CHIEF and CHEEF

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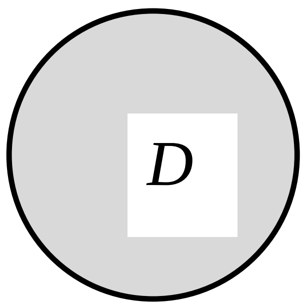
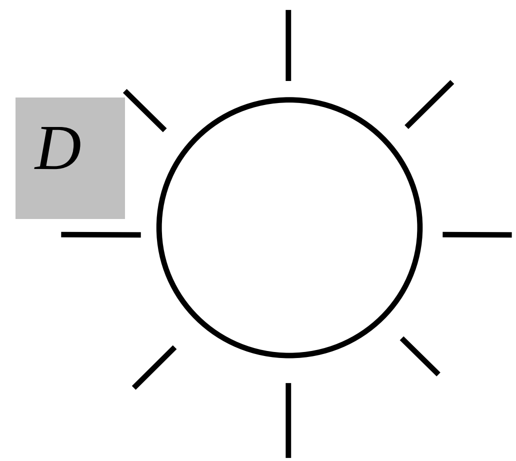
Interior and Exterior Helmholtz problems



interior eigenproblem

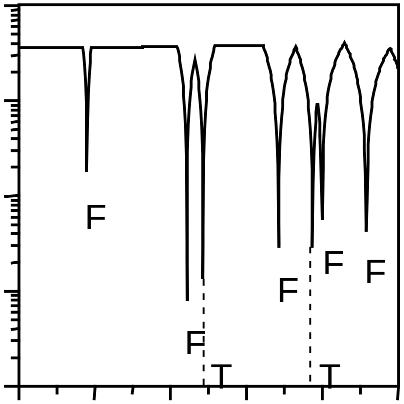


exterior radiation/scatting problem

Problems using BEM

Interior problem

Spurious eigenvalues

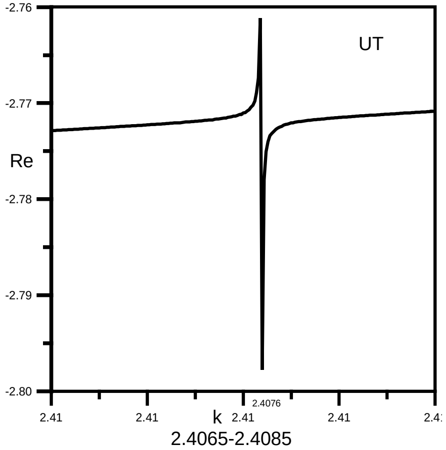


(Real-part or Imaginary-part BEM)

Complex-valued BEM (OK)

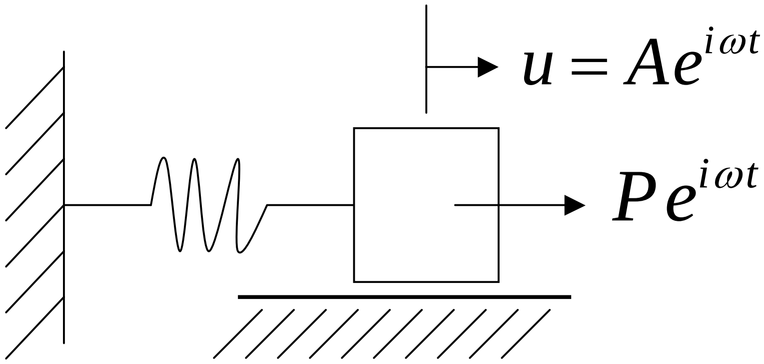
Exterior problem

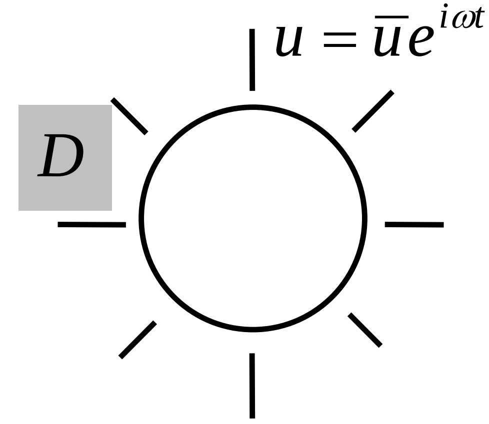
Fictitious eigenvalues



(UT or LM method)

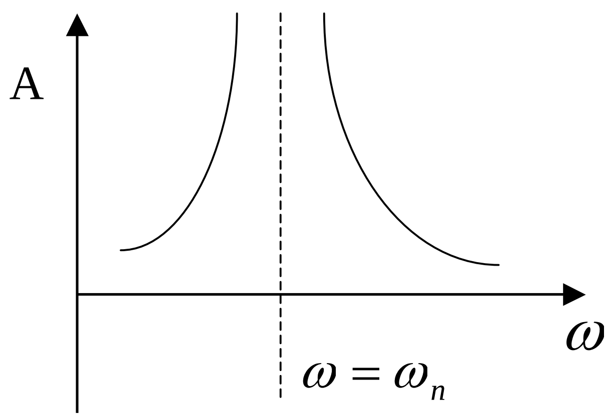
Dual BEM (Burton & Miller) OK

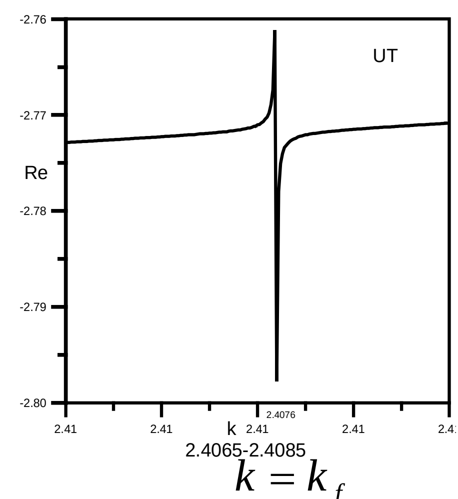




Physical resonance

Numerical resonance



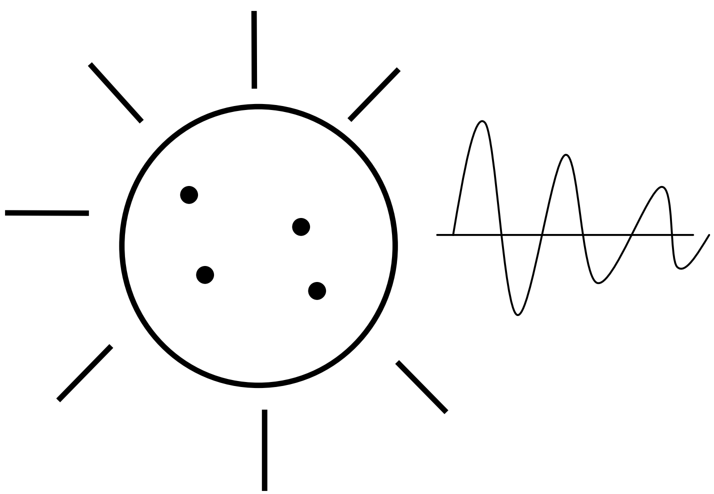


ω_n : natural frequency

k_f : fictitious wave number

CHIEF

Combined Helmholtz <u>*Interior*</u> integral Equation Formulation

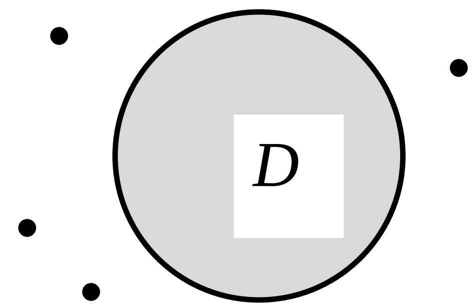


• Sampling points

US Navy: 10 interior points

CHEEF

Combined Helmholtz <u>*Exterior*</u> integral Equation Formulation



• Sampling points

Adopted one or two points

Analytical study on failure points

UT equation for boundary point

$$\pi u = C.P.V\int_B T(s,x)u(s)dB(s) - R.P.V\int_B U(s,x)t(s)dB(s)$$

for the Dirichlet problem and discretizing the boundary *B*,

$$[U(k)]\{t\} = 0 \Rightarrow \begin{cases} k_T, \{t_T\} & \textit{True eigenvalues, boundary eigenmodes} \\ k_S, \{t_S\} & \textit{Spurious eigenvalues, boundary eigenmodes} \end{cases} \quad (1)$$

The true eigenvalues satisfy $J_n(k\rho) = 0$

The spurious eigenvalues satisfy $Y_n(k\rho) = 0$

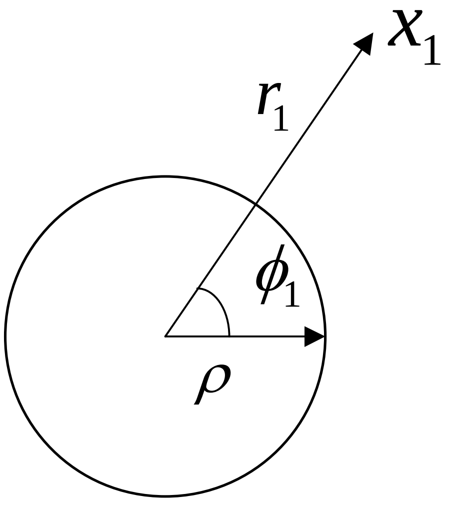
where $[U(k)]$ is the influence coefficient matrix and $\{t\} = \{e^{inm\Delta\theta}\}$

ρ is radius of the circular cavity.

1. UT equation for one exterior point to detect spurious solutions with multiplicity 1,

$0=\int_B U(s,x)t(s)dB(s)$ for exterior point,

$$\tilde{v}_1^T(k)\tilde{t}=0 \qquad (2)$$



Combining Eq.(1) and (2) equation, we have

$$\begin{bmatrix} U(k) \\ \tilde{v}_1^T(k) \end{bmatrix}\{t\}=\{0\}$$

where $\tilde{v}_1^T=(v_1^1,v_1^2,v_1^3,\cdots,v_1^{2N})$ is the row vector of the influence matrix by collocating the exterior point x_1 .

(a). If the spurious eigenvalue is a single root,

the additional constrain $\tilde{v}_1^T\{t\}$ provides the discriminant Δ

$$\Delta = [v_1^T]\{t\} = \pi^2 r_1 Y_0(k_S r_1) J_0(k_S \rho) e^{in\phi_1}$$

Since $J_0(k_S\rho) = 0$, we have

if $Y_0(k_S r_1) = 0$ failure point, unfortunately

if $Y_0(k_S r_1) \neq 0$ lucky point,

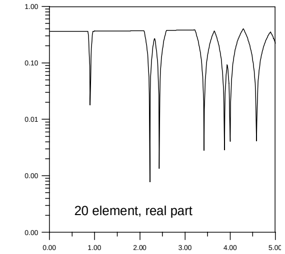
when $k_S r_1$ is the zero for $Y_0(x)$, *i.e.*,

$$Y_0(k_S r_1) = 0 = Y_0(k_0)$$

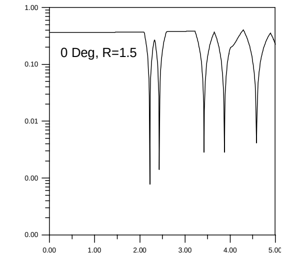
$k_S r_1 = k_0$, then $r_1 = \dfrac{k_0}{k_S}$ sympathy point,

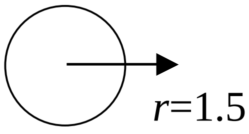
where k_0 is the zero for $Y_0(x)$.

Untreatment

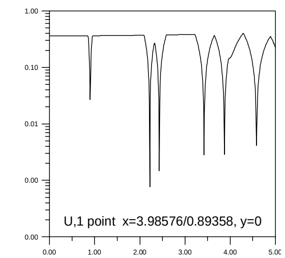


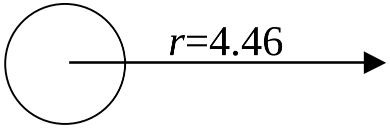
Lucky point





Unfortunate point





(b). If the spurious eigenvalues are double roots, rank reduces by two.

But one additional point provided only one constraint.

Therefore, one point can not filter out the double spurious roots, so another independent equation is need.

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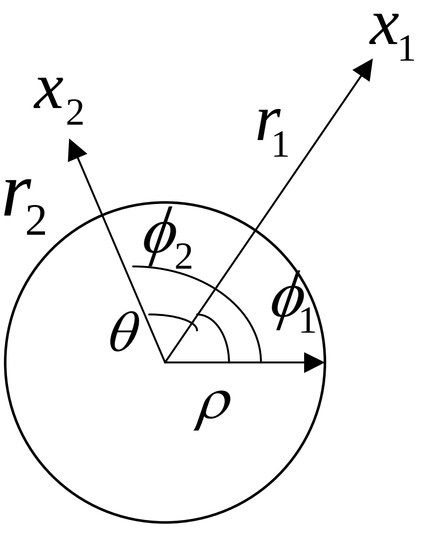
2. UT equation for two exterior points to detect spurious equation with multiplicity 2,

$$\begin{bmatrix} U(k) \\ \tilde{v}_1^T(k) \\ \tilde{v}_2^T(k) \end{bmatrix} \{t\} = \{0\}$$



where $\tilde{v}_2^T$ is the row vector of the influence matrix by collocating the second exterior point at x_2 .

When spurious eigenvalues are the double roots, we have

$$\begin{bmatrix} \tilde{v}_1^T \\ \tilde{v}_2^T \end{bmatrix} \{t\} = \begin{bmatrix} \tilde{v}_1^T \\ \tilde{v}_2^T \end{bmatrix} \{\alpha \tilde{b}_1 + \beta \tilde{b}_2\} = \begin{Bmatrix} \tilde{v}_1^T \tilde{b}_1 \alpha + \tilde{v}_1^T \tilde{b}_2 \beta \\ \tilde{v}_2^T \tilde{b}_1 \alpha + \tilde{v}_2^T \tilde{b}_2 \beta \end{Bmatrix}$$

$$= \begin{bmatrix} \tilde{v}_1^T \tilde{b}_1 & \tilde{v}_1^T \tilde{b}_2 \\ \tilde{v}_2^T \tilde{b}_1 & \tilde{v}_2^T \tilde{b}_2 \end{bmatrix} \begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} \qquad (3)$$

where

$$\tilde{v}_1^T \tilde{b}_1 = \pi^2 r_1 Y_n(kr_1) J_n(k\rho) e^{in\phi_1}$$

$$\tilde{v}_1^T \tilde{b}_2 = \pi^2 r_1 Y_n(kr_1) J_n(k\rho) e^{-in\phi_1}$$

$$\tilde{v}_2^T \tilde{b}_1 = \pi^2 r_2 Y_n(kr_2) J_n(k\rho) e^{in\phi_2}$$

$$\tilde{v}_2^T \tilde{b}_2 = \pi^2 r_2 Y_n(kr_2) J_n(k\rho) e^{-in\phi_2}$$

Since the spurious double roots make the rank less than 2, the additional two points must provide independent constraints,

$$\begin{bmatrix} \pi^2 r_1 Y_n(kr_1) J_n(k\rho) e^{in\phi_1} & \pi^2 r_1 Y_n(kr_1) J_n(k\rho) e^{-in\phi_1} \\ \pi^2 r_2 Y_n(kr_2) J_n(k\rho) e^{in\phi_2} & \pi^2 r_2 Y_n(kr_2) J_n(k\rho) e^{-in\phi_2} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = 0$$

If they are dependent, we have

$$\det \begin{vmatrix} \pi^2 r_1 Y_n(kr_1) J_n(k\rho) e^{in\phi_1} & \pi^2 r_1 Y_n(kr_1) J_n(k\rho) e^{-in\phi_1} \\ \pi^2 r_2 Y_n(kr_2) J_n(k\rho) e^{in\phi_2} & \pi^2 r_2 Y_n(kr_2) J_n(k\rho) e^{-in\phi_2} \end{vmatrix} = 0$$

$$\Delta = r_1 r_2 Y_n(kr_1) Y_n(kr_2) J_n(k\rho) J_n(k\rho) (e^{in(\phi_1-\phi_2)} - e^{-in(\phi_1-\phi_2)}) = 0$$

$$e^{in(\phi_1-\phi_2)} - e^{-in(\phi_1-\phi_2)} = 2i\sin(n(\phi_1-\phi_2)) = 2i\sin(n\theta) = 0$$

The discriminant Δ indicates,

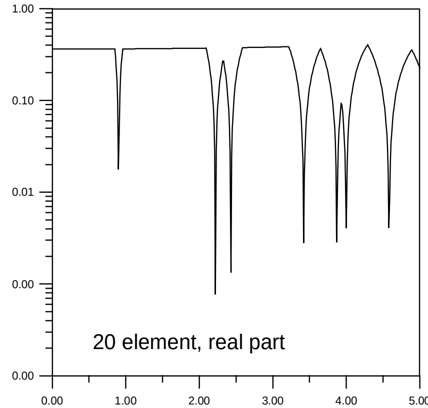
1. If only one additional point is chosen, the failure criterion to detect Y_n spurious roots is $Y_n(k_s r) = Y_n(k_0)$.

2. If the angle between the two points is $2\pi n$, we will fail to filter out the double spurious root.

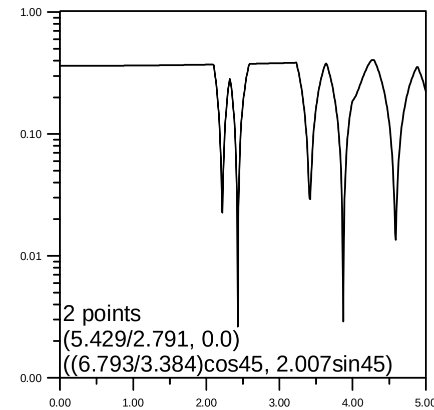
3. If the two points make $Y_n(k_S r_1) = 0$ and $Y_n(k_S r_2) = 0$, $n=0,1,2..$

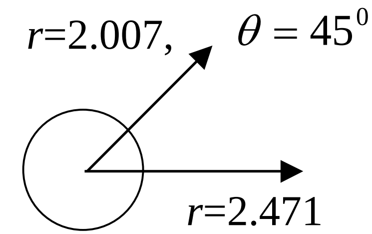
   then we will fail to filter out the double spurious root.

Sensitivity analysis for the collocating radius

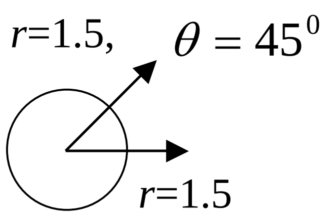


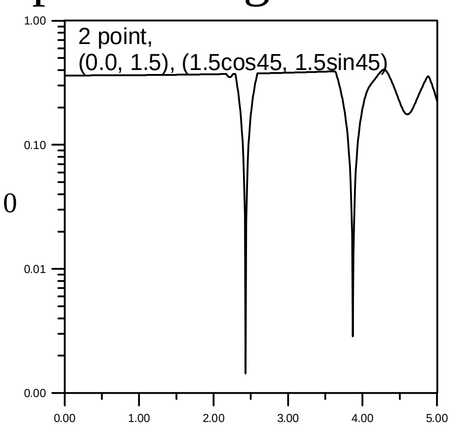
Untreatment, the true and spurious eigenvalues



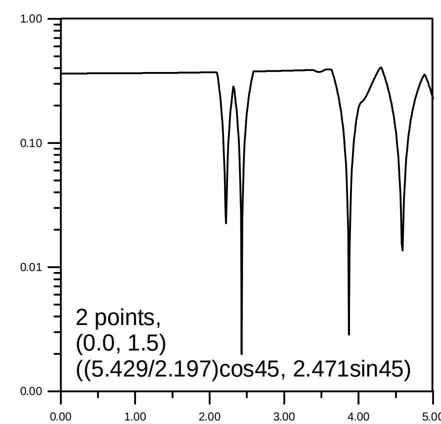


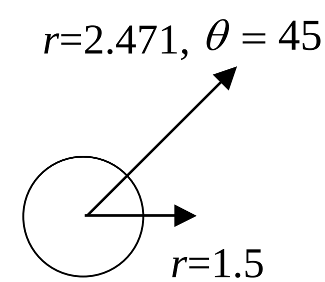
Two unlucky points





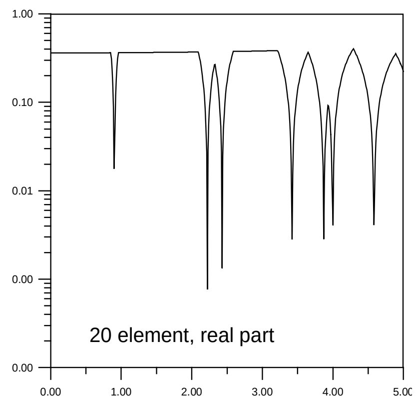
Two lucky points



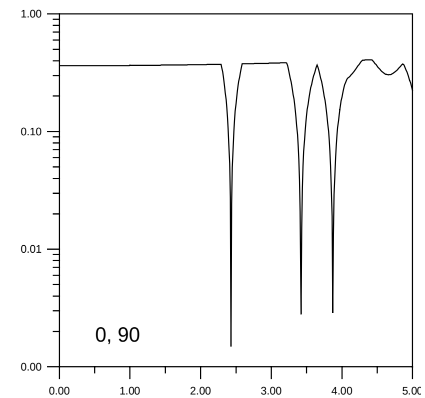


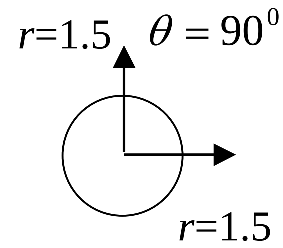
One lucky and one unlucky points

Sensitivity analysis for the intersecting angle

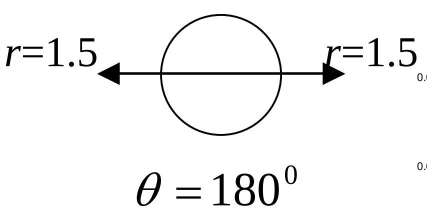


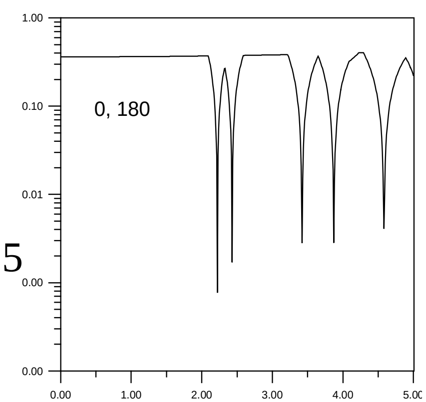
Untreatment, true and spurious eigenvalues



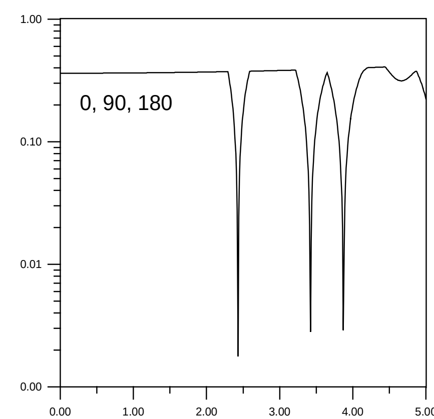


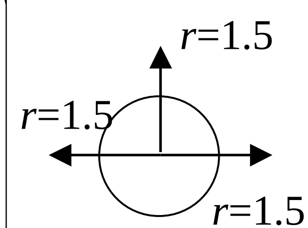
The intersecting angle 90^0





The intersecting angle 180^0





Three points with the angle $0^0, 90^0, 180^0$

Conclusions

- Real-part BEM is reviewed.
- CHEEF method was proposed and compared with CHIEF.
- Failure points are analytical studied and tested numerically.

1. If only one additional point is chosen, the failure criterion to detect Y_n spurious roots is $Y_n(k_s r) = Y_n(k_0)$.
2. If the angle between the two points is $2\pi n$, we will fail to filter out the double spurious root.
3. If the two points make $Y_n(k_S r_1) = 0$ and $Y_n(k_S r_2) = 0$, *n=0,1,2*..

   then we will fail to filter out the double spurious root.
4. The third point is not necessary, if proper choice is made.