中華民國第七屆結構工程研討會
The 7th National Conference on Structure Engineering
Chung-Li, Taiwan, R. O. C., 22-24 Aug. 2004

邊界元素法中退化問題的統一推導

A Unified Formulation for Degenerate Problems in BEM

陳正宗 [1] 陳桂鴻 [2] 陳義麟 [3]

Jeng-Tzong Chen, I-Lin Chen, Kue-Hong Chen

[1] 國立海洋大學河海工程學系教授

[2] 稻江科技暨管理學院資訊管理學系助理教授

[3] 國立高雄海洋科技大學造船工程學系副教授

[1]Department of Harbor and River Engineering, National Taiwan Ocean University

[2]Department of Information Management, Toko University

[3]Department of Naval Architecture, National Kaohsiung Marine University

摘要

本文提出邊界元素法中有關退化問題的統一觀點，包含了退化邊界、退化尺度、假根、虛擬頻率及角點問題。對於這些退化問題我們可以歸納均源自於影響係數矩陣的秩降現象，為探討這些解不唯一問題採用 Fredholm 二擇一定理以及奇異值分解法的技巧來處理，而 SVD 補充行與補充列技巧亦將同時被考慮。邊界元素法所建構的影響係數矩陣在經過奇異值分解後，產生之左、右酉矩陣與真假根、虛擬頻率與虛擬模態之相關性，以 Fredholm 二擇一定理來檢驗。我們成功地舉數個例題來驗證統一推導的可行性。吳大猷獎助計畫的第一年計畫針對退化邊界以及退化尺度的問題，第二年則著重於單連通及多連通問題的真假特徵值問題。最後，虛擬頻率的問題將於第三年來探討。本文將研究成果所發表論文作一彙整。

關鍵字: 邊界元素法； 退化尺度； 退化邊界； 退化核； 假根；虛擬頻率；Fredholm 二擇一定理； 奇異值分解法-補充行與補充列； 虛擬模態。

Abstract

We provide a perspective on the degenerate problems, including degenerate boundary, degenerate scale, spurious eigensolution, fictitious frequency and corner problems, in the boundary integral formulation. All the degenerate problems are found to originate from the rank deficiency in the influence matrix. Both the Fredholm alternative theorem and singular value decomposition (SVD) technique are employed to study the degenerate problems. Updating terms and updating documents of the SVD technique are utilized. The roles of right and left unitary matrices of SVD in BEM and their relations to true, spurious and fictitious modes are examined by using the Fredholm alternative theorem. A unified method for dealing with the degenerate problems in BEM is proposed. Several examples are given to show the validity of the unified formulation. In the first year, of Wu Ta-You Award Project, degenerate boundary and degenerate scale problems were studied. For the second year, we focused on the spurious eigenvalues for both simply-connected and multiply-connected problems. Finally, the fictitious frequencies were revisited in the third year. The published papers are summarized in the references.

Key Words: boundary element method; degenerate scale; degenerate boundary; spurious eigenvalue; fictitious frequency; Fredholm alternative theorem; SVD updating term and document; fictitious mode.

E-Mail：jtchen@mail.ntou.edu.tw
Tel：02-24622192 ext. 6140 or 6177

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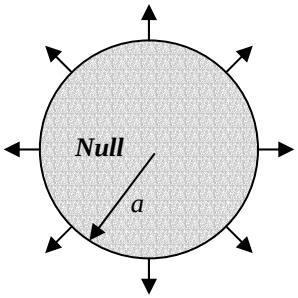
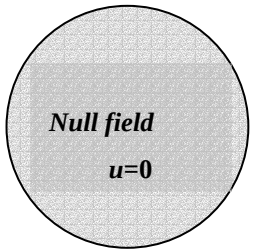
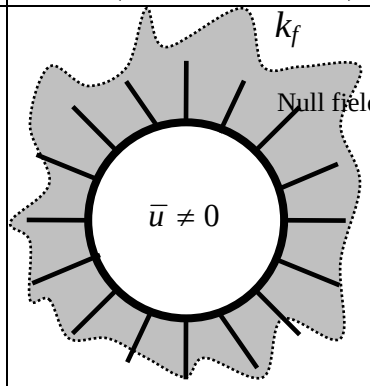
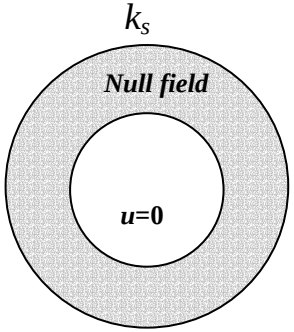
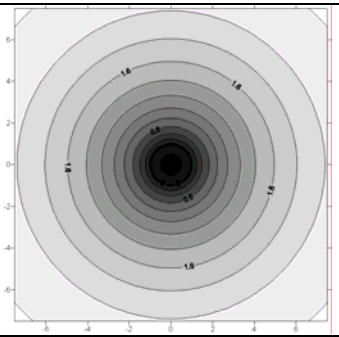
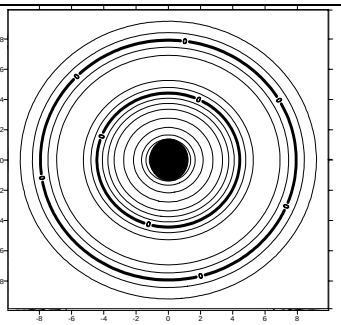
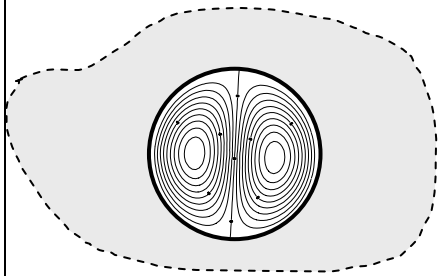
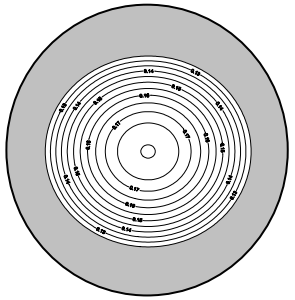
Table 1 Pitfalls of BEM and its treatment

Pitfalls in BEM	Treatment
Degenerate boundary [1~21]	Dual BEM (hypersingularity) Multi-domain BEM Conventional BEM + SVD
Degenerate scale [22~25]	Addition of rigid body term Hypersingular formulation CHEEF method SVD updating technique Pseudo-inverse
Fictitious frequency [26~32]	Burton and Miller approach CHIEF method SVD updating technique GSVD
Spurious eigenfrequency [33~48]	Burton and Miller method SVD updating technique Dual BEM Subdomain method GSVD CHEEF method (simply-connected) CHIEF method (multiply-connected)
Corner singularity [49~51]	Double node technique Hypersingular formulation

Table 2 Null fields in the nonuniqueness problem using BEM and BIE

	Degenerate scale (potential problem)	Spurious eigenvalues (interior acoustics)	Fictitious frequencies (exterior acoustics)	Spurious eigenvalues (multiply-connected acoustics and plate)
Figure sketch	 Special scale (a=1) $\nabla^2 u=0$	 **Simply-connected domain** $\nabla^2 u = -k^2 u$	 $\nabla^2 u+k^2 u=0$	 **Multiply-connected domain** $\nabla^2 u = -k^2 u$ (*acoustics*) $\nabla^4 u = \lambda^4 u$ (*plate*)
Null-field				
Rank deficiency