Support motion of a finite bar with an external spring and a damper

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ABSTRACT

In this study, the vibration problem of a finite bar with an external spring and a damper on one side subject to the support motion is analytically solved by using the method of characteristics in conjunction with the diamond rule. Special case to only spring end is also compared with the method of mode superposition. Agreement is made. Two systems, non-conservative and conservative cases, with damper and without damper, respectively, are studied. For the conservative system, both the mode superposition method and method of characteristics are employed to solve the problem, while the non-conservative system with a damper or a damper and a spring is solved by using the method of diamond rule only to avoid the complex eigen-system. For the zero-valued and infinite-valued spring stiffness, two special cases of clamped and free end are also considered. The effect of damper on the vibration response is also addressed.

Keywords: damper, support motion, diamond rule, mode superposition, characteristics

1. INTRODUCTION

Wave propagation is very important in physics and mechanics, because there are various engineering problems which can be modeled by using the wave equation. Many researchers have solved this problem by using various methods, *e.g.*, the mode superposition technique [1], the method of separation variables [2, 3, 4], the method of quasi-static decomposition [3, 5, 6], the method of the diamond rule [3, 7] or the so called method of characteristics, the image method [6], the finite element method (FEM) [8], the boundary element method (BEM) [9], and the meshless method [10], etc..

The Rayleigh-damped Bernoulli-Euler beam and the string subjected to multi-support excitation have been studied by using many methods including Stokes transformation and Cesaro sum [3, 5, 6]. D'Alembert's solution can provide an exact solution for an infinite string. Method of characteristics (Diamond rule) can be found in the textbook of Farlow [11]. It is widely employed to solve various kinds of problems, *e.g.*, water hammer [12]. The diamond rule which is based on D'Alembert's solution was proposed by John [13] in 1975 and was mainly used to solve the wave problem. The diamond rule has been employed to solve the one-dimensional vibration problem of an infinite or a semi-infinite string attached by a mass, a spring, or a damper [7], a finite string [3] and a finite bar with an external spring subjected to a support motion [5]. Besides, the animation was also given in [7].

Although the mode superposition method in conjunction with the quasi-static decomposition is a popular approach for solving the support-motion problem, it becomes tedious when the vibration system contains a damper. Three reasons can be explained. One is that the quasi-static solution is not straight forward to be obtained. Another is that the orthogonal relation of complex modes is not easily found. The other is that a complex eigen-system is required. The present solution free of mode superposition is possible since we can employ the method of characteristics in conjunction with the diamond rule for the real response in the time domain.

In this paper, we will extend the finite bar with an external spring [4] to a spring and a damper together. Three special cases of conservative system, the free end, the clamped end and the spring only, were verified by using both methods, the mode superposition method in conjunction with the quasi-static decomposition and the method of characteristics using the diamond rule. Consistency check will be done. For the non-conservative system with a damper, only the method of the characteristics is used to avoid the complex eigen-system. Finally, the effect of damper on the vibration response will be addressed in more detail.

2. PROBLEM STATEMENTS AND METHODS OF SOLUTION

Here, we consider a finite bar with different boundary types as shown in Figure 1. The governing equation for the vibration problem of finite bar is shown below:

$$c^2 \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial^2 u(x,t)}{\partial t^2}, \quad 0 < x < L, \ t > 0, \qquad (1)$$

where $c=\sqrt{E/\rho}$ and $u(x,t)$ denote the wave speed and displacement in the x direction, respectively. The symbols E, ρ and L denote Young's modulus, the density and the length of bar, respectively. The initial displacement and velocity conditions are

$$u(x,t)\big|_{t=0}=\phi(x)=0, \qquad (2)$$

$$\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=\varphi(x)=0, \qquad (3)$$

where $\phi(x)$ and $\varphi(x)$ are initial displacement and velocity functions, respectively.

The boundary condition at the left hand side ($x=0$) can be expressed by the specified support motion as follows:

$$u(0,t)=a(t). \qquad (4)$$

The boundary condition at the right hand side is given from the different boundary types in Figure 1 (a)-(e) as follows:

$$AE\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=L}=-c_d\left.\frac{\partial u(x,t)}{\partial t}\right|_{x=L}-k\,u(x,t)\big|_{x=L}, \text{ damper + spring}, \qquad (5)$$

$$AE\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=L}=-c_d\left.\frac{\partial u(x,t)}{\partial t}\right|_{x=L}, \text{ damper only}, \qquad (6)$$

$$AE\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=L}=-k\,u(x,t)\big|_{x=L}, \text{ spring only}, \qquad (7)$$

$$u(L,t)=0, \text{ clamped end}, \qquad (8)$$

$$AE\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=L}=0, \text{ free end}, \qquad (9)$$

where k denotes the spring constant, c_d denotes the damping coefficient, and A is the area of cross section.

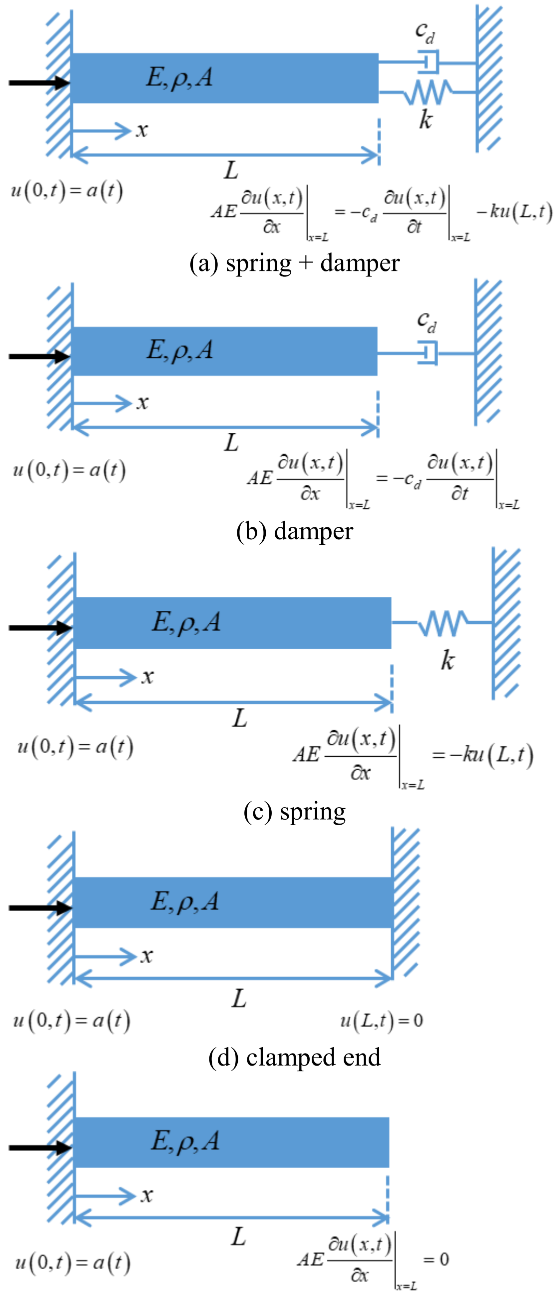


Figure 1 A finite bar with different boundary types subjected to a support motion (a) spring + damper (b) damper (c) spring (d) clamped end (e) free end

2.1 Method 1: mode superposition approach in conjunction with the quasi-static decomposition method

The solution can be decomposed into two parts:

$$u(x,t)=U(x,t)+\sum_{n=1}^{\infty}q_n(t)u_n(x), \qquad (10)$$

where $U(x,t)$ denotes the quasi-static solution, and the natural modes $u_n(x)$ weighted by the generalized coordinate, $q_n(t)$, is the generalized coordinate of dynamic contribution due to the inertia effect. The quasi-static part $U(x,t)$, satisfies the governing equation

$$AE\frac{\partial^2 U(x,t)}{\partial x^2}=0,\ 0<x<L, \tag{11}$$

and is subject to time-dependent boundary conditions at the left hand side:

$$U(0,t)=a(t), \tag{12}$$

and is subject to different time-dependent boundary conditions at the right hand side in Figure 1 (c)-(e):

$$AE\frac{\partial U(x,t)}{\partial x}\bigg|_{x=L}=-kU(x,t)\big|_{x=L},\ \text{spring end}, \tag{13}$$

$$U(x,t)\big|_{x=L}=0,\ \text{clamped end}, \tag{14}$$

$$AE\frac{\partial U(x,t)}{\partial x}\bigg|_{x=L}=0,\ \text{free end}. \tag{15}$$

By solving Eq. (11) subject to boundary conditions, we have the quasi-static solution,

$$U(x,t)=a(t)\left(1-\frac{k}{AE+kL}x\right),\ \text{spring end}, \tag{16}$$

$$U(x,t)=a(t)\left(1-\frac{x}{L}\right),\ \text{clamped end}, \tag{17}$$

$$U(x,t)=a(t),\ \text{free end}. \tag{18}$$

The nth natural mode, $u_n(x)$, satisfies the governing equation

$$u_n''(x)+\lambda_n^2 u_n(x)=0,\ n=1,2\cdots, \tag{19}$$

subject to the boundary conditions at the left hand side:

$$u_n(0)=0,\ \text{at the clamped end of } x=0, \tag{20}$$

and subject to the boundary conditions at the right hand side:

$$AE\frac{\partial u_n(x)}{\partial x}\bigg|_{x=L}=-ku_n(x)\big|_{x=L},\ \text{spring end}, \tag{21}$$

$$u_n(L)=0,\ \text{clamped end}, \tag{22}$$

$$AE\frac{\partial u_n(x)}{\partial x}\bigg|_{x=L}=0,\ \text{free end}. \tag{23}$$

By solving Eq. (19) subject to boundary conditions, we have the nth natural mode $u_n(x)$ of the eigenvalue λ_n,

$$u_n(x)=\begin{cases}\sin(\lambda_n^s x),\ n=1,2\cdots,\ \text{spring end}\\ \sin(\lambda_n^c x),\ n=1,2\cdots,\ \text{clamped end},\\ \sin(\lambda_n^f x),\ n=1,2\cdots,\ \text{free end}\end{cases} \tag{24}$$

where the superscripts, s, c and f denote spring end, clamped end and free end, respectively. The corresponding eigen-equations are obtained,

$$\lambda_n=\begin{cases}\lambda_n^s=\dfrac{-k\tan(\lambda_n L)}{AE},\ n=1,2\cdots,\ \text{spring end}\\ \lambda_n^c=\dfrac{n\pi}{L},\ n=1,2\cdots,\ \text{clamped end}\\ \lambda_n^f=\dfrac{(2n-1)\pi}{2L},\ n=1,2\cdots,\ \text{free end}\end{cases}, \tag{25}$$

and the corresponding natural frequency is

$$\omega_n=\lambda_n\sqrt{E/\rho},\ n=1,2\cdots. \tag{26}$$

The orthogonality of the eigenfunction is

$$\int_0^L u_n(x)u_m(x)dx=\delta_{nm}N_n,\ n=1,2,3\cdots,\ m=1,2,3\cdots, \tag{27}$$

where δ_{nm} is the Kronecker delta and

$$N_n=\frac{L}{2}-\frac{\sin(2\lambda_n L)}{4\lambda_n}. \tag{28}$$

Substituting Eq. (10) into Eq. (1), we obtain

$$\sum_{n=1}^{\infty}\left[\ddot{q}_n(t)+\omega_n^2 q_n(t)\right]u_n(x)=-\ddot{U}(x,t). \tag{29}$$

After considering the initial conditions, we have

$$N_n q_n(0)=-\int_0^L U(x,0)u_n(x)dx=F_n(0),\ (\text{n no sum}), \tag{30}$$

$$N_n \dot{q}_n(0)=-\int_0^L \dot{U}(x,0)u_n(x)dx=\dot{F}_n(0),\ (\text{n no sum}). \tag{31}$$

Regarding the detailed derivation of $q_n(t)$ and the series solution for the displacement, readers can consult with the paper of Chen et al. [4].

2.2 Method 2: method of characteristics in conjunction with the diamond rule

By employing the method of characteristic line, we can assume the general solution of 1D wave equation in Eq. (1) as

$$u(x,t)=P(x+ct)+Q(x-ct), \tag{32}$$

where $P(x+ct)$ and $Q(x-ct)$ are specified functions to match initial conditions in Eqs. (2) and (3). The functions $P(x+ct)$ and $Q(x-ct)$ represent a left-going-traveling wave and a right-going-traveling wave, respectively. By satisfying Eqs. (2) and (3) for Eq. (32), the D'Alembert's solution for a certain region is expressed as

$$u(x,t)=\frac{1}{2}\left[\phi(x+ct)+\phi(x-ct)\right]+\frac{1}{2c}\int_{x-ct}^{x+ct}\varphi(\tau)d\tau, \tag{33}$$

where $\phi(x)$ and $\varphi(x)$ are functions of initial displacement and velocity, respectively. Two groups of characteristic lines from Eq. (33) are included in the solution of the wave equation. Moreover, the two groups of parallel characteristic lines can form a parallelogram in the space-time plane as shown in Figure 2. Based on the D'Alembert's solution, we have the equation of the diamond rule [4, 6], as shown below:

$$u_A+u_B=u_C+u_D, \tag{34}$$

where u_A, u_B, u_C and u_D denote the displacement at the four points A, B, C and D, respectively. Several parallel characteristic lines separate the domain into many regions of the space-time plane as shown in Figure 3. The diagrams of calculating the displacement by using the diamond rule in the regions I, II, III, IV, V and VI are given in Figure 4. The displacements in the former six regions are given below:

$$u_{\text{I}}(x,t)=0,\ (x,t)\in \text{I}, \tag{35}$$

$$u_{\text{II}}(x,t)=a\left(\frac{ct-x}{c}\right),\ (x,t)\in \text{II}, \tag{36}$$

$$u_{\text{III}}(x,t)=r_1\left(\frac{x+ct-L}{c}\right),\ (x,t)\in \text{III}, \tag{37}$$

$$u_{\text{IV}}(x,t)=a\left(\frac{ct-x}{c}\right),\ (x,t)\in \text{IV}, \tag{38}$$

$$u_{\text{V}}(x,t)=a\left(\frac{ct-x}{c}\right),\ (x,t)\in \text{V}, \tag{39}$$

$$u_{\mathrm{VI}}(x,t)=a\left(\frac{ct-x}{c}\right)-a\left(\frac{x+ct-2L}{c}\right)+r_2\left(\frac{x+ct-L}{c}\right),\ (x,t)\in \mathrm{VI}, \quad (40)$$

where the simple form of Eqs. (39)-(40) is due to zero $r_1(t)$ in silent regions of I and III and zero initial displacement $\phi(x)$ and velocity $\varphi(x)$. Following the same procedure, the marching scheme of the time-space plane region and solution can be done. Then, $r_1(t)$ and $r_2(t)$ denote the displacements of $u(L,t),\ 0\le t\le L/c$ and $u(L,t),\ L/c\le t\le 2L/c$, respectively, which can be obtained from the condition of force equilibrium at $x = L$,

$$AE\left.\frac{\partial u_{\mathrm{III}}(x,t)}{\partial x}\right|_{x=L}=-c_d\left.\frac{\partial u_{\mathrm{III}}(x,t)}{\partial t}\right|_{x=L}-k\,u_{\mathrm{III}}(x,t)\big|_{x=L}, \quad (41)$$

$$AE\left.\frac{\partial u_{\mathrm{VI}}(x,t)}{\partial x}\right|_{x=L}=-c_d\left.\frac{\partial u_{\mathrm{VI}}(x,t)}{\partial t}\right|_{x=L}-k\,u_{\mathrm{VI}}(x,t)\big|_{x=L}, \quad (42)$$

for the case of spring and damper end. Thus, we can determine $r_1(t)$ by using Eq. (37) to satisfy Eq. (41). The displacement at $x = L$, $u_{\mathrm{III}}(L,0)$ and $u_{\mathrm{I}}(L,0)$, must satisfy the displacement continuity. Then, we have

$$r_1(t)=0,\ 0\le t\le L/c. \quad (43)$$

Similarly, the response of $r_2(t)$ can be obtained by using Eq. (40) to satisfy Eq. (42). By solving the corresponding first-order ODE for $r_2(t)$ at the end of spring and damper as shown below:

$$r_2'(t)+\frac{kc}{AE+c_dc}r_2(t)=\frac{2AE}{AE+c_dc}a'\left(\frac{ct-L}{c}\right),\ L/c\le t\le 2L/c, \quad (44)$$

we can obtain

$$r_2(t)=e^{-\frac{kc}{AE+c_dc}t}\int e^{\frac{kc}{AE+c_dc}t}\cdot\frac{2AE}{AE+c_dc}a'\left(\frac{ct-L}{c}\right)dt,\ L/c\le t\le 2L/c, \quad (45)$$

after using the integral factor, where the undetermined constant can be determined by satisfying the displacement continuity of solution in the region IV and VI at $(x,t)=(L,L/c)$ as shown in Figure 4.

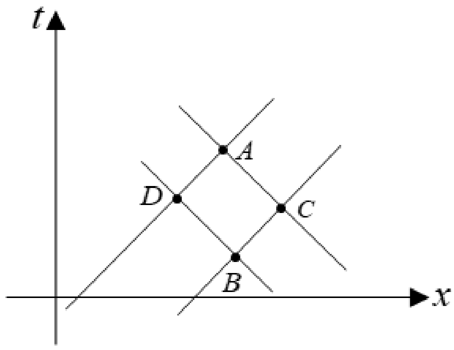


Figure 2 The diamond rule of $u_A+u_B=u_C+u_D$

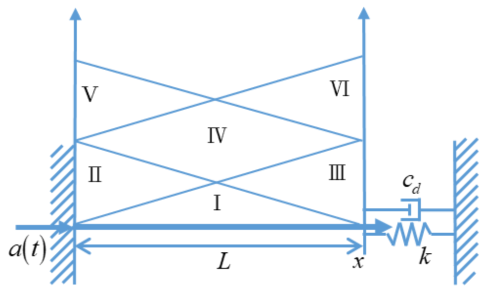


Figure 3 Space-time regions separated by using the characteristic line.

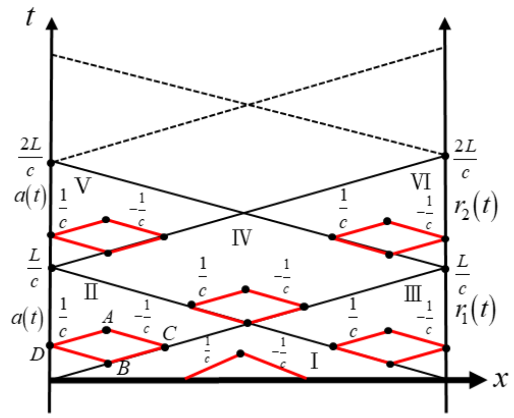


Figure 4 Space-time regions, I, II, III, IV, V and VI and the diamond rule.

3. ILLUSTRATIVE EXAMPLES

Case (a) of a finite bar with an external spring and a damper subjected to a support motion is considered. The model parameters are given as follows: $c=1\,m/s$, $AE=1\,N$, $L=7\,m$, $k=2\,N/m$ and $c_d=2\,N\cdot s/m$. By setting the support motion,

$$a(t)=\sin(t), \quad (46)$$

the solution of this approach can be obtained as shown in the following subsection.

(a) Method of characteristics in conjunction with the diamond rule

By substituting model parameters c, A, E, L, k, c_d and Eq. (46) into Eqs. (35)-(40), we have

$$u_{\mathrm{I}}(x,t)=0,\ (x,t)\in \mathrm{I}, \quad (47)$$

$$u_{\mathrm{II}}(x,t)=\sin(t-x),\ (x,t)\in \mathrm{II}, \quad (48)$$

$$u_{\mathrm{III}}(x,t)=0,\ (x,t)\in \mathrm{III}, \quad (49)$$

$$u_{\mathrm{IV}}(x,t)=\sin(t-x),\ (x,t)\in \mathrm{IV}, \quad (50)$$

$$u_{\mathrm{V}}(x,t)=\sin(t-x),\ (x,t)\in \mathrm{V}, \quad (51)$$

$$u_{\mathrm{VI}}(x,t)=\sin(t-x)-\sin(x+t-14)+r_2(x+t-7),\ (x,t)\in \mathrm{VI}, \quad (52)$$

where

$$r_2(t)=\frac{\cos(7-t)-\sin(7-t)}{2}-\frac{e^{(7-t)}}{2},\ 7\le t\le 14. \quad (53)$$

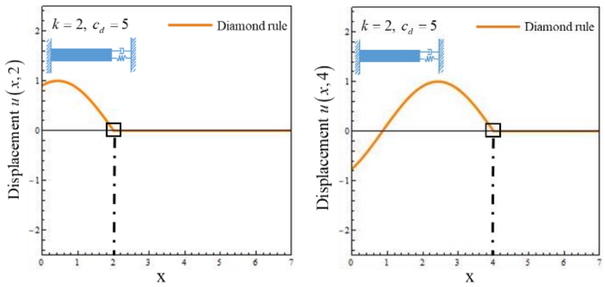
The displacement profiles with the silent area for $t = 2$ and 4 sec. by using the diamond rule are shown in Figure 5 (a)-(b), respectively. It matches the silent response begins at $x = 2$ and 4 m to the end of bar ($x = 7$ m), for the time when $t = 2$ and 4 sec.. In Figure 6, shadow regions, I and III, denote the dead zone. It is found that the slopes are discontinuous at $x = 2$ and 4 m when $t = 2$ and 4 sec., respectively. Two discontinuities occur at the location of (2,2) and (4,4) in the x-t plane as shown in Figure 6. As theoretically predicted, the two discontinuities of the slope really occur at the positions of (2,2) and (4,4), on the characteristic line. The displacement profiles for $t = 8$ and 10 sec. by using the diamond rule are shown in Figure 7 (a)-(b), respectively. It is found that the slopes are discontinuous at $x = 6$ and 4 m when $t = 8$ and 10 sec., respectively. Two slope discontinuities occur at the locations of (6,8) and (4,10) in the *x*-*t* plane as shown in Figure 8. This finding matches well from the mathematical requirement that the discontinuity must occur at the position on the characteristic line [14]. The displacement response history at x = 5 m by using the diamond rule is also shown in Figure 9.

The displacement profiles for the case of only damper at t = 8 and 10 sec. by using the diamond rule are shown in Figure 10 (a)-(b), respectively.

(b) Mode superposition approach and the diamond rule

Now, we consider the special case (c) of zero damping (only spring end) and we can compare the solution with that of using the mode superposition method [4]. The result is shown in Table 1. Convergence test for the displacement profile and time history are both verified. To reach the accuracy for comparing with the exact solution obtained by using the diamond rule, a number of terms for the series are required for the displacement profile and the time history as shown in Table 2 where the time history response and the displacement profile are obtained by using 2, 4, 6, 8, 10 and 15 modes. The error of 0.1 % is reached by using 15 modes.

Besides, more special cases of infinite-valued (case d) and zero-valued (case e) spring stiffness are employed to model the clamped and free end, respectively. This results are shown in Table 3 and Table 4 for clamped and free ends, respectively. To check the ability of capturing the silent area (dead zone) for the mode superposition method, the response is shown in Figure 11, where it indicates that silent area is well captured if a sufficient number of modes is considered. In this case, 15 modes are required.



(a) t = 2 sec (b) t = 4 sec
Figure 5 Displacement profiles by using the diamond rule (spring + damper)

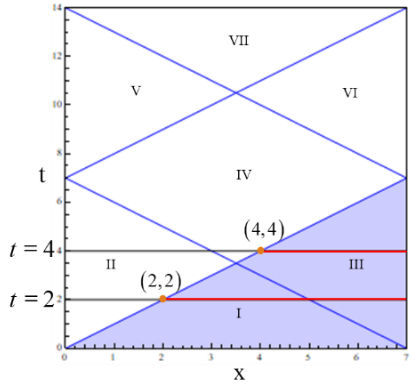
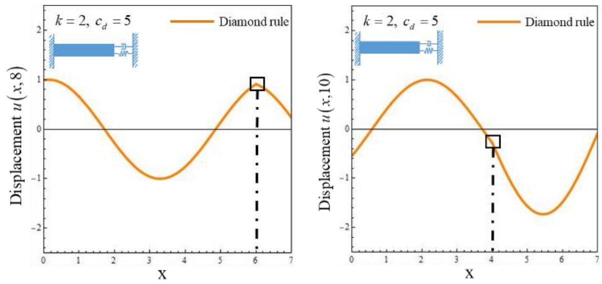


Figure 6 The locations of slope discontinuities at (2,2) and (4,4), where the shadow region denotes the dead zone



(a) t = 8 sec (b) t = 10 sec
Figure 7 Displacement profiles by using the diamond rule (spring + damper)

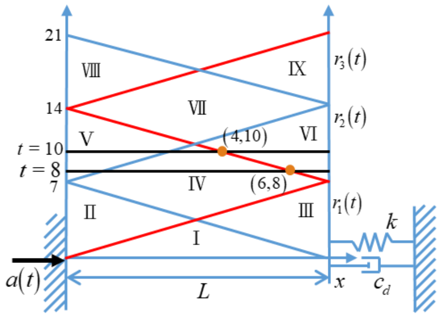


Figure 8 The locations of slope discontinuities (6,8) and (4,10)

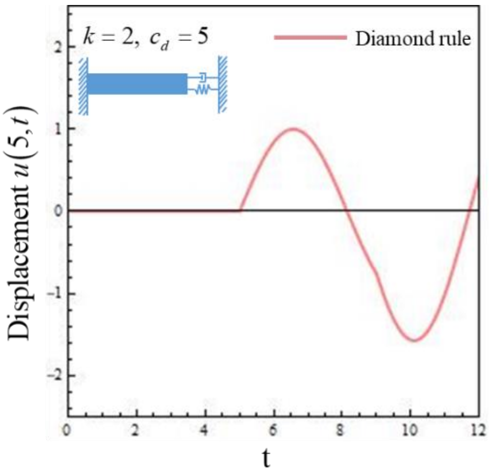
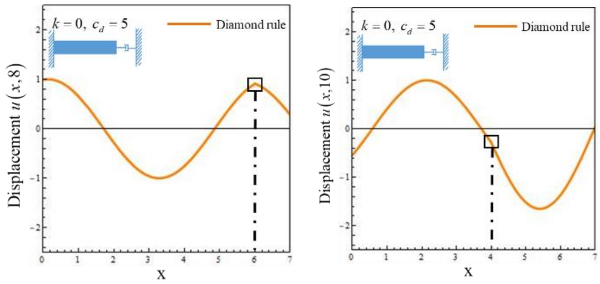


Figure 9 Displacement history at x = 5 m



(a) $t = 8$ sec (b) $t = 10$ sec

Figure 10 Displacement profiles by using the diamond rule (damper only)

Table 1 The vibration response at t=10 sec (spring only)

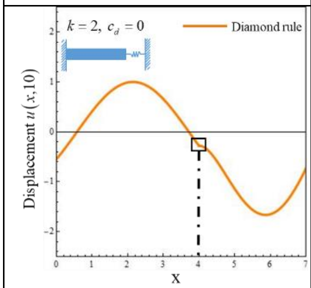
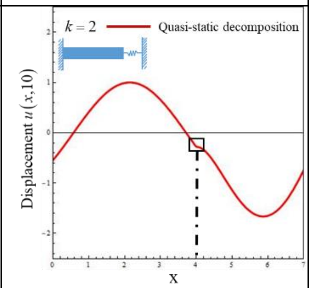
Present method	Chen et al. [4]
Diamond rule	Mode superposition method
	

Table 2 Convergence of the displacement profile and time history and the relative error plot

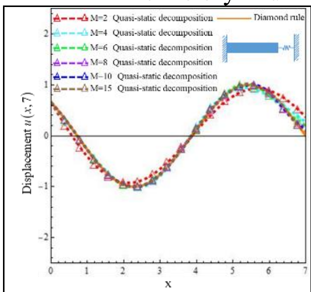
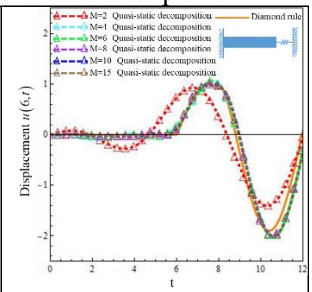
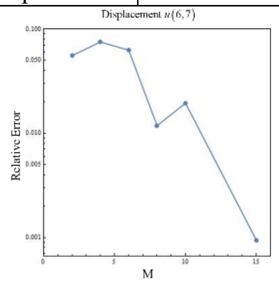
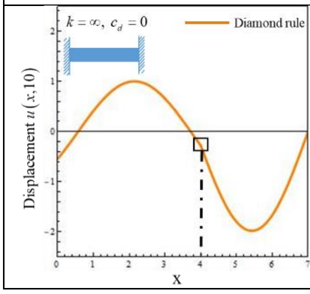
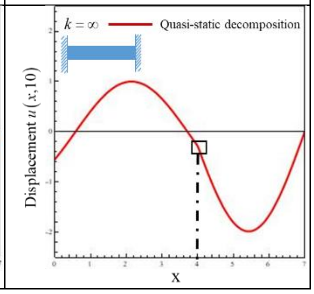
 Convergence of the displacement profile	 Convergence of the time history
 Relative error	

Table 3 The vibration response profile at t =10 sec (clamped end, $k=\infty$)

Present method	Chen et al. [4]
Diamond rule	Mode superposition method
	

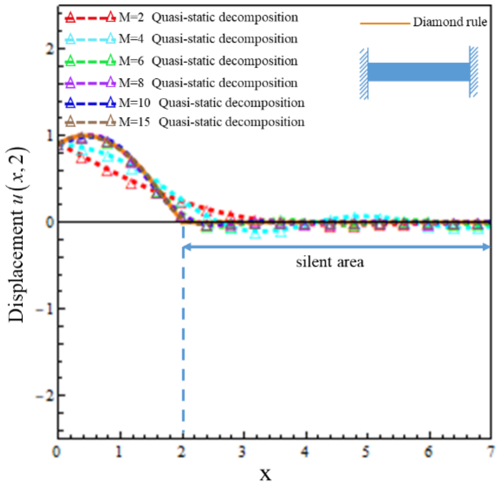
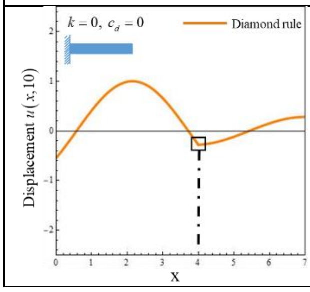
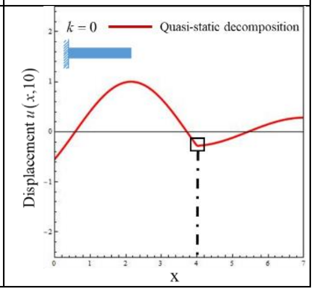


Figure 11 Convergence of the silent area at t =2 sec

Table 4 The vibration response profile at t =10 sec (free end, $k=0$)

Present method	Chen et al. [4]
Diamond rule	Mode superposition method
	

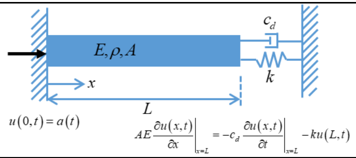
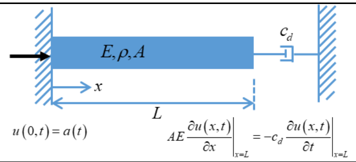
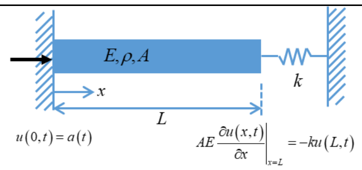
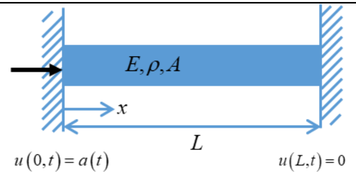
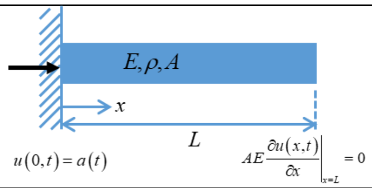
4. CONCLUSION

In this paper, we have analytically solved the direct problem of the longitudinal vibration analysis of a finite bar with an external spring and a damper on one side and the support motion on the other clamped side by using the diamond rule. The slope discontinuity occurs at the position on the characteristic line as mathematically predicted. The effect of damper on the vibration response is also addressed. To avoid the complex eigenvalues in the frequency domain for traditionally solving the problem with the damped boundary, the method of characteristics in conjunction with the diamond rule was successfully employed to solve the problem containing the damped boundary in the time domain. Table 5 summaries the comparison of advantages and disadvantages of the two approaches. Finally, the five systems by using either the mode superposition method or approach of the diamond rule, is summarized in Table 6.

Table 5 Comparison of the two approaches for the vibration problem of a finite rod

Method / Item analysis	Mode superposition method in conjunction with the quasi-static decomposition	Method of characteristics in conjunction with the diamond rule
Solution form	Series solution (continuous)	Exact solution (continuous)
Advantage	Without dividing the space-time region to represent the corresponding displacement response	1. Without the truncation error of finite term of series sum 2. It can analytically capture the dead zone 3. General approach for either conservative or non-conservative system 4. Suitable for support excitation of short duration, *e.g.*, earthquake input
Disadvantage	3. Error due to truncation series in the real computation 2. Convergence test is required 3. Complex eigenvalue and eigenequation are required for a damped system	Previous stage error propagates to the later response

Table 6 Comparison of five systems, including conservative and non-conservative cases, using two approaches

Diamond rule	Conservative cases	Mode superposition method
V		X
V		X
	Non-conservative cases	
V		V
V		V
V		V
X : not available in this paper since complex eigen-system is required		

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含邊彈簧及阻尼器之有限桿支承運動

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摘要

本研究使用鑽石法則解決了含邊彈簧及邊阻尼器之有限桿受支承運動的振動問題。在僅有邊界彈簧的特例中同時與模態疊加法進行比。分別探討非保守系統(含邊阻尼)與保守系統(不含邊阻尼)。對於保守系統而言，使用模態疊加法和特徵線法兩種方法均可進行求解比對。然而對於非保守系統(含邊阻尼、含邊阻尼與邊彈簧)，本文避開複數特徵系統而僅使用鑽石法則進行求解。針對彈簧剛度為無限大(固定端)以及零(自由端)的例子也進行探討。阻尼器對振動反應的影響也一併討論。

關鍵詞：阻尼，支承運動，鑽石法則，模態疊加，特徵線