A STUDY ON LAPLACE PROBLEMS OF INFINITE PLANE WITH MULTIPLE CIRCULAR HOLES

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Outlines

- Problem statement
- Motivation and Literatures review
- Boundary integral equations
- Degenerate kernels
- Present method
- Numerical examples
- Conclusions

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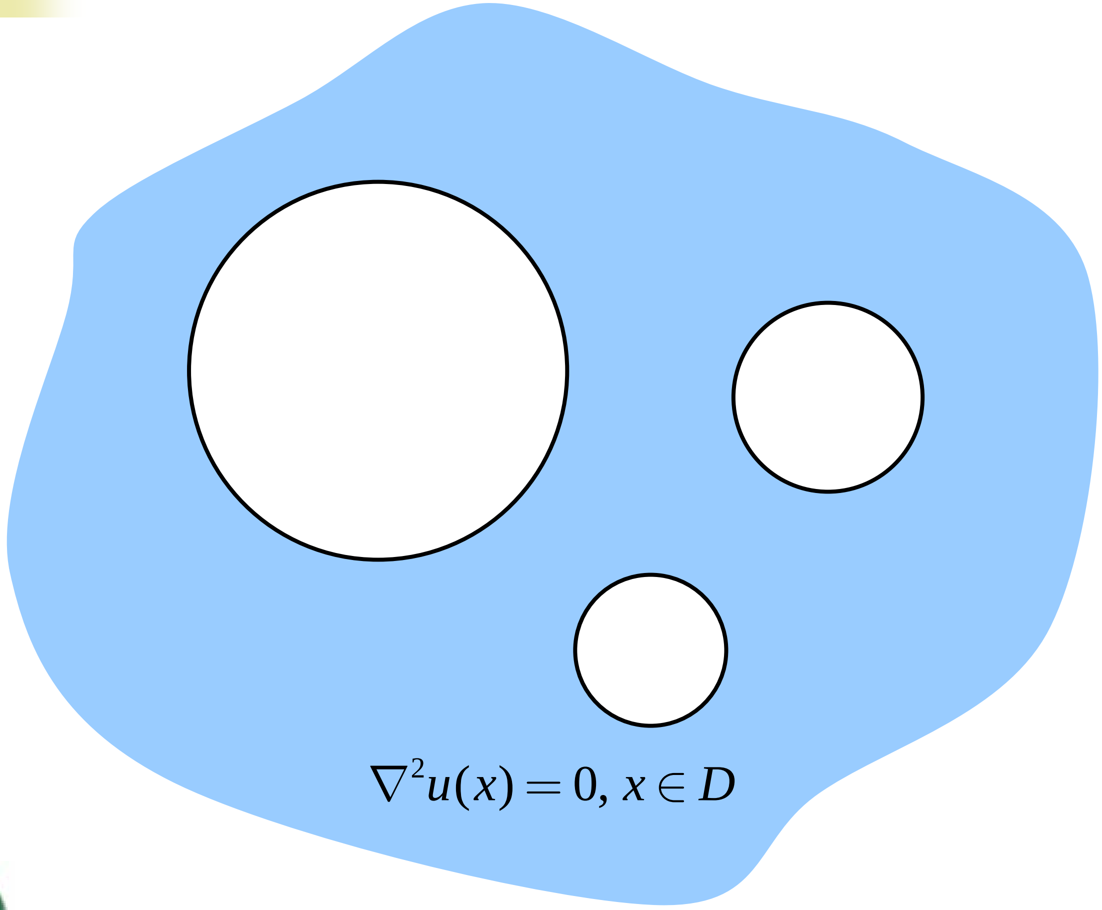
Problem statement



$$\nabla^2 u(x) = 0, x \in D$$

where

∇^2 : Laplacian operator

x : field point

D : domain of interest

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Motivation

Laplace's problems with circular holes:

- Electrostatic fields of wires
- Torsion bar with circular holes
- Steady state heat conduction of tube
- Flow of incompressible fluid around cylinders

Literatures review

- Ling (1943)- torsion bar of circular tubes
- Caulk *et al.* (1983)-steady state heat conduction with circular holes
- Bird and Steele (1992)- harmonic and biharmonic problems with circular holes
- Mogilevskaya *et al.* (2002)- elasticity problems with circular boundaries

However, they didn't employ the null-field integral equation and degenerate kernels to fully capture the circular boundary, although they all used Fourier series expansion.

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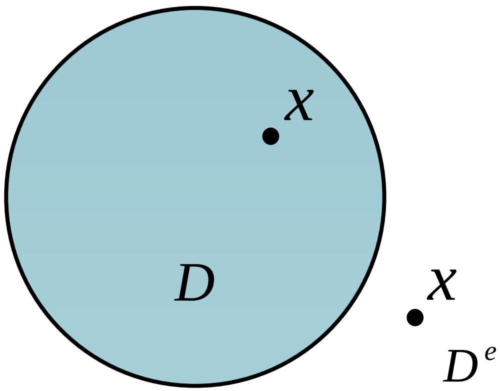
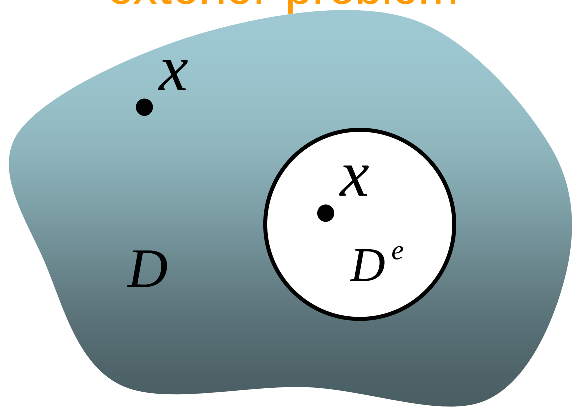
Boundary integral equations

interior problem



exterior problem



where

$$U(s,x)=\ln|x-s|=\ln r$$

$$T(s,x)=\frac{\partial U(s,x)}{\partial n_s}$$

$$t(s)=\frac{\partial u(s)}{\partial n_s}$$

$$2\pi u(x)=\int_B T(s,x)u(s)dB(s)-\int_B U(s,x)t(s)dB(s),\ x\in D$$

$$0=\int_B T(s,x)u(s)dB(s)-\int_B U(s,x)t(s)dB(s),\ x\in D^e$$

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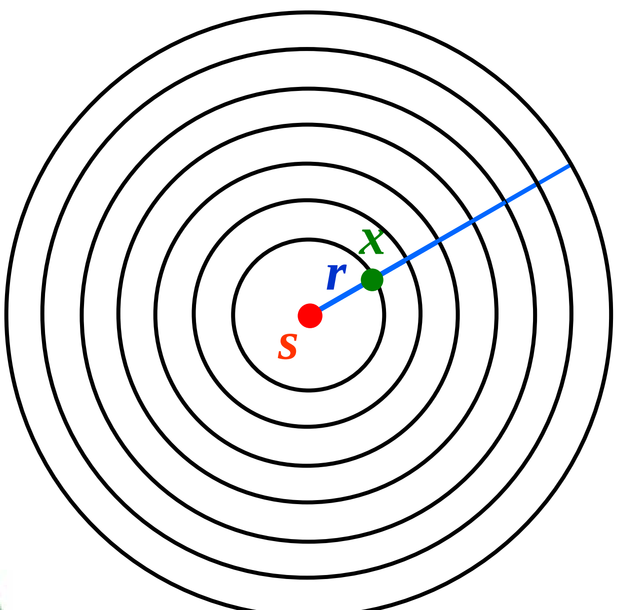
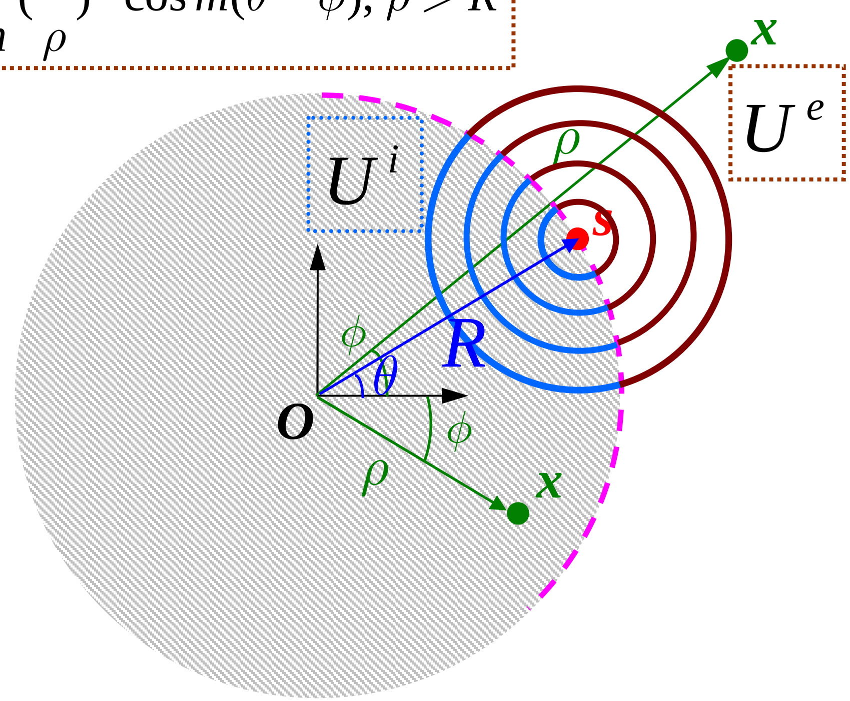
Degenerate kernels

$$U(s,x)=\ln r=\begin{cases} U^{i}(s,x)=\ln R-\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{\rho}{R}\right)^{m}\cos m(\theta-\phi),\ R>\rho \\ U^{e}(s,x)=\ln \rho-\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{R}{\rho}\right)^{m}\cos m(\theta-\phi),\ \rho>R \end{cases}$$

$s=(R,\theta)$

$x=(\rho,\phi)$





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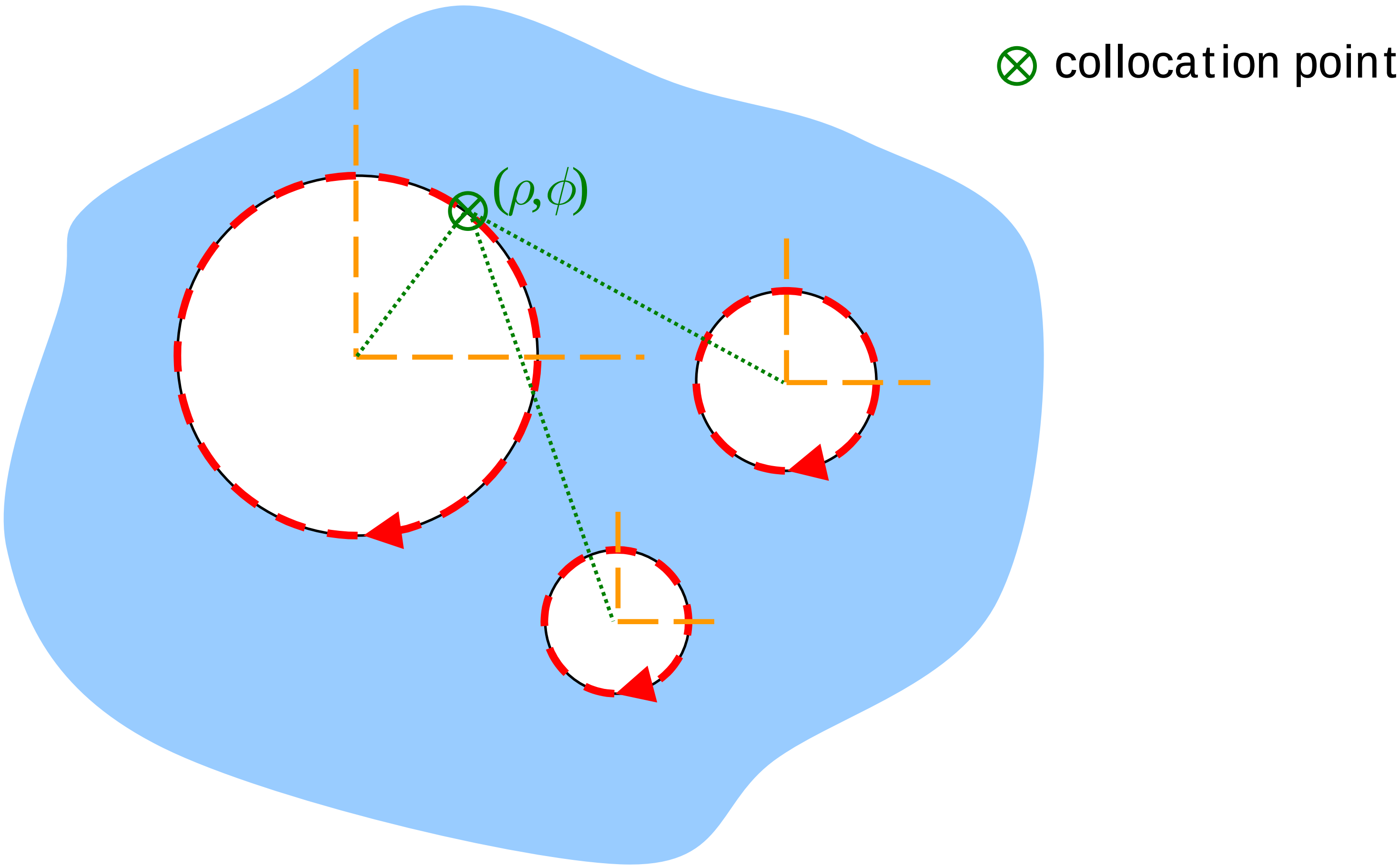
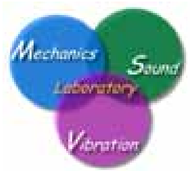
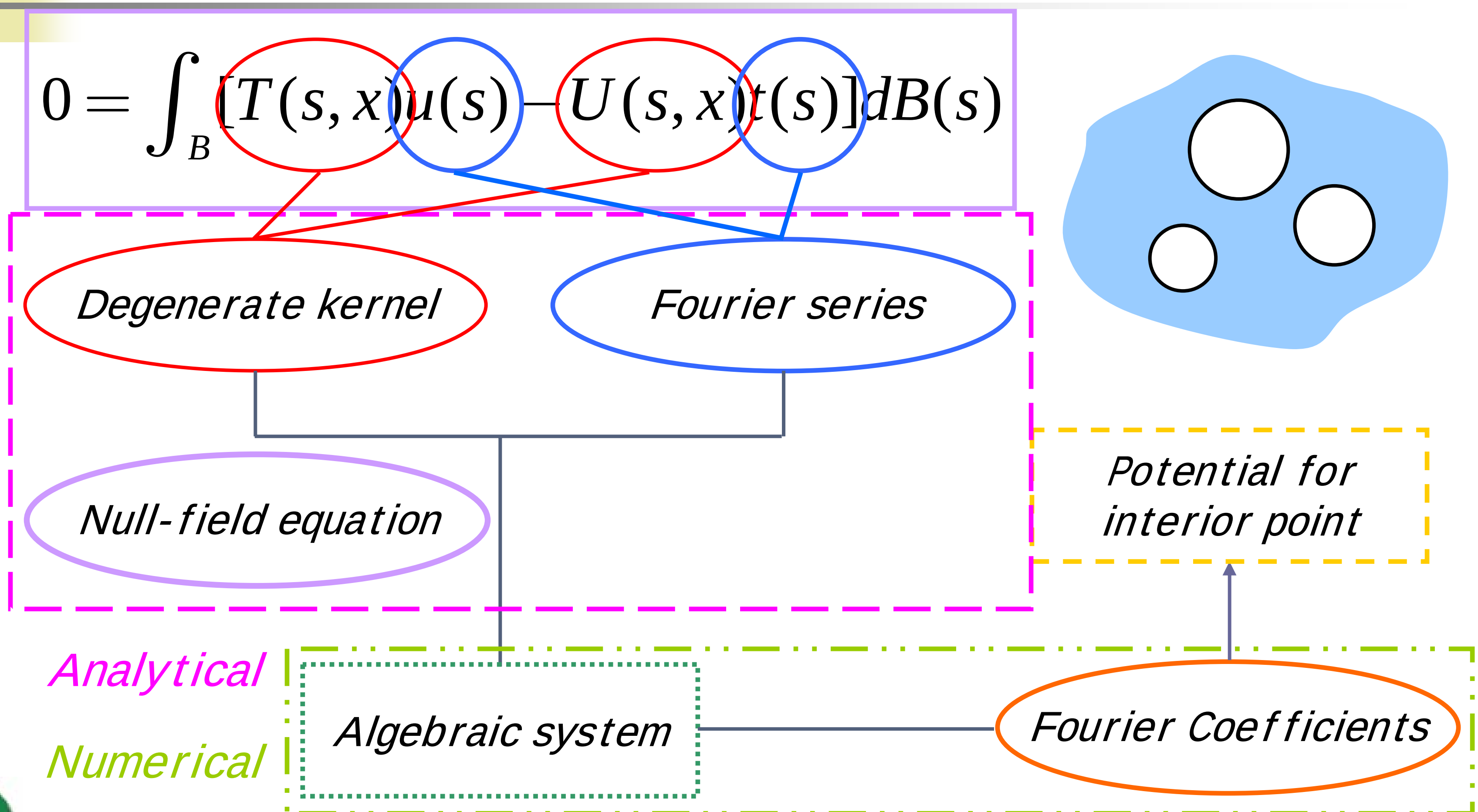
Adaptive observer system





Flowchart of present method

$$0 = \int_B [T(s,x)u(s) - U(s,x)t(s)]dB(s)$$

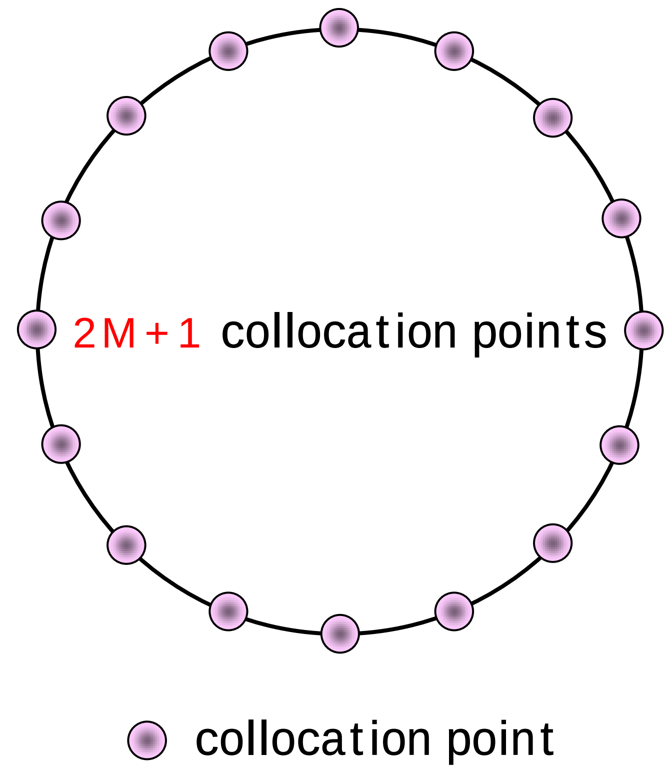


Collocation points

$$u(s) = a_0 + \sum_{n=1}^{M} (a_n \cos n\theta + b_n \sin n\theta)$$

$$t(s) = p_0 + \sum_{n=1}^{M} (p_n \cos n\theta + q_n \sin n\theta)$$

2M+1 unknown Fourier coefficients



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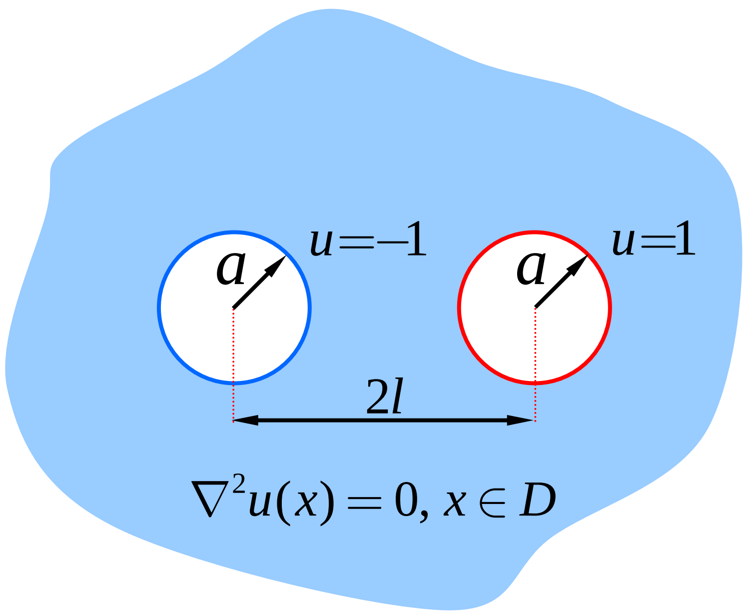
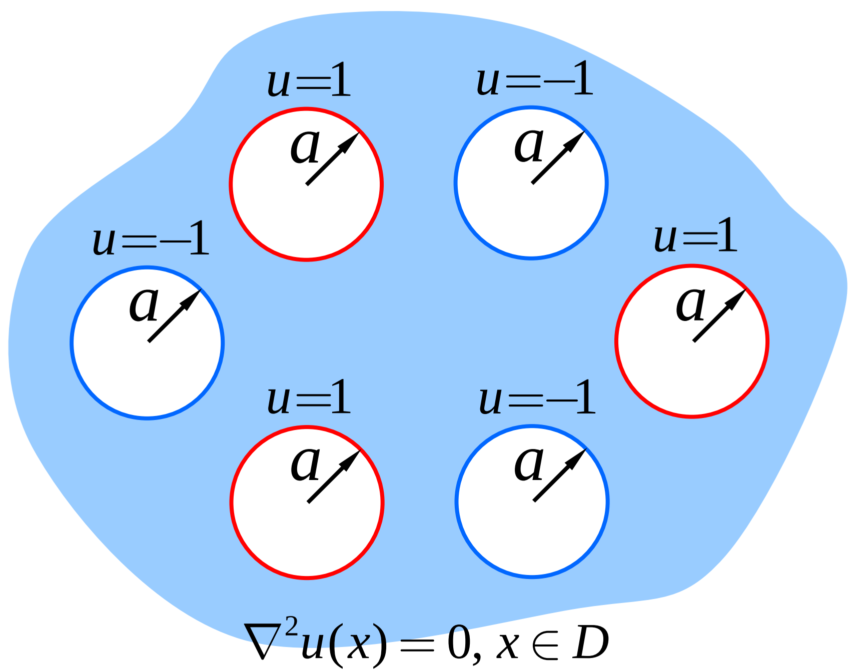
Numerical examples

Case 1 [Lebedev *et al.*]

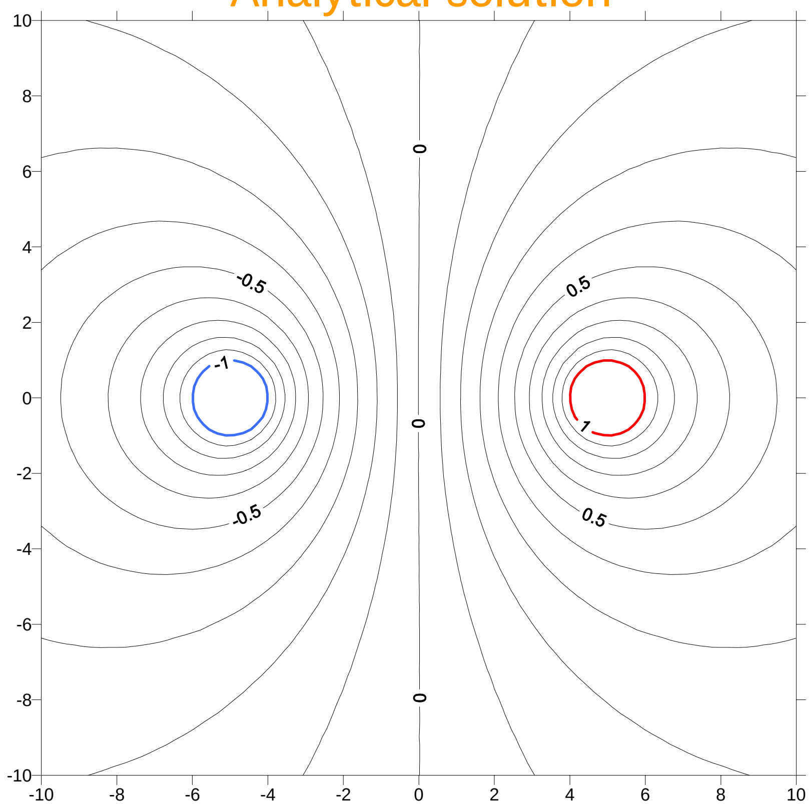


Case 2 [Onishi *et al.*]

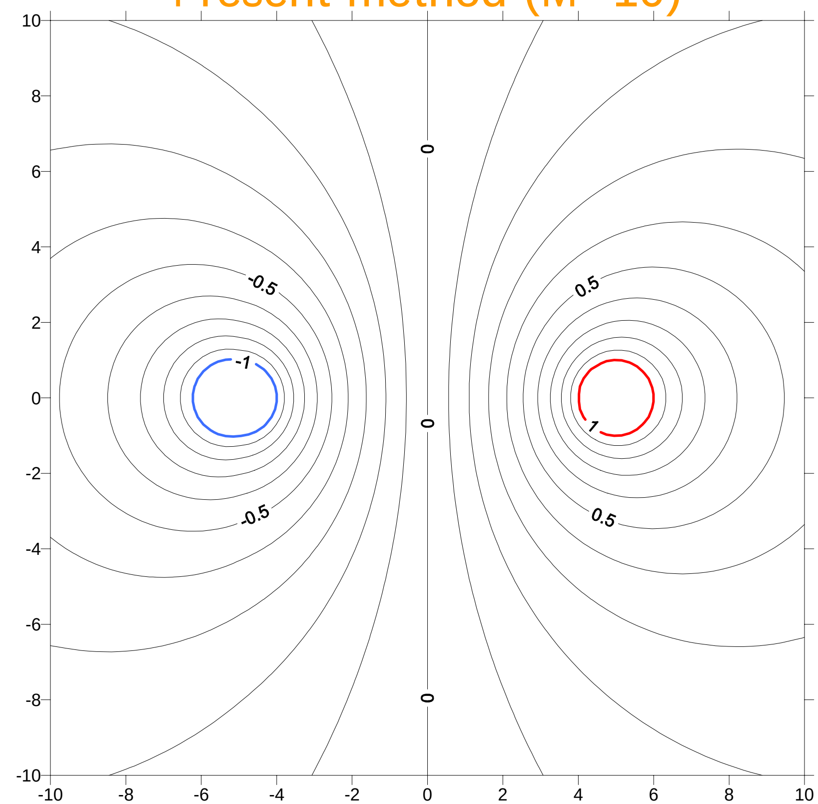


Contour of potential (case 1)

Analytical solution

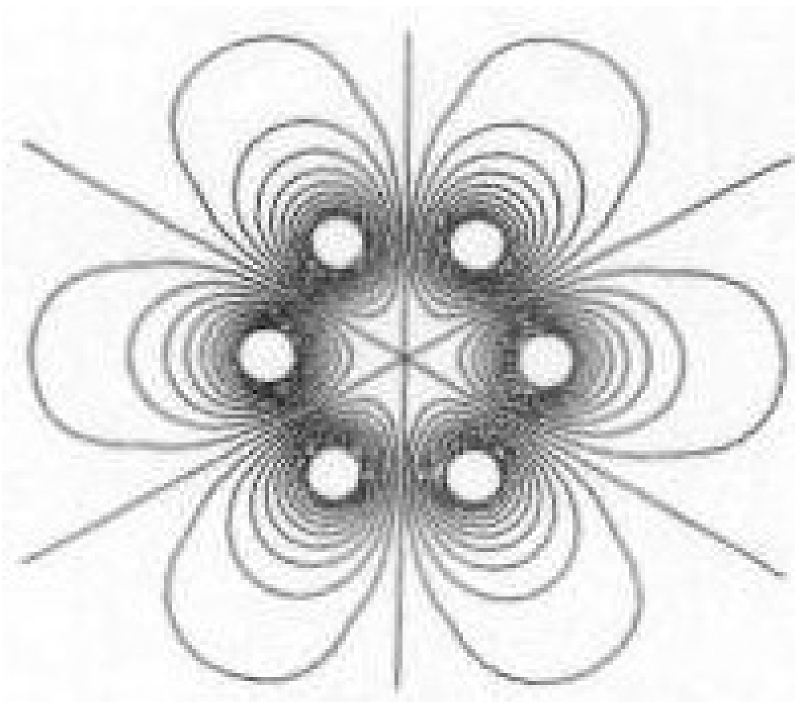


Present method (M=10)



Contour of potential (case 2)

Onishi (1991)



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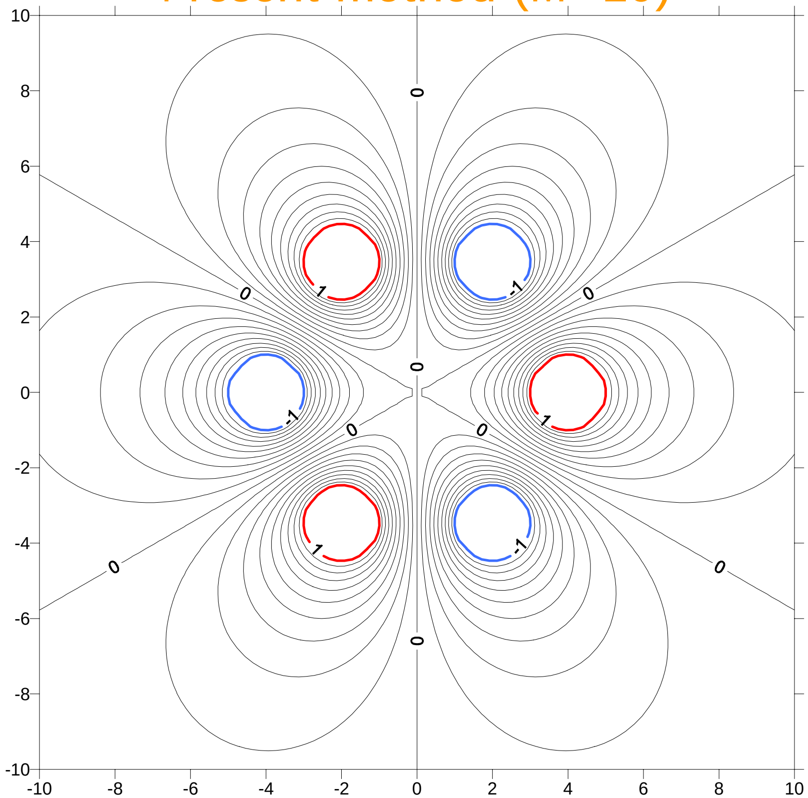

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Conclusions

- A numerical procedure using degenerate kernels, Fourier series and null-field integral equation has been successfully proposed to solve problems with circular boundaries.
- The method shows great generality and versatility for problems with multiple circular holes of arbitrary radii and position.
- Numerical results agree well with available exact solutions and Onishi's data for only few terms of Fourier series.

The end

Thanks for your kind attentions.

Your comment is much appreciated.