Dual boundary element analysis for cracked bars under torsion

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Motivation of this research

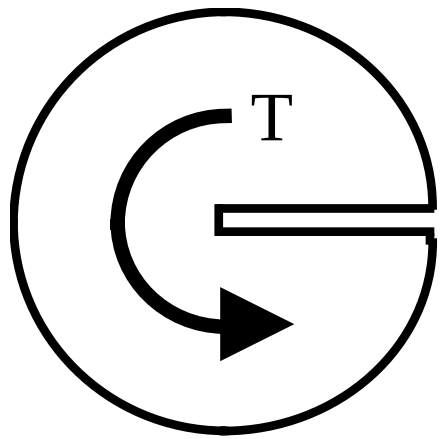
- Discretization on the given boundaries only
- Regularization for the ill-posedness
- Transformation of domain integral to boundary integral for torsional rigidity
- discussions on the torsion rigidity due to the influence of different crack lengthes and orientations

Statement of the problem

Sain-Venant torsional problem

Governing equation:

$$\nabla^2 \overline{\psi}^*(x) = 0 \quad , \quad x \in D$$

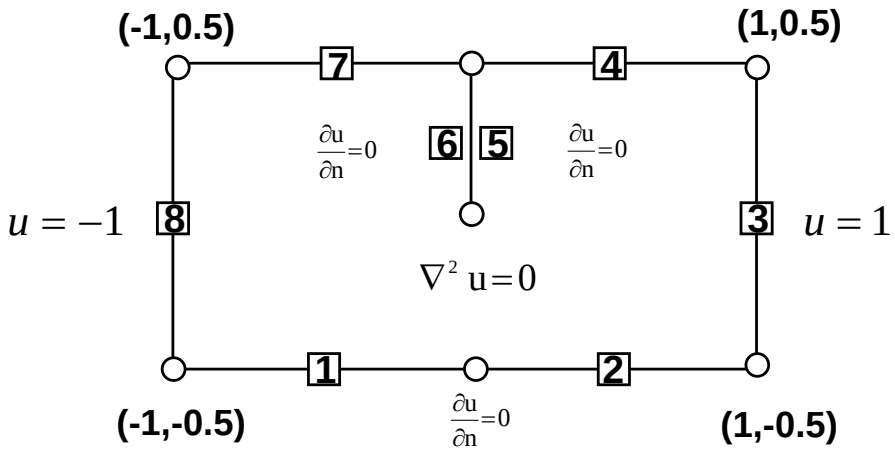


Boundary conditions

$$\psi^*(x) = f(x) \quad , \quad x \in Bu$$

$$\frac{\partial \psi^*(x)}{\partial n(x)} = g(x) \quad , \quad x \in Bt$$

Ill-posedness in the conventional BEM modelling and its regularization using dual BEM



○ **geometry node**

[N] **the Nth constant or linear element**

$$[U]\{t\}=[T]\{u\}$$

$$[L]\{t\}=[M]\{u\}$$

[U] =

	5(+)				6(+)			
5(+)	−1.693	−0.045	0.471	0.347	−0.054	−0.054	0.039	-0.335
	−0.045	−1.693	−0.335	0.039	−0.054	−0.054	0.347	0.471
	0.445	−0.335	−1.693	−0.335	0.019	0.019	0.445	0.703
	0.347	0.039	−0.335	−1.693	−0.281	−0.281	-0.045	0.471
6(+)	−0.081	−0.081	0.063	−0.638	−1.193	−1.193	-0.638	0.063
	−0.081	−0.081	0.063	−0.638	−1.193	−1.193	-0.638	0.063
	0.039	0.347	0.471	−0.045	−0.281	−0.281	-1.693	-0.334
	−0.335	0.445	0.703	0.445	0.019	0.019	-0.335	-1.693

[T] =

$n(s)$		5(+)				6(-)		
	−π	0.000	0.588	0.519	−0.321	0.321	0.927	1.107
	0.000	−π	1.107	0.927	0.321	−0.321	0.519	0.588
	0.219	1.107	−π	1.107	0.464	−0.464	0.219	0.490
	0.519	0.927	1.107	−π	0.785	−0.785	0.000	0.588
5(+)	0.927	0.927	0.888	1.326	−π	−π	1.326	0.888
6(+)	0.927	0.927	0.888	1.326	−π	−π	1.326	0.888
	0.927	0.519	0.588	0.000	−0.7854	0.785	-π	1.107
	1.107	0.219	0.490	0.219	−0.464	0.464	1.107	-π

[L] =

$n(x)$		5(+)				6(+)		
	π	0.000	0.184	0.519	0.458	0.458	0.927	0.805
	0.000	π	0.805	0.927	0.458	0.458	0.519	0.184
	0.612	0.805	π	0.805	0.464	0.464	0.612	0.490
	0.519	0.927	0.805	π	0.347	0.347	0.000	0.184
5(+)	−0.511	0.511	0.888	1.417	π	−π	-1.417	-0.888
6(-)	0.511	−0.511	−0.888	−1.417	−π	π	1.417	0.888
	0.927	0.519	0.184	0.000	0.347	0.347	π	0.805
	0.805	0.612	0.490	0.612	0.464	0.464	0.805	π

[M] =

$n(x)$	$n(s)$	5(+)				6(-)		
	4.000	−1.333	−0.205	−0.061	0.600	−0.600	-0.800	-1.600
	−1.333	4.000	−1.600	−0.800	−0.600	0.600	-0.061	-0.205
	−0.282	−1.600	4.000	−1.600	−0.400	0.400	-0.282	-0.236
	−0.061	−0.800	−1.600	4.000	−1.000	1.000	-1.333	-0.205
5(+)	0.853	−0.853	−0.715	−3.765	8.000	−8.000	3.765	0.715
6(-)	−0.853	0.853	0.715	3.765	−8.000	8.000	-3.765	-0.715
	−0.800	−0.062	−0.205	−1.333	1.000	−1.000	4.000	-1.600
	−1.600	−0.282	−0.235	−0.282	0.400	−0.400	-1.600	4.000

Results and discussions

- Index of Ill-posedness
- Torsional rigidity for different crack lengthes
- Torsional rigidity for different crack orientations

Transformation of domain integral into boundary integral for torsional rigidity

$$M_z = \iint_A (x\tau_{yz} - y\tau_{xz})\, dxdy$$

$$= -\alpha G \oint_B \tilde{\psi} \bullet \frac{\partial \psi}{\partial n} dB - \frac{\alpha G}{16} \oint \frac{\partial \{(x^2 + y^2)^2\}}{\partial n} dB$$

$$where \quad \psi = \frac{1}{2}(x^2 + y^2) \quad , \quad \psi^* = \psi + \tilde{\psi}$$

$$\tau_{yz} = -\alpha G \frac{\partial \psi}{\partial x} \quad , \quad \tau_{xz} = \alpha G \frac{\partial \psi}{\partial y}$$

Conclusions

- The torsion problem with a crack have been successfully solved by using dual BEM
- The domain integrals for torsion rigidity are transformed to boundary integrals
- The torsional rigidity for different crack lengthes and crack orientations are examined and compared with analytical results and FEM results