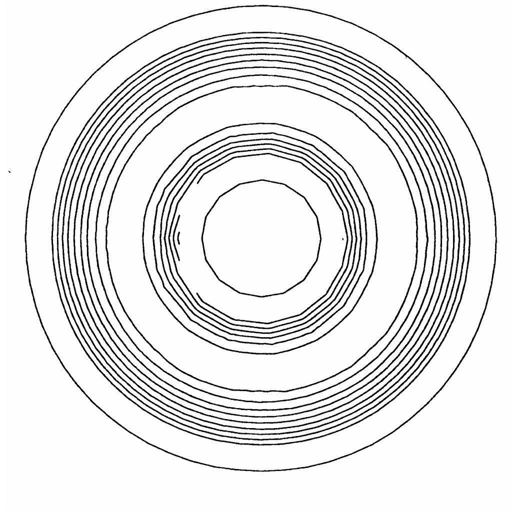
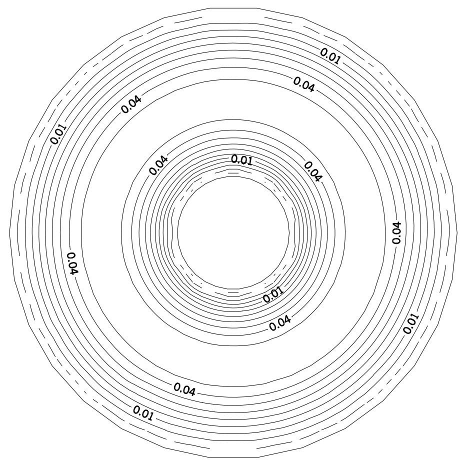
Mode 1 (k_1=2.05)







Present method **FEM** **BEM**

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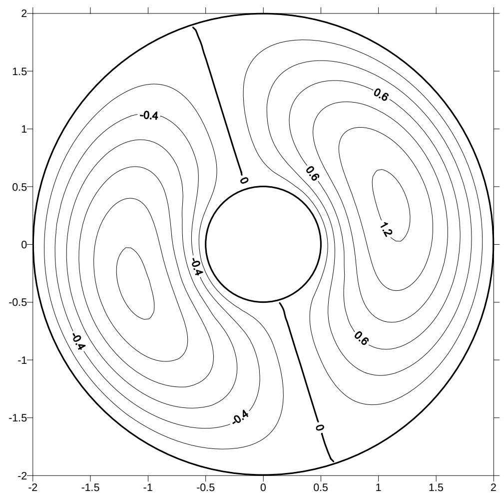
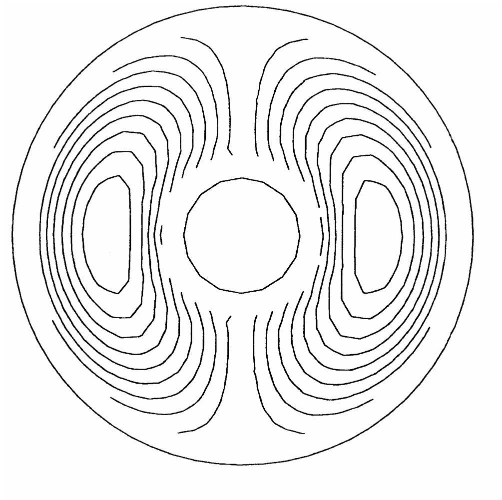
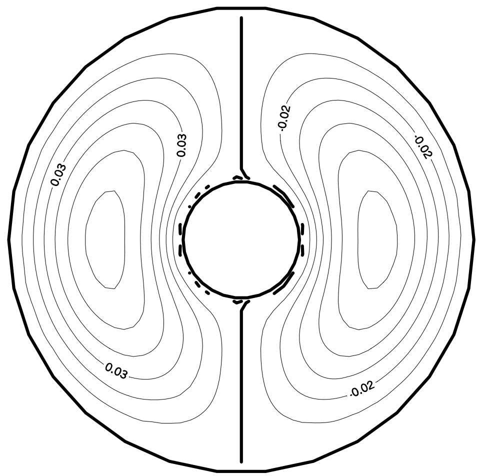
Mode 2 (k_2=2.23)

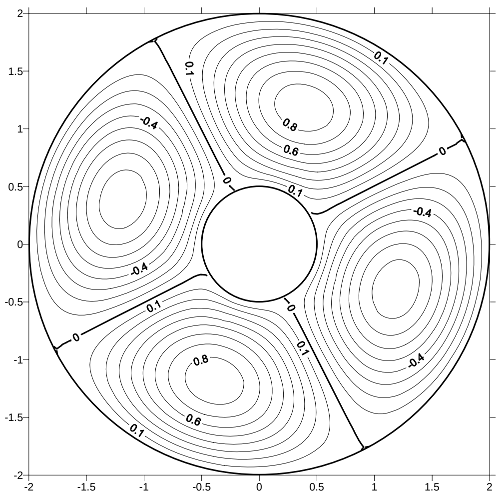




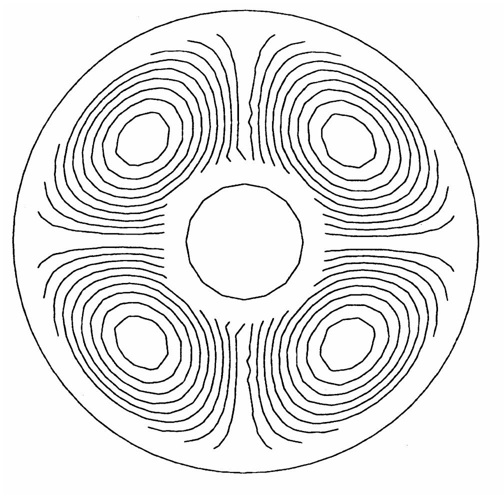


Present method **FEM** **BEM**

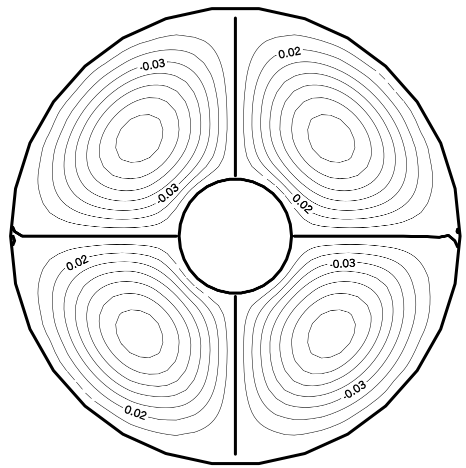
Mode 3 (k_3=2.66)



Present method



FEM



BEM

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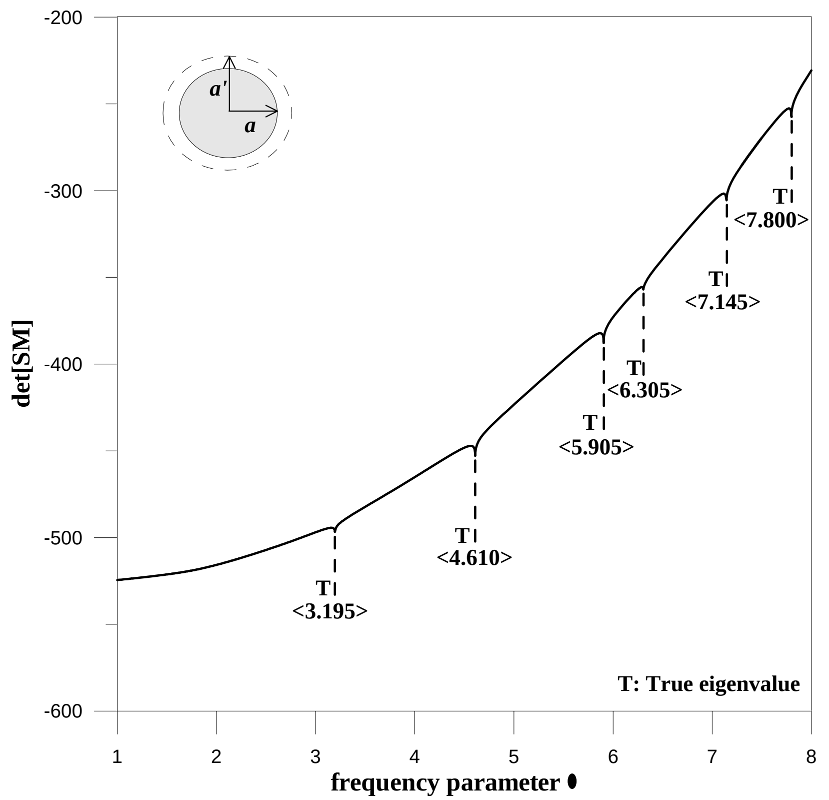
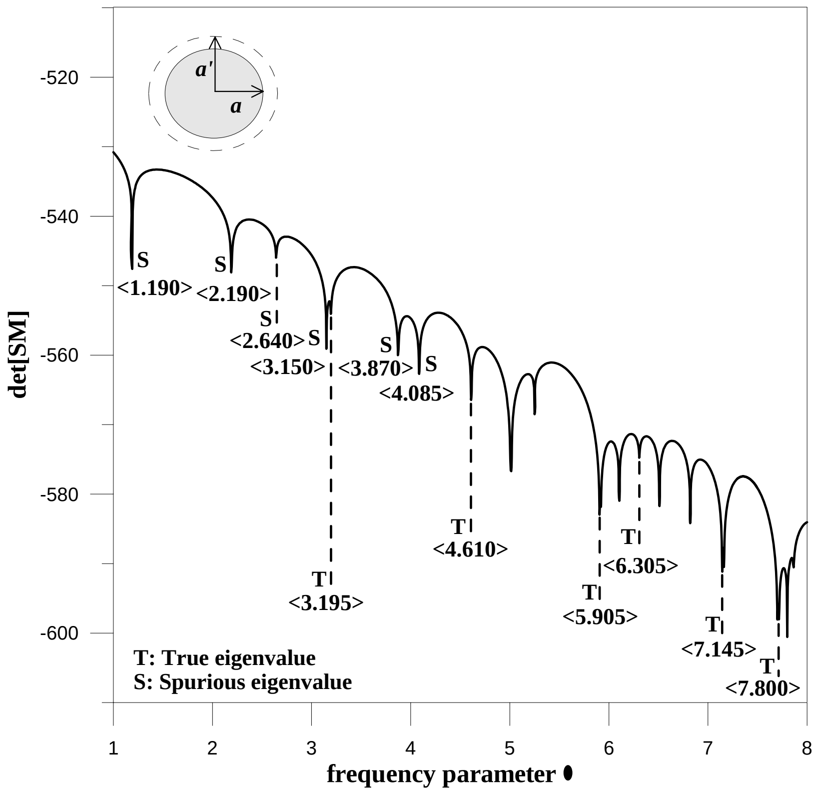
Clamped plate

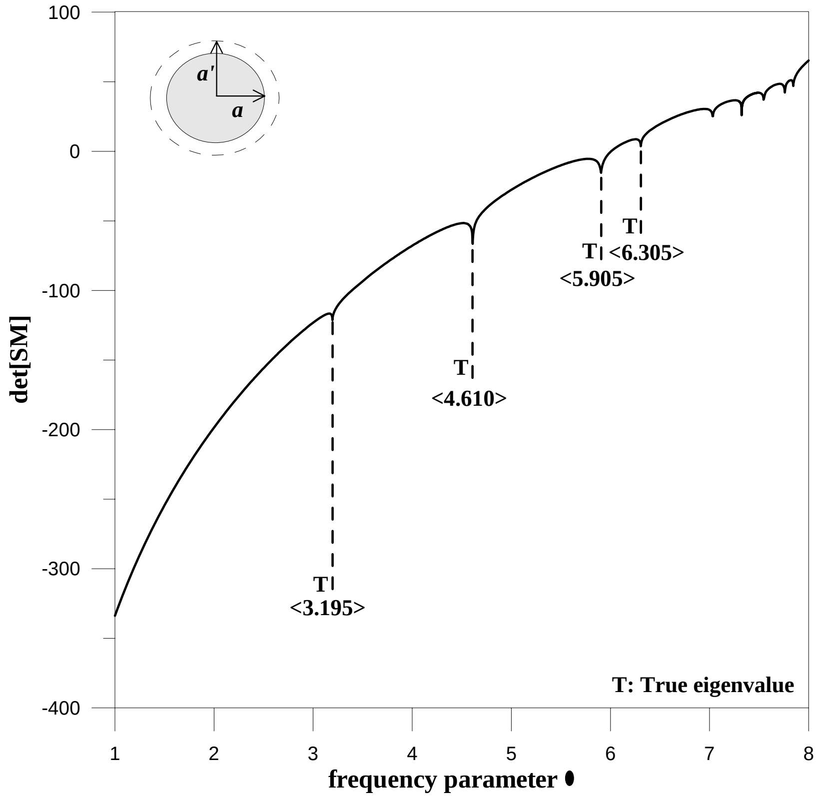


Complex-valued MFS

Clamped plate

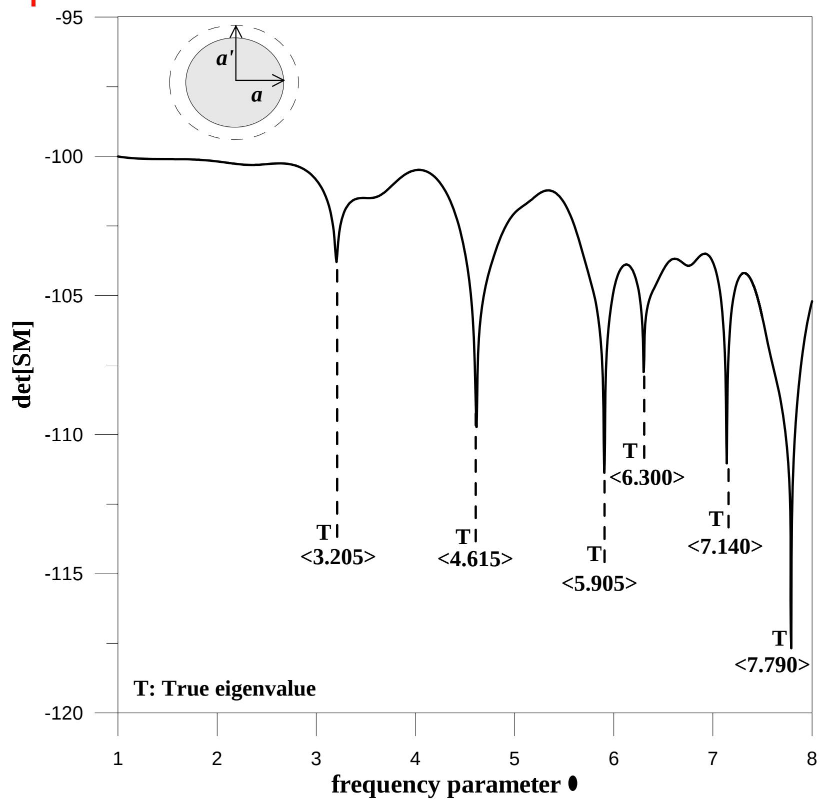


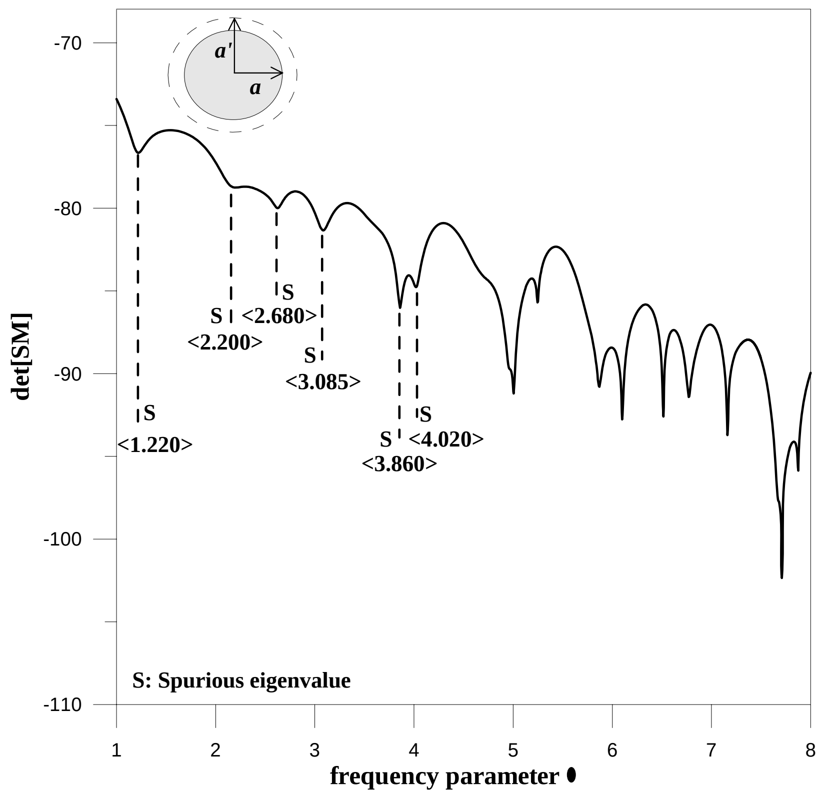
Real-part MFS



Imaginary-part MFS

Clamped plate

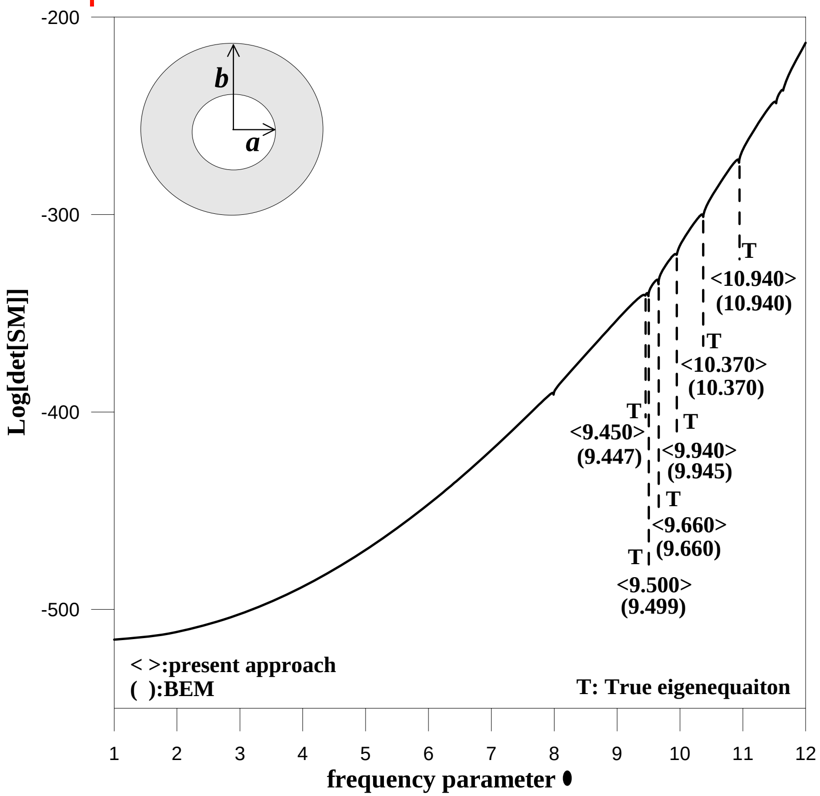




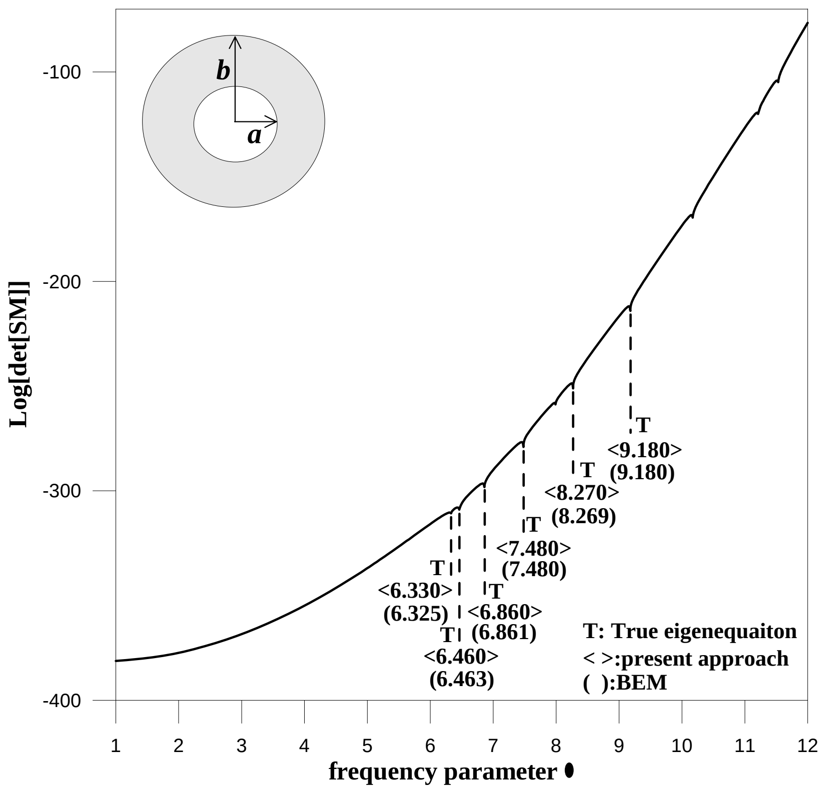
Real-part MFS +SVD updating document

Real-part MFS +SVD updating term

Multiply-connected plate



Clamped-clamped boundary



Simply-supported-simply-supported boundary

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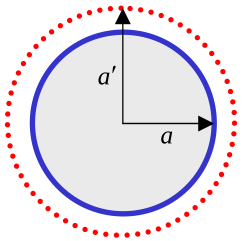
Method of solution

Membrane

For the Dirichlet problem (*u*=0) (Single-layer)



$$\{0\} = [U_{ij}]\{\phi_j\} \Rightarrow \det[U_{ij}]=0$$

Plate

For the clamped boundary condition ($u=0,\ \theta = 0$) $(U - \Theta)$

$$\{0\} = [U]\{\phi\} + [\Theta]\{\psi\}$$
$$\{0\} = [U_\theta]\{\phi\} + [\Theta_\theta]\{\psi\}$$

$$\Rightarrow \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} U & \Theta \\ U_\theta & \Theta_\theta \end{bmatrix} \begin{Bmatrix} \phi \\ \psi \end{Bmatrix}$$

$$\det[SM_1^c] = 0$$

Degenerate kernels

Membrane

$$U(s,x)=\begin{cases} U^{i}(R,\theta;\rho,\phi)=\sum_{\ell=-\infty}^{\infty} J_{\ell}(k\rho)[iJ_{\ell}(kR)-Y_{\ell}(kR)]\cos(\ell(\theta-\phi)), & R>\rho, \\ U^{e}(R,\theta;\rho,\phi)=\sum_{\ell=-\infty}^{\infty} J_{\ell}(kR)[iJ_{\ell}(k\rho)-Y_{\ell}(k\rho)]\}\cos(\ell(\theta-\phi)), & R<\rho, \end{cases}$$

Plate

$$U^{i}(R,\theta;\rho,\phi)=\sum_{m=-\infty}^{\infty}\frac{1}{8\lambda^{2}}\{J_{m}(\lambda\rho)[Y_{m}(\lambda R)-iJ_{m}(\lambda R)]+\frac{2}{\pi}(-1)^{m}I_{m}(\lambda\rho)[(-1)^{m}K_{m}(\lambda R)-iI_{m}(\lambda R)]\}\cos(m(\theta-\phi)),\quad R>\rho,$$

$$U^{e}(R,\theta;\rho,\phi)=\sum_{m=-\infty}^{\infty}\frac{1}{8\lambda^{2}}\{J_{m}(\lambda R)[Y_{m}(\lambda\rho)-iJ_{m}(\lambda\rho)]+\frac{2}{\pi}(-1)^{m}I_{m}(\lambda R)[(-1)^{m}K_{m}(\lambda\rho)-iI_{m}(\lambda\rho)]\}\cos(m(\theta-\phi)),\quad R<\rho,$$