國立台灣海洋大學
National Taiwan Ocean University

交通大學
National Chiao Tung University

National Taiwan Ocean University
MSVLAB
Department of Harbor and River Engineering

交大土木
Civil Engineering

Dual BEM since 1986

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Oct. 18, 2006

Presentation for Dept. Civil Engrg, NCTU

Outlines

- **Overview of BEM and dual BEM**
- **Mathematical formulation**
  Hypersingular BIE
- **Nonuniqueness and its treatments**
  Degenerate scale
  True and spurious eigensolution (interior prob.)
  Fictitious frequency (exterior acoustics)
- **Conclusions**

Top ten countries of BEM and dual BEM

- BEM

  USA, China, UK, Japan, Germany, France,
  Taiwan (547), Canada, South Korea, Italy (No.7)

- Dual BEM (Made in Taiwan)

  UK, USA, Taiwan (69), China, Germany,
  France, Japan, Australia, Brazil, Slovenia (No.3)

(ISI information Oct.17, 2006)

台灣加油 FEM Taiwan (No.9/1311)

Top three scholars on BEM and dual BEM

- BEM

  Aliabadi M H (UK, Queen Mary College)

  Mukherjee S (USA, Cornell Univ.)

  Chen J T (Taiwan, Ocean Univ.) 54 篇

  Tanaka M (Japan, Shinshu Univ.)

- Dual BEM (Made in Taiwan)

  Aliabadi M H (UK, Queen Mary Univ. London)

  Chen J T (Taiwan, Ocean Univ.) 43 篇

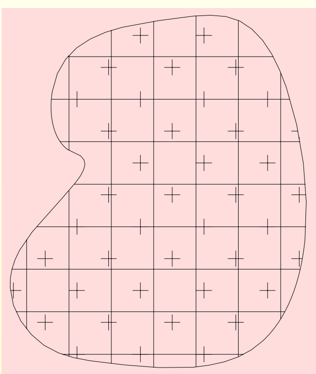
  Power H (UK, Univ Nottingham)

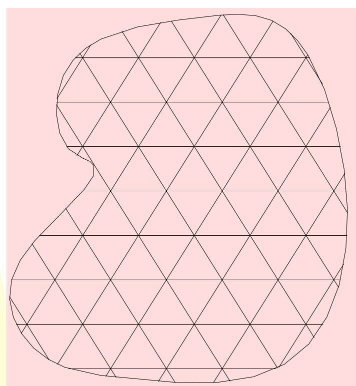
(ISI information Oct.17, 2006)

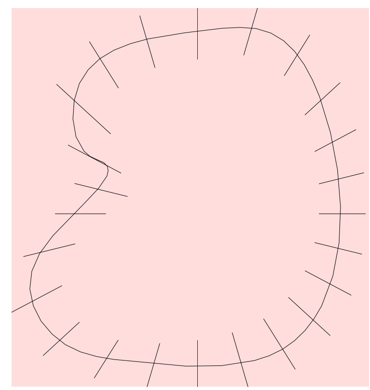
NTOU/MSV 加油

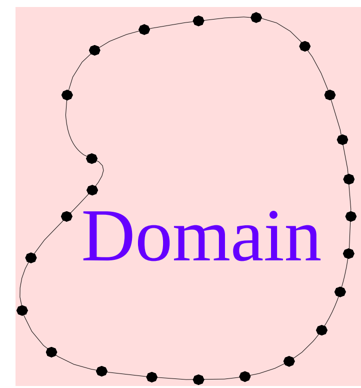
Overview of numerical methods

- Numerical Methods
  - Mesh Methods
    - Finite Difference Method
    - Finite Element Method
    - Boundary Element Method
  - Meshless Methods









Boundary

DE

PDE-
variational

IE

MFS
Trefftz method
MLS, EFG

Numeber of Papers of FEM, BEM and FDM

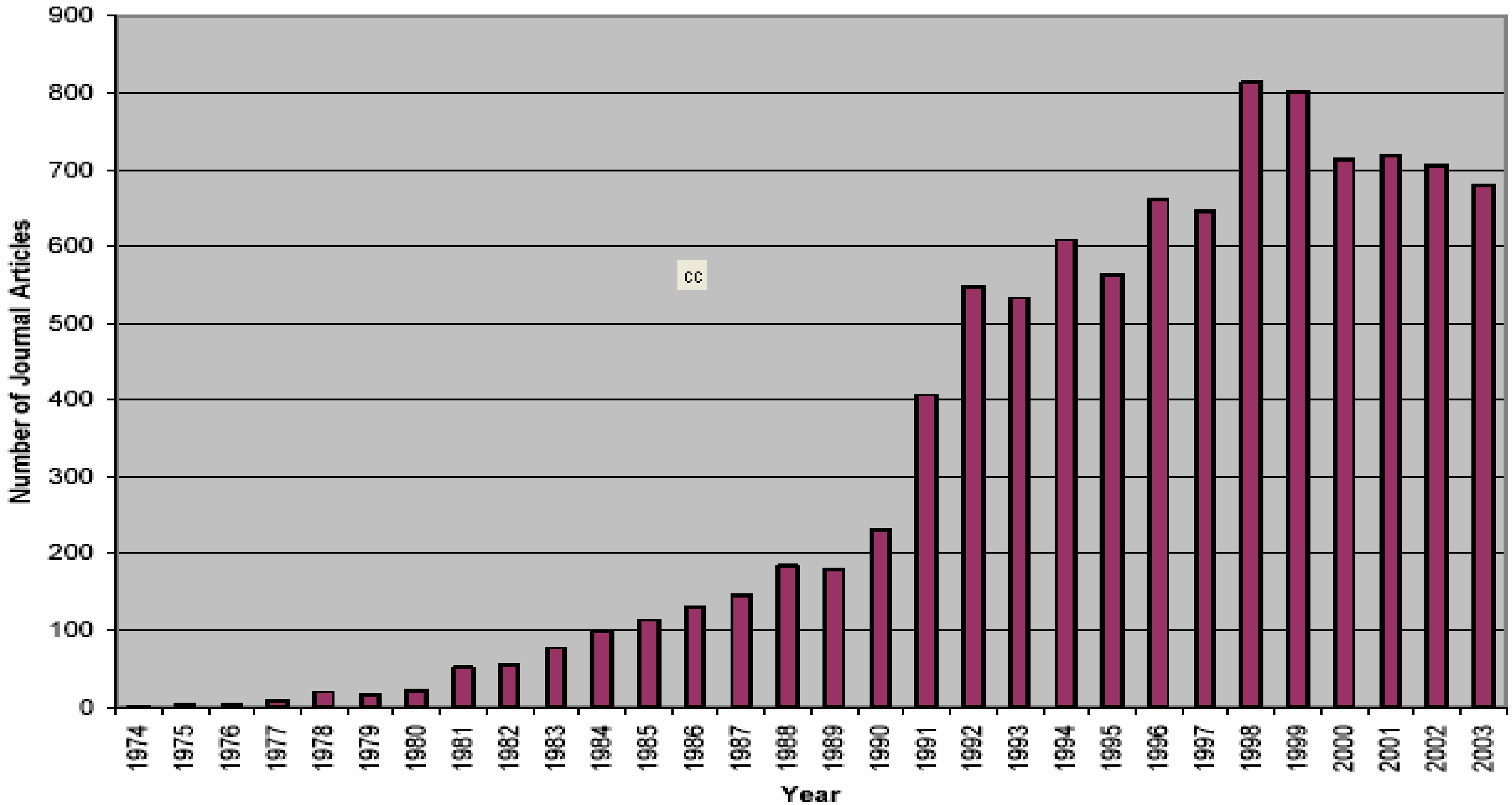
Table 1
Bibliographic database search based on the Web of Science

Numerical method	Search phrase in topic field	No. of entries	
FEM	'Finite element' or 'finite elements'	66,237	6
FDM	'Finite difference' or 'finite differences'	19,531	2
BEM	'Boundary element' or 'boundary elements' or 'boundary integral'	10,126	1
FVM	'Finite volume method' or 'finite volume methods'	1695	
CM	'Collocation method' or 'collocation methods'	1615	

Refer to Appendix A for search criteria. (Search date: May 3, 2004).

(Data form Prof. Cheng A. H. D.)

Growth of BEM/BIEM papers



(data from Prof. Cheng A.H.D.)

Advantages of BEM

- Discretization dimension reduction
- Infinite domain (half plane)
- Interaction problem
- Local concentration

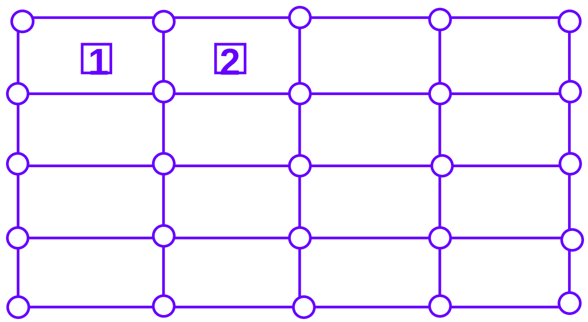
Disadvantages of BEM

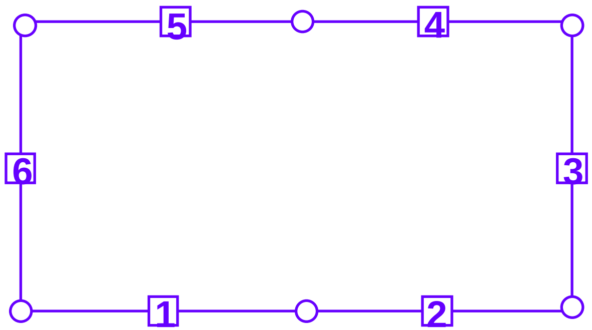
- Integral equations with singularity 北京清華
- Full matrix (nonsymmetric)

What Is Boundary Element Method ?

- **Finite element method**

Boundary element method





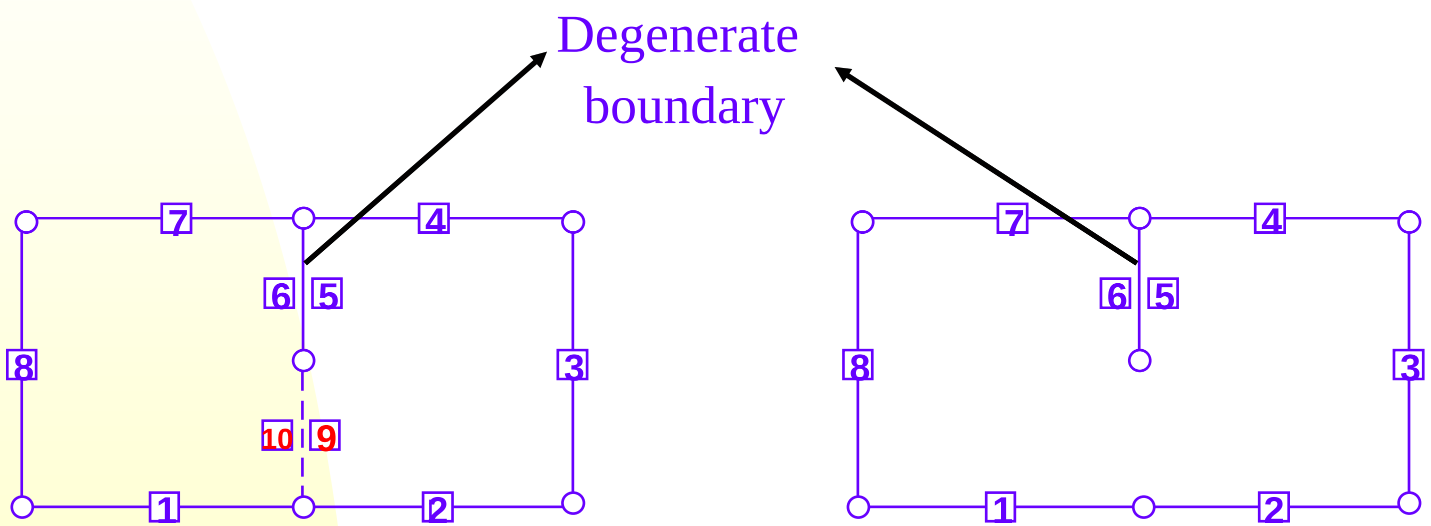
geometry node

N **the Nth constant or linear element**

Dual BEM

Why hypersingular BIE is required

(potential theory)



Artifical boundary introduced !
BEM

Dual integral equations needed !
Dual BEM

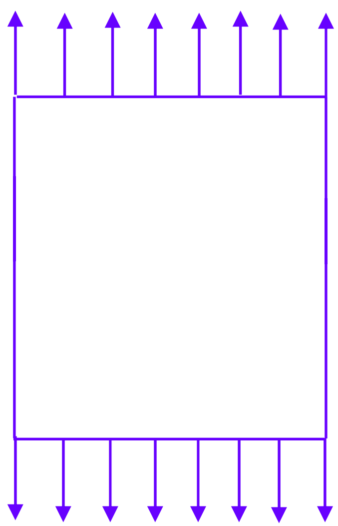
Dual Integral Equations by Hong and Chen(1984-1986)

Singular integral equation → **Hypersingular integral equation**

Cauchy principal value → **Hadamard principal value**

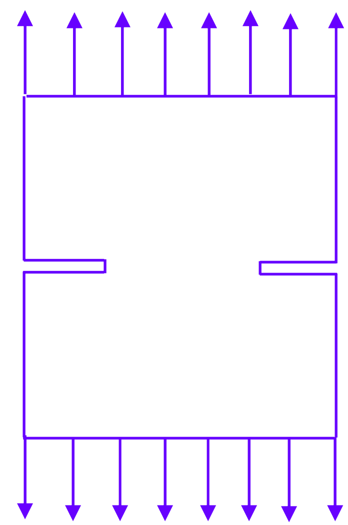
Boundary element method → **Dual boundary element method**

1969



normal boundary

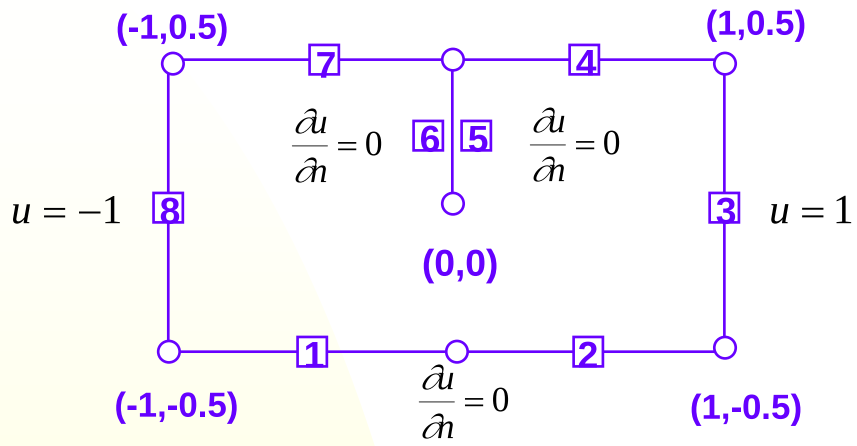
1986



2006

degenerate boundary

Degenerate boundary



○ geometry node

N the Nth constant or linear element

$$[U]\{t\}=[T]\{u\}$$

$$[L]\{t\}=[M]\{u\}$$

$$\begin{array}{ccc} & \begin{array}{cccccccc} & & & & 5(+) & 6(+) & & \end{array} & \\ [U]= & \begin{bmatrix} -1.693 & -0.045 & 0.471 & 0.347 & -0.054 & -0.054 & 0.039 & -0.335 \\ -0.045 & -1.693 & -0.335 & 0.039 & -0.054 & -0.054 & 0.347 & 0.471 \\ 0.445 & -0.335 & -1.693 & -0.335 & 0.019 & 0.019 & 0.445 & 0.703 \\ 0.347 & 0.039 & -0.335 & -1.693 & -0.281 & -0.281 & -0.045 & 0.471 \\ -0.081 & -0.081 & 0.063 & -0.638 & -1.193 & -1.193 & -0.638 & 0.063 \\ -0.081 & -0.081 & 0.063 & -0.638 & -1.193 & -1.193 & -0.638 & 0.063 \\ 0.039 & 0.347 & 0.471 & -0.045 & -0.281 & -0.281 & -1.693 & -0.334 \\ -0.335 & 0.445 & 0.703 & 0.445 & 0.019 & 0.019 & -0.335 & -1.693 \end{bmatrix} & \begin{array}{c} \\ \\ \\ \\ 5(+) \\ 6(+) \\ \\ \\ \end{array} \end{array}$$

$n(s)$

$$\begin{array}{ccc} & \begin{array}{cccccccc} & & & & 5(+) & 6(-) & & \end{array} & \\ [T]= & \begin{bmatrix} -\pi & 0.000 & 0.588 & 0.519 & -0.321 & 0.321 & 0.927 & 1.107 \\ 0.000 & -\pi & 1.107 & 0.927 & 0.321 & -0.321 & 0.519 & 0.588 \\ 0.219 & 1.107 & -\pi & 1.107 & 0.464 & -0.464 & 0.219 & 0.490 \\ 0.519 & 0.927 & 1.107 & -\pi & 0.785 & -0.785 & 0.000 & 0.588 \\ 0.927 & 0.927 & 0.888 & 1.326 & -\pi & -\pi & 1.326 & 0.888 \\ 0.927 & 0.927 & 0.888 & 1.326 & -\pi & -\pi & 1.326 & 0.888 \\ 0.927 & 0.519 & 0.588 & 0.000 & -0.7854 & 0.785 & -\pi & 1.107 \\ 1.107 & 0.219 & 0.490 & 0.219 & -0.464 & 0.464 & 1.107 & -\pi \end{bmatrix} & \begin{array}{c} \\ \\ \\ \\ 5(+) \\ 6(+) \\ \\ \\ \end{array} \end{array}$$

$n(x)$

$$\begin{array}{ccc} & \begin{array}{cccccccc} & & & & 5(+) & 6(+) & & \end{array} & \\ [L]= & \begin{bmatrix} \pi & 0.000 & 0.184 & 0.519 & 0.458 & 0.458 & 0.927 & 0.805 \\ 0.000 & \pi & 0.805 & 0.927 & 0.458 & 0.458 & 0.519 & 0.184 \\ 0.612 & 0.805 & \pi & 0.805 & 0.464 & 0.464 & 0.612 & 0.490 \\ 0.519 & 0.927 & 0.805 & \pi & 0.347 & 0.347 & 0.000 & 0.184 \\ -0.511 & 0.511 & 0.888 & 1.417 & \pi & -\pi & -1.417 & -0.888 \\ 0.511 & -0.511 & -0.888 & -1.417 & -\pi & \pi & 1.417 & 0.888 \\ 0.927 & 0.519 & 0.184 & 0.000 & 0.347 & 0.347 & \pi & 0.805 \\ 0.805 & 0.612 & 0.490 & 0.612 & 0.464 & 0.464 & 0.805 & \pi \end{bmatrix} & \begin{array}{c} \\ \\ \\ \\ 5(+) \\ 6(-) \\ \\ \\ \end{array} \end{array}$$

$n(s)$

$n(x)$

$$\begin{array}{ccc} & \begin{array}{cccccccc} & & & & 5(+) & 6(-) & & \end{array} & \\ [M]= & \begin{bmatrix} 4.000 & -1.333 & -0.205 & -0.061 & 0.600 & -0.600 & -0.800 & -1.600 \\ -1.333 & 4.000 & -1.600 & -0.800 & -0.600 & 0.600 & -0.061 & -0.205 \\ -0.282 & -1.600 & 4.000 & -1.600 & -0.400 & 0.400 & -0.282 & -0.236 \\ -0.061 & -0.800 & -1.600 & 4.000 & -1.000 & 1.000 & -1.333 & -0.205 \\ 0.853 & -0.853 & -0.715 & -3.765 & 8.000 & -8.000 & 3.765 & 0.715 \\ -0.853 & 0.853 & 0.715 & 3.765 & -8.000 & 8.000 & -3.765 & -0.715 \\ -0.800 & -0.062 & -0.205 & -1.333 & 1.000 & -1.000 & 4.000 & -1.600 \\ -1.600 & -0.282 & -0.235 & -0.282 & 0.400 & -0.400 & -1.600 & 4.000 \end{bmatrix} & \begin{array}{c} \\ \\ \\ \\ 5(+) \\ 6(-) \\ \\ \\ \end{array} \end{array}$$

How to get additional constraints

$$f(x) = (x-a)^2 Q(x) + px + q$$

$$f(a) = pa + q, \quad when\ x = a$$

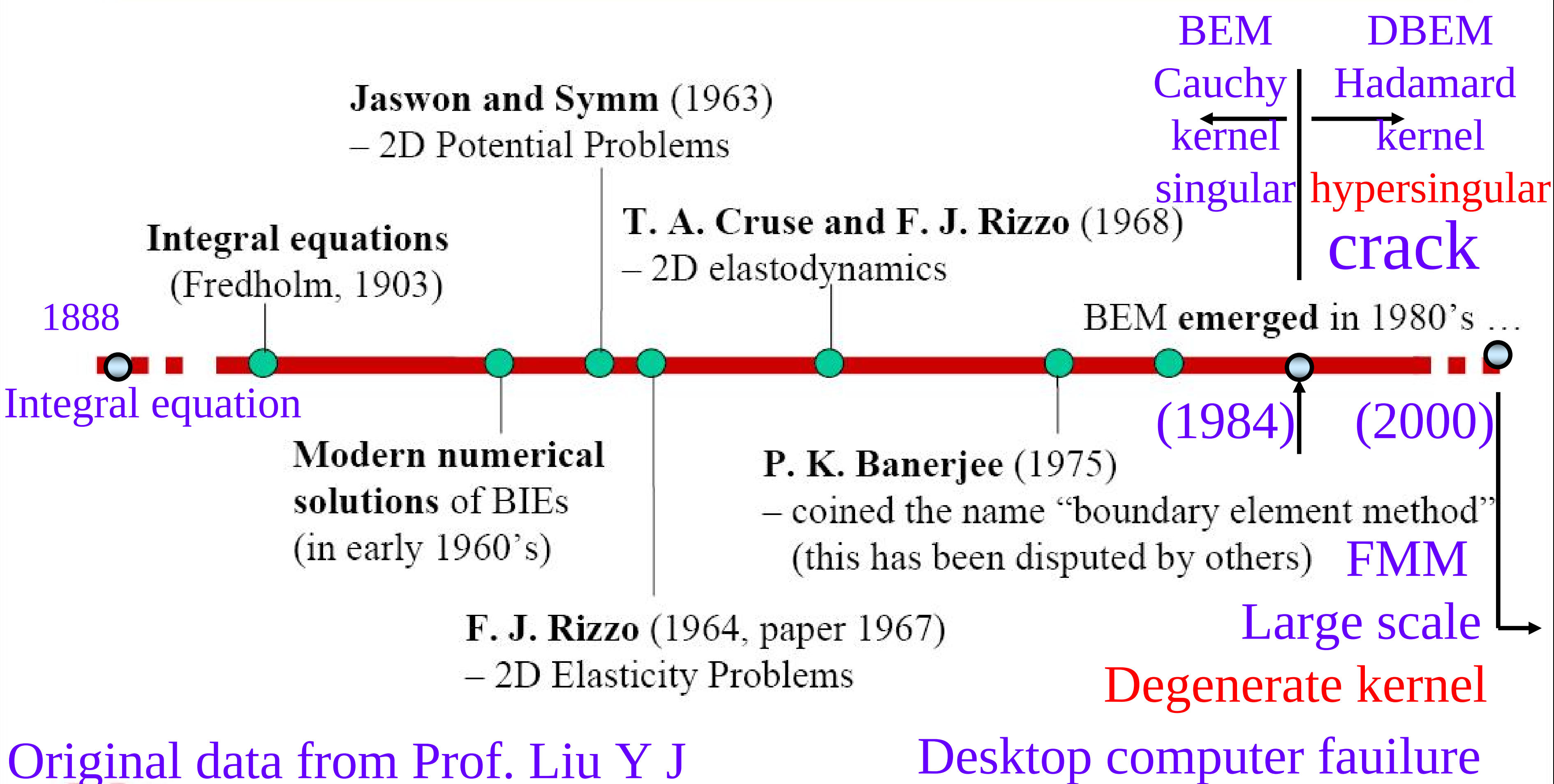
The constraint equation is not enough to determine coefficients of p **and** q**,**

Another constraint equation is required

$$f'(x) = 2(x-a)Q(x) + (x-a)^2 Q'(x) + p$$

$$f'(a) = p, \quad when\ \ x = a$$

A Brief History of the BEM

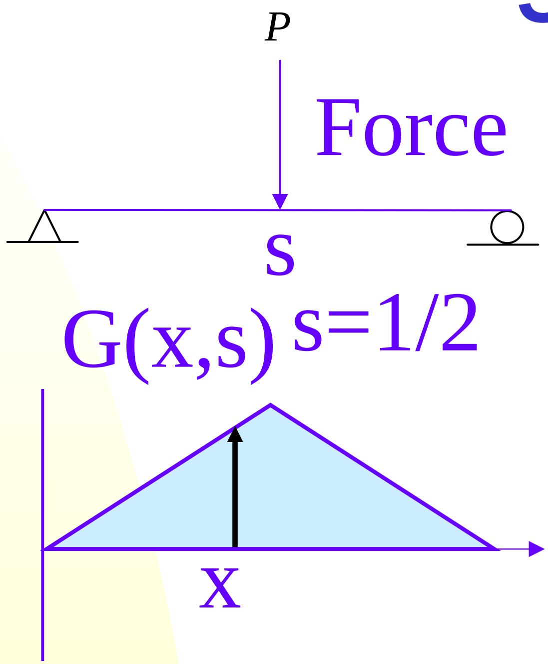


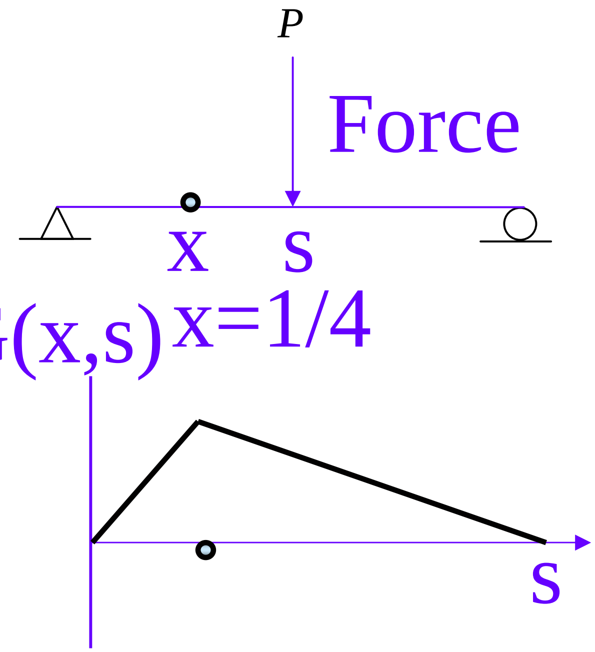
Fundamental solution

- Field response due to source (space)
- Green's function
- Casual effect (time)

$$K(x,s;t,\tau)$$

Green’s function, influence line and moment diagram

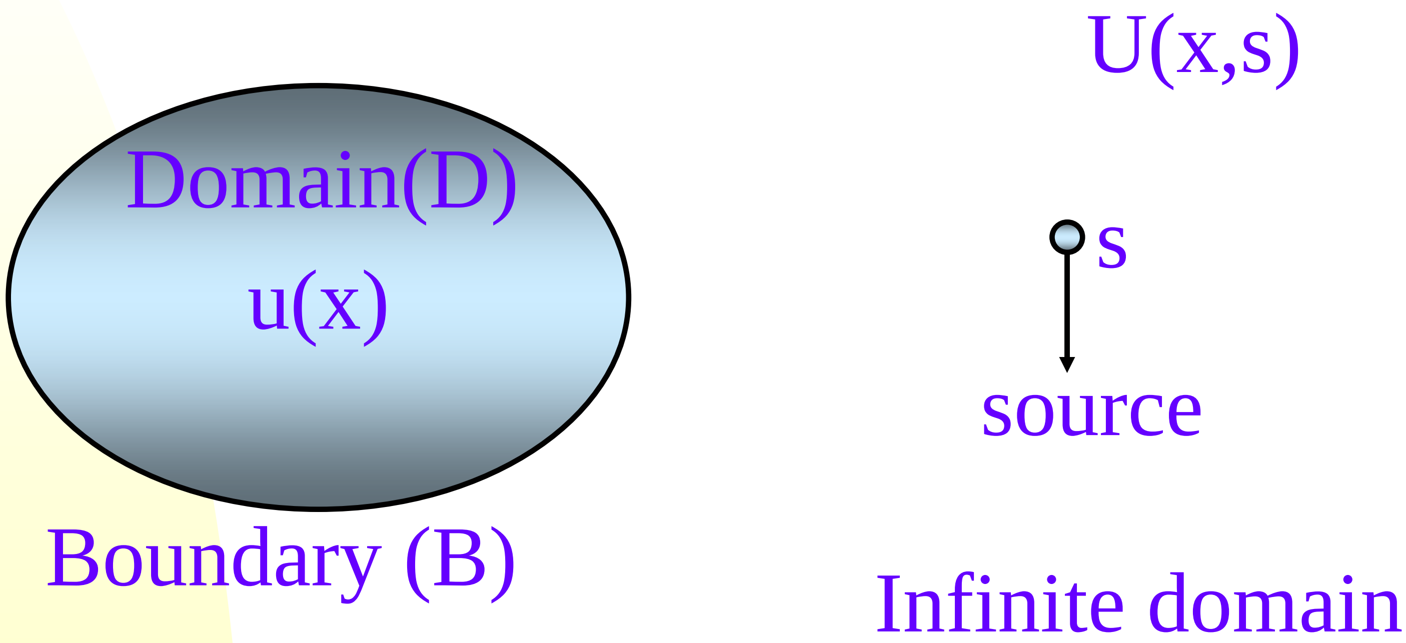




Moment diagram
s:fixed
x:observer

Influence line
s:moving
x:observer(instrument)

Two systems u and U



Dual integral equations for a domain point (Green's third identity for two systems, u and U)

Primary field

$$2\pi \; u(x) = \int_B T(s,x) \; u(s) \; dB(s) - \int_B U(s,x) \; t(s) \; dB(s), \quad x \in D$$

Secondary field

$$2\pi \; t(x) = \int_B M(s,x) \; u(s) \; dB(s) - \int_B L(s,x) \; t(s) \; dB(s), \quad x \in D$$

where $U(s,x)=\ln(r)$ **is the fundamental solution.**

$$T(s,x) \equiv \frac{\partial U}{\partial n_s} \qquad L(s,x) \equiv \frac{\partial U}{\partial n_x} \qquad M(s,x) \equiv \frac{\partial^2 U}{\partial n_s \partial n_x}$$

$$t = \frac{\partial u}{\partial n}$$

Dual integral equations for a boundary point (x push to boundary)

Singular integral equation

$$\pi u(x) = C.P.V.\int_B T(s,x)u(s)\,dB(s) - R.P.V.\int_B U(s,x)t(s)\,dB(s),\ x \in B$$

Hypersingular integral equation

$$\pi t(x) = H.P.V.\int_B M(s,x)u(s)\,dB(s) - C.P.V.\int_B L(s,x)t(s)\,dB(s),\ x \in B$$

where $U(s,x)$ **is the fundamental solution.**

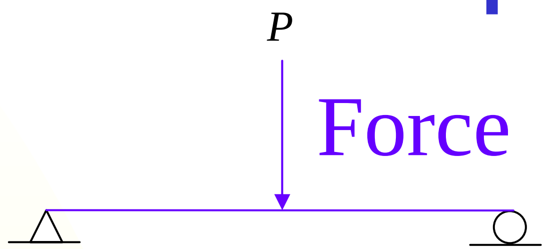
$$T(s,x) \equiv \frac{\partial U}{\partial n_s} \qquad L(s,x) \equiv \frac{\partial U}{\partial n_x} \qquad M(s,x) \equiv \frac{\partial^2 U}{\partial n_s \partial n_x}$$

Potential theory

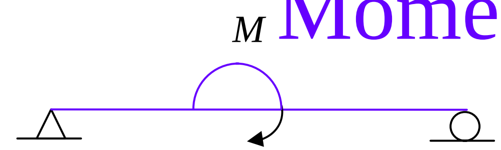
- Single layer potential (U)
- Double layer potential (T)
- Normal derivative of single layer potential (L)
- Normal derivative of double layer potential (M)

Physical examples for potentials

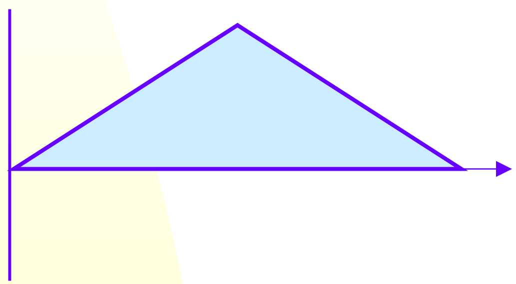
Force



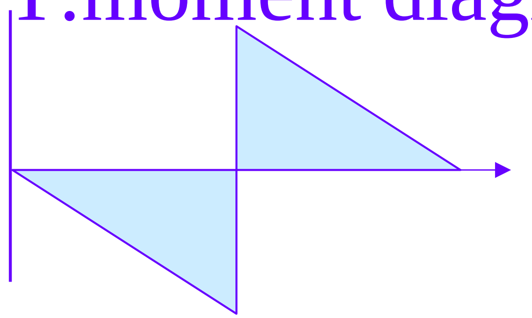
Moment



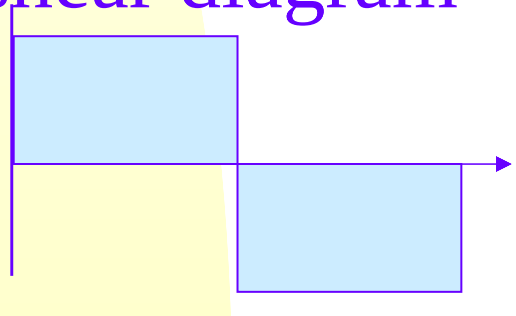
U:moment diagram



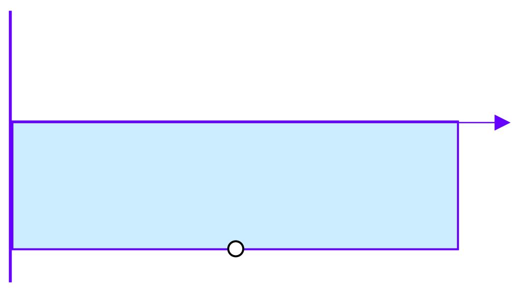
T:moment diagram



L:shear diagram

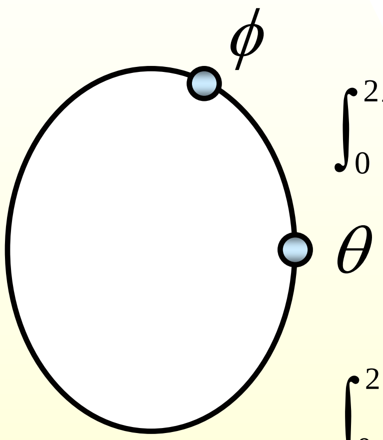


M:shear diagram



Order of pseudo-differential operators

- Single layer potential (U) --- (-1)
- Double layer potential (T) --- (0)



$$\int_0^{2\pi} T(\phi,\theta)u(\theta)d\theta = \pi u\left(\phi\right) + CPV\int_0^{2\pi} T(\phi,\theta)u(\theta)d\theta$$

- Normal derivative of single layer potential (L) --- (0)

$$\int_0^{2\pi} L(\phi,\theta)t(\theta)d\theta = -\pi\ t\left(\phi\right) + CPV\int_0^{2\pi} L(\phi,\theta)t(\theta)d\theta$$

- Normal derivative of double layer potential (M) --- (1)

$$\int_0^{2\pi} M(\phi,\theta)u(\theta)d\theta = \mathrm{M}(u)$$

$\mathrm{M}(\mathrm{M}(u)) = -u''$ Pseudo differential operator

$D(D(u)) = u''$ Real differential operator

Calderon projector

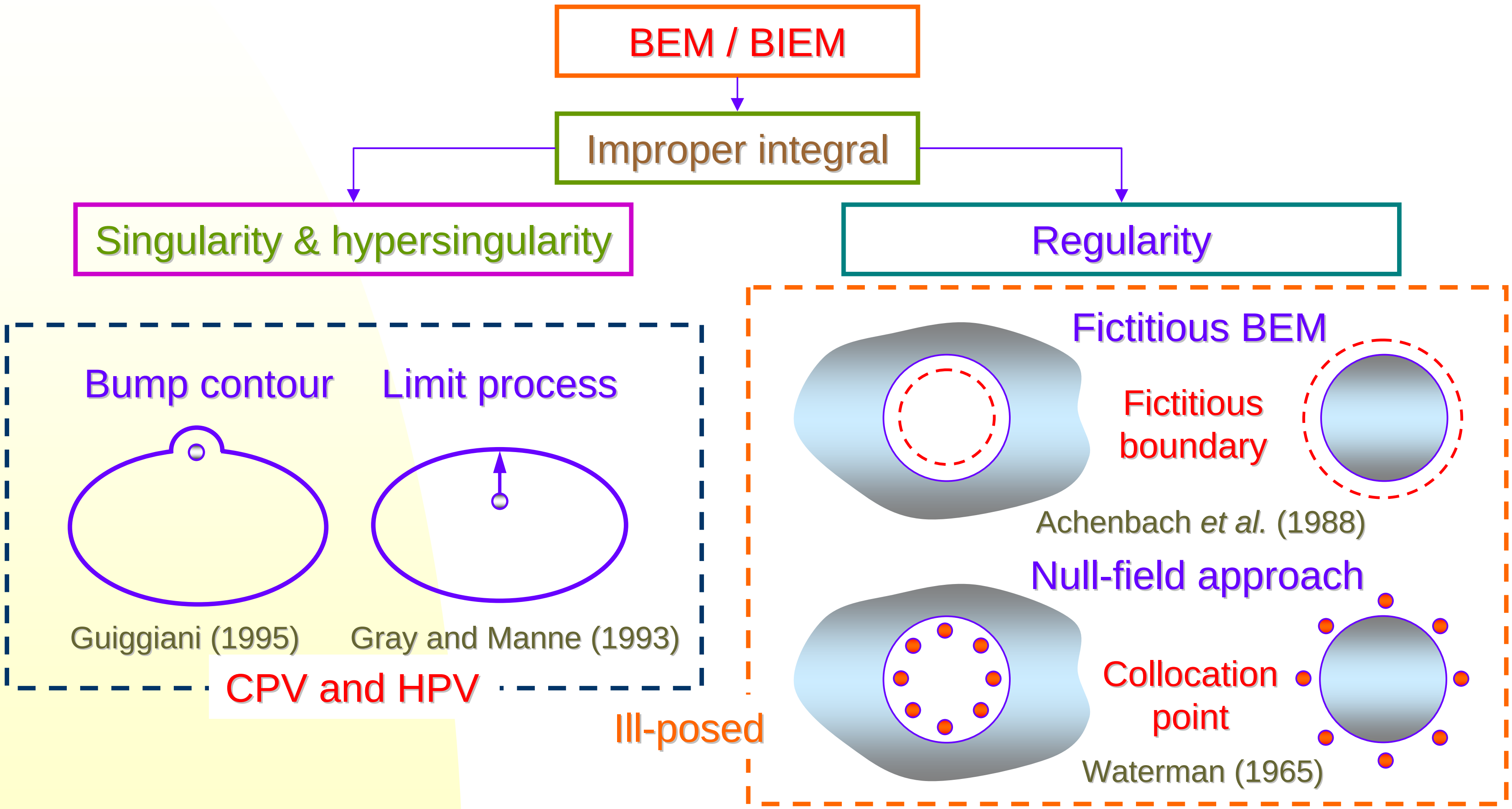
$$-\frac{1}{4}[I]+[T]^2-[U][M]=[0]$$

$$[U][L]=[T][M]$$

$$[M][T]=[L][M]$$

$$-\frac{1}{4}[I]+[L]^2-[M][U]=[0]$$

How engineers avoid singularity



Definitions of R.P.V., C.P.V. and H.P.V. using bump approach

- **R.P.V. (Riemann principal value)**

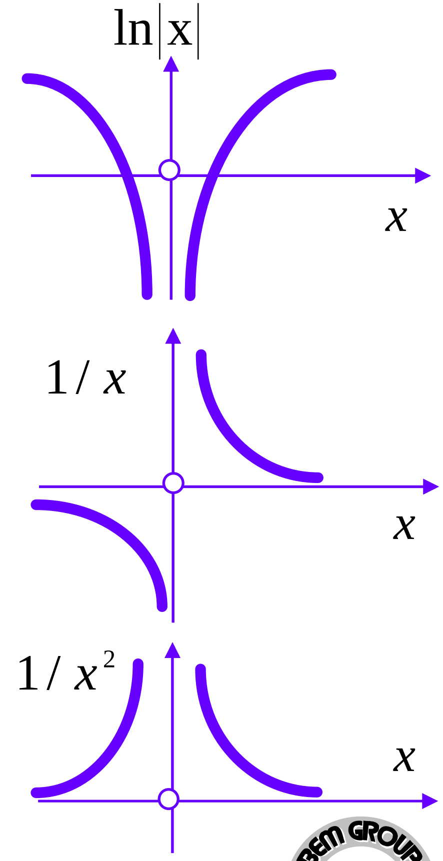
$$R.P.V.\int_{-1}^{1} \ln|x|\,dx = (x\ln|x| - x)\Big|_{x=-1}^{x=1} = -2$$

- **C.P.V.(Cauchy principal value)**

$$C.P.V.\int_{-1}^{1} \frac{1}{x}\,dx = \lim_{\varepsilon\to 0}\int_{-1}^{-\varepsilon} + \int_{\varepsilon}^{1} \frac{1}{x}\,dx = 0$$

- **H.P.V.(Hadamard principal value)**

$$H.P.V.\int_{-1}^{1} \frac{1}{x^2}\,dx = \lim_{\varepsilon\to 0}\int_{-1}^{-\varepsilon} + \int_{\varepsilon}^{1} \frac{1}{x^2}\,dx - \frac{2}{\varepsilon} = -2$$



Principal value in who's sense

- Common sense
- Riemann sense
- Lebesgue sense
- Cauchy sense
- Hadamard sense (elasticity)
- Mangler sense (aerodynamics)
- Liggett and Liu's sense

  The singularity that occur when the base point and field point coincide are not integrable. (1983)

Two approaches to understand HPV

$$H.P.V.\int_{-1}^{1}\frac{1}{x^2}\,dx=\lim\int_{-1}^{-\varepsilon}+\int_{\varepsilon}^{1}\frac{1}{x^2}\,dx-\frac{2}{\varepsilon}=\text{-}2$$

Differential first and then trace operator

$$\lim_{y\to 0}\int_{-1}^{1}\frac{1}{x^2+y^2}\,dx=\text{-}2$$

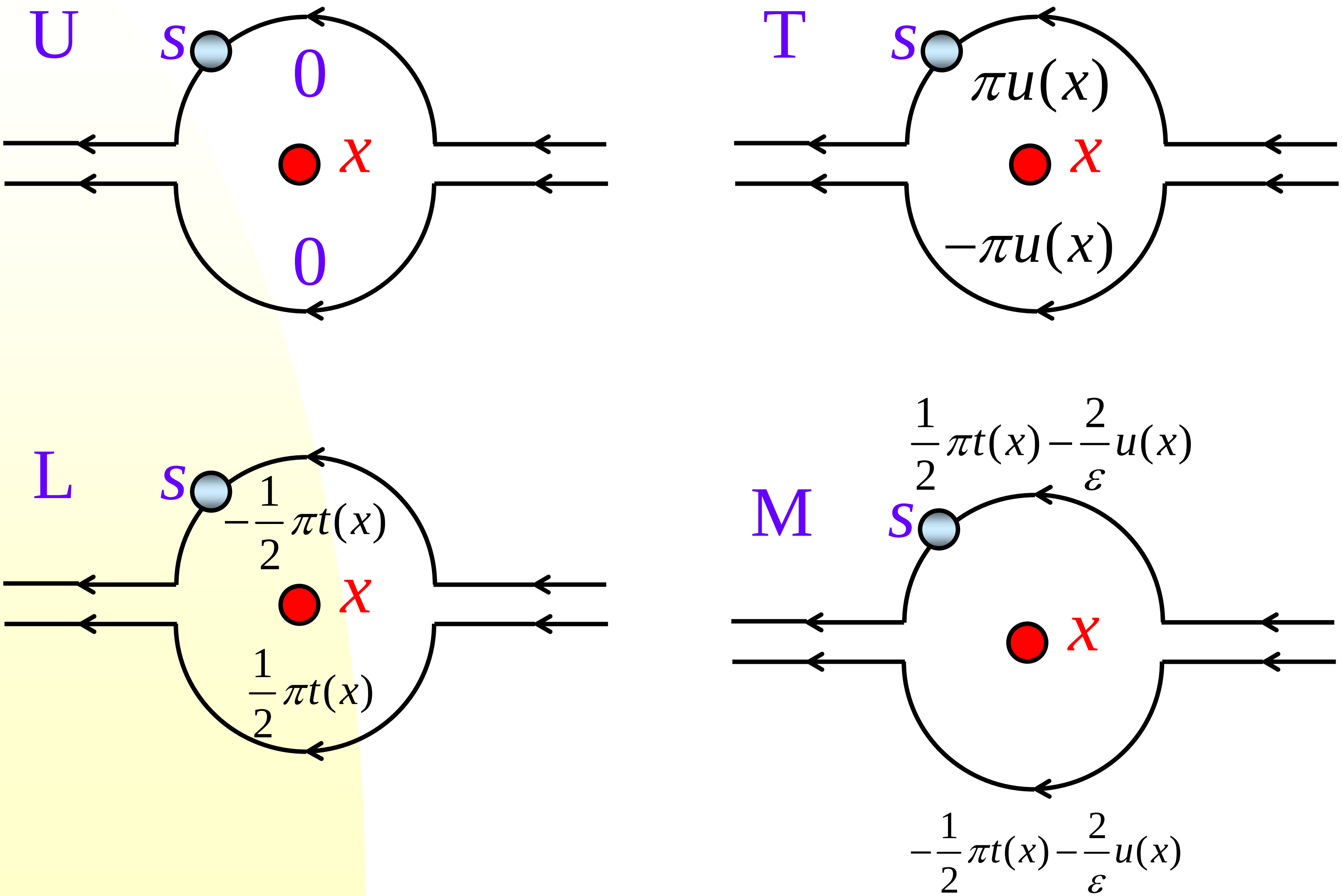
(Limit and integral operator can not be commuted)

Trace first and then differential operator

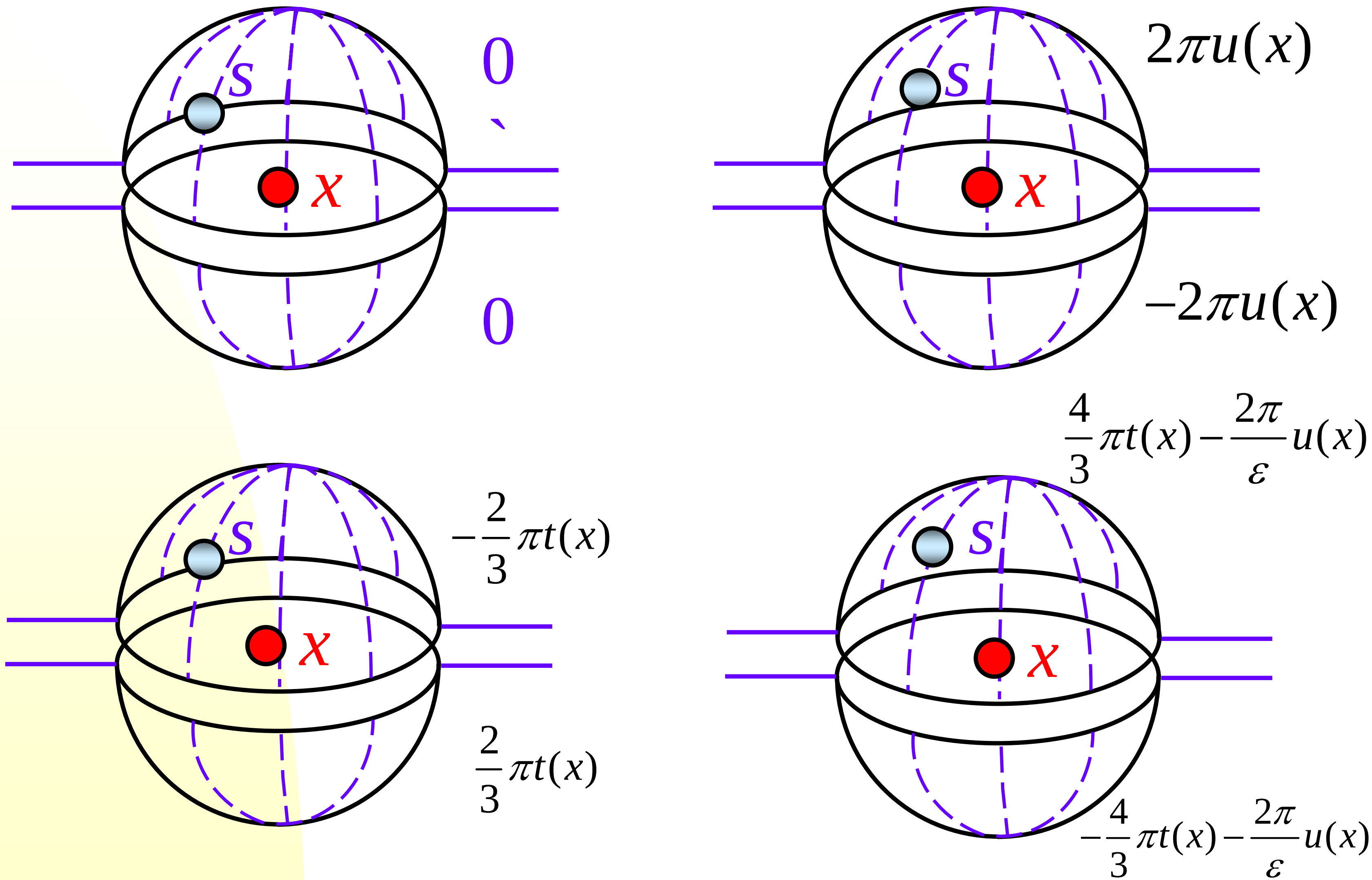
$$\frac{d}{dt}\{CPV\int_{-1}^{1}\frac{-1}{x-t}dx\}\Big|_{t=0}=-2$$

(Leibnitz rule should be considered)

Bump contribution (2-D)



Bump contribution (3-D)



Successful experiences since 1986

Solid rocket motor (工蜂火箭)

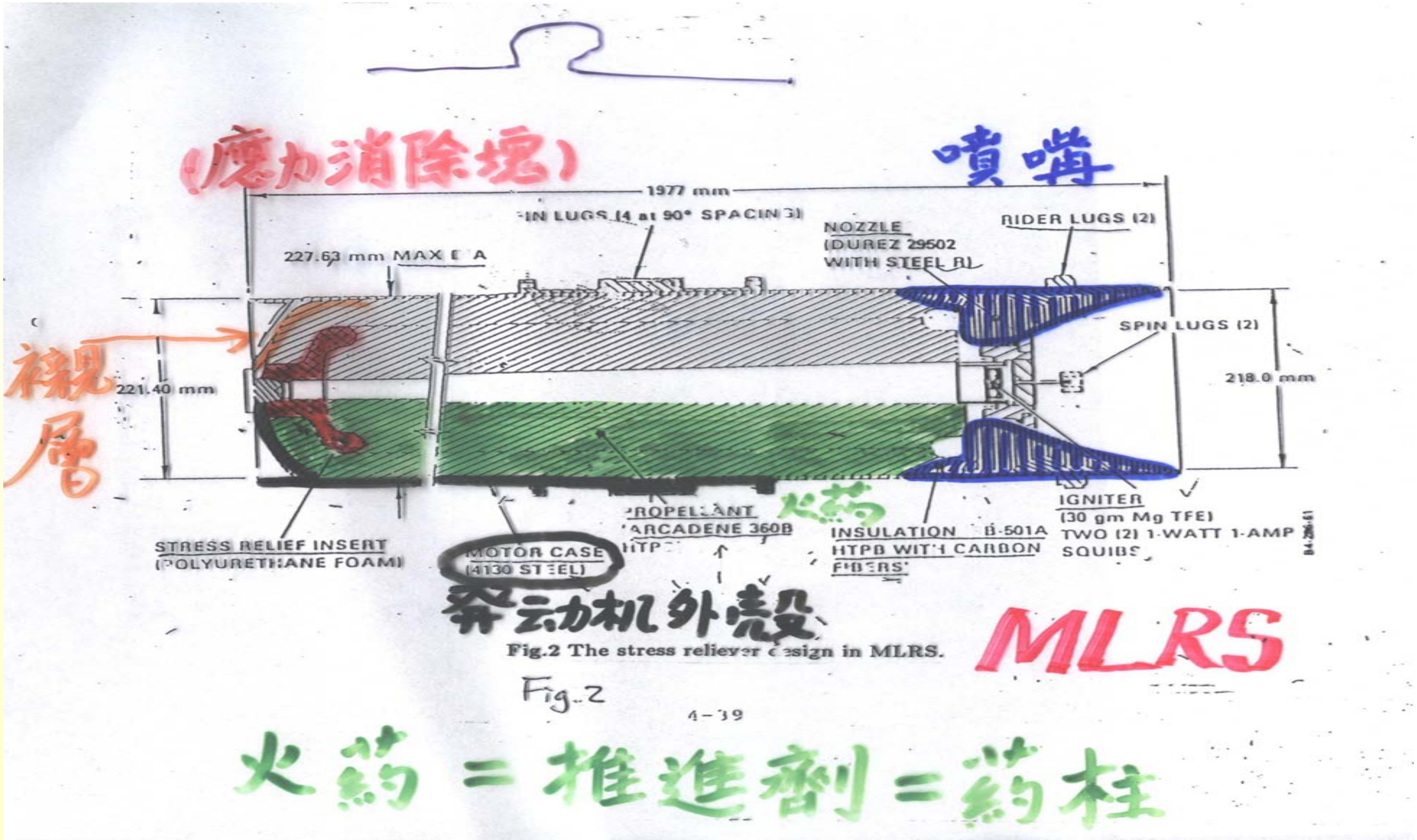


Fig.2 The stress reliever design in MLRS.

X-ray detection (三溫暖測試)

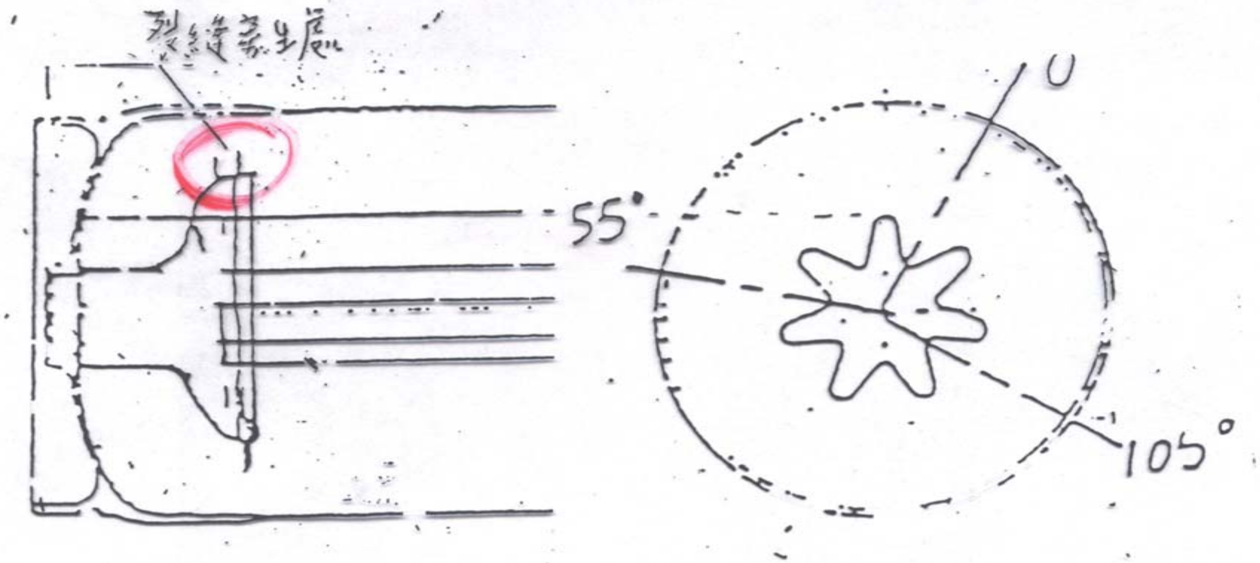
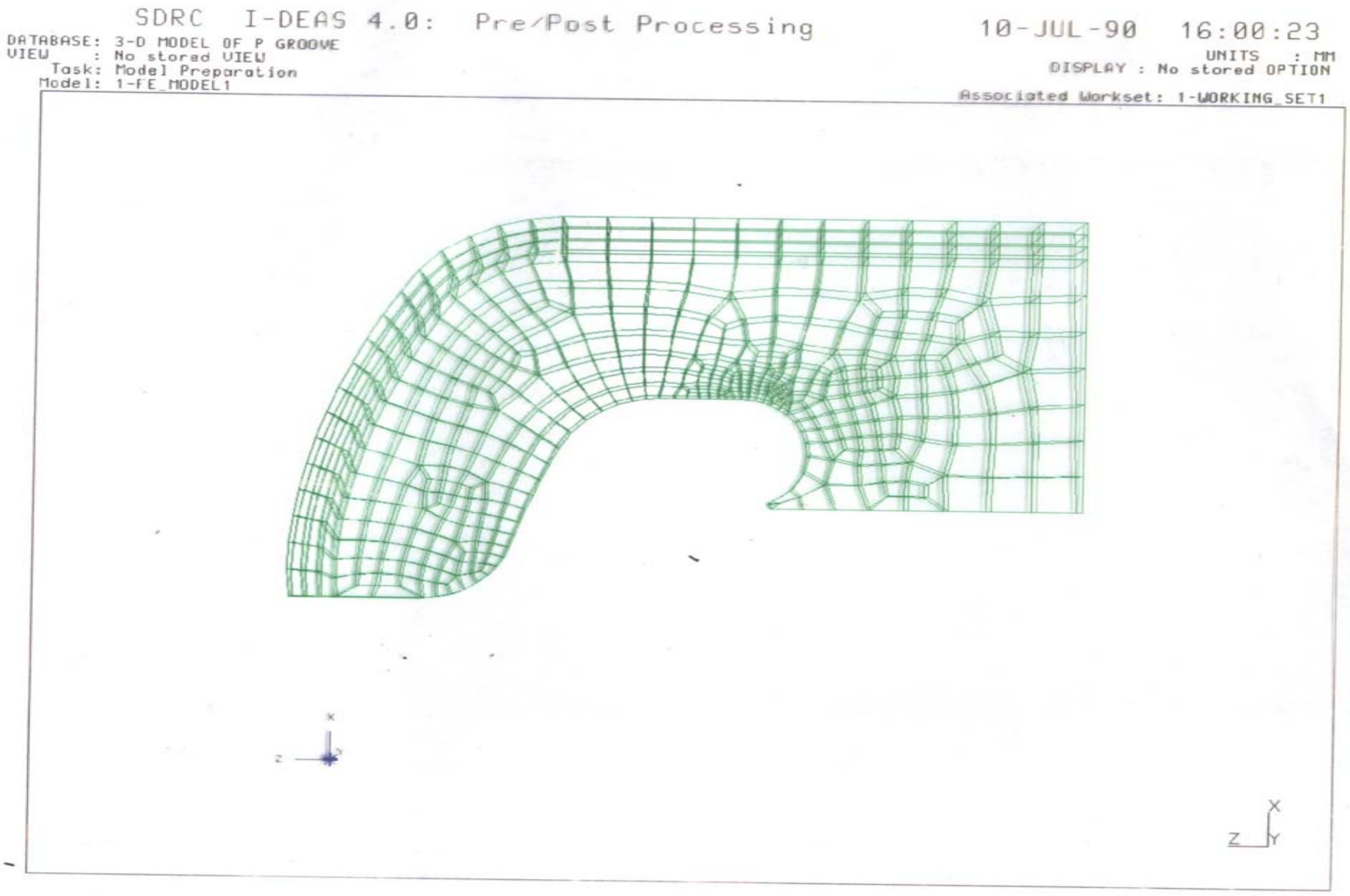


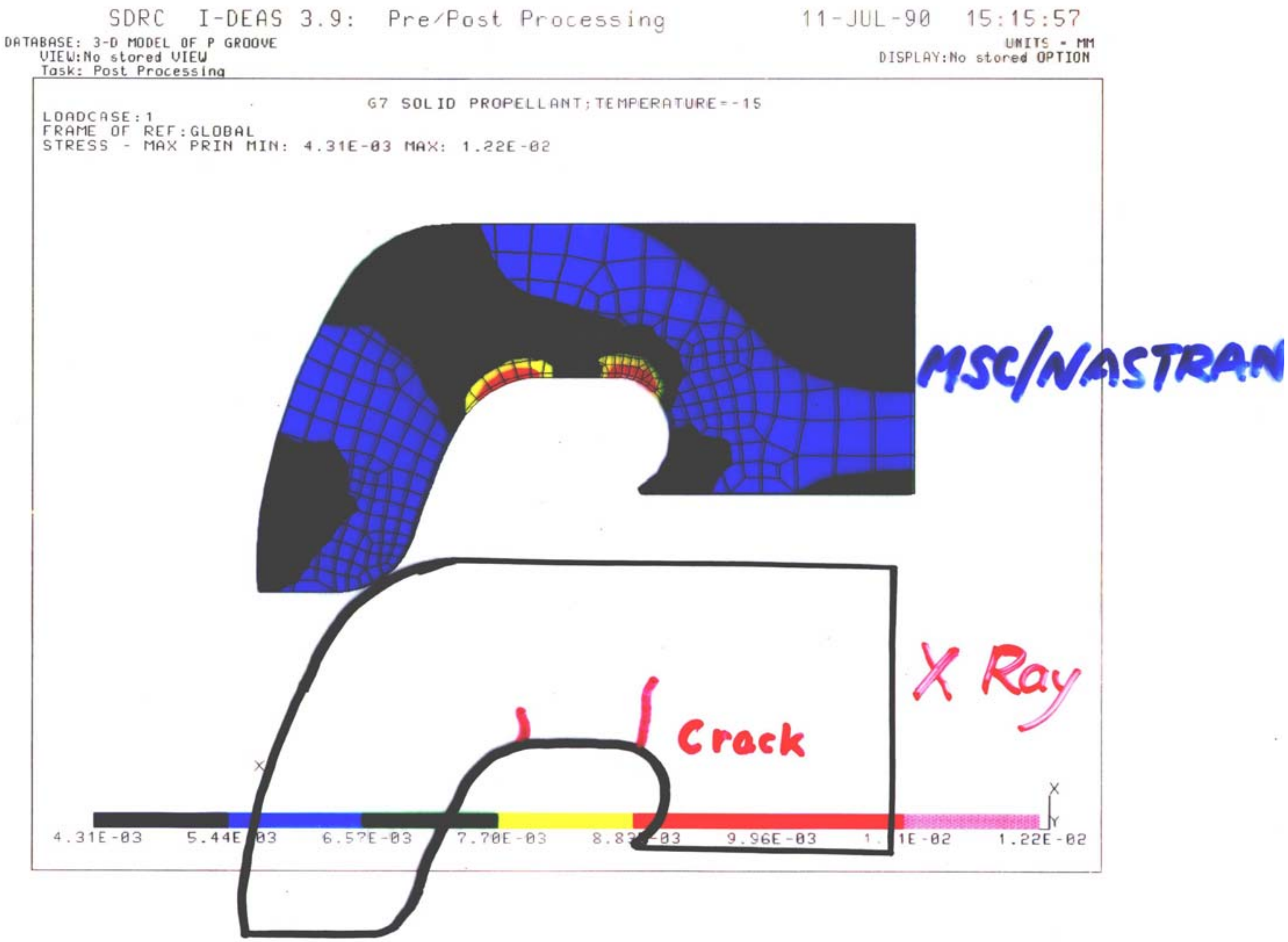
Fig. [illegible] DT X-ray results.

圖一應力消除塊設計圖 [12,13]

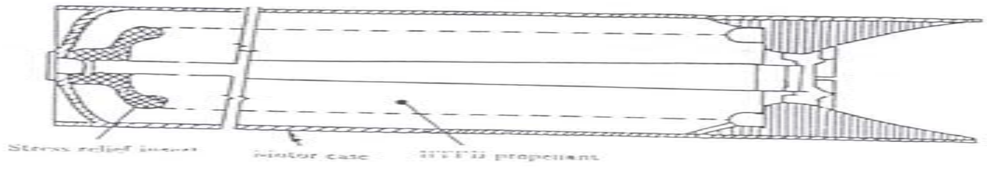
FEM simulation

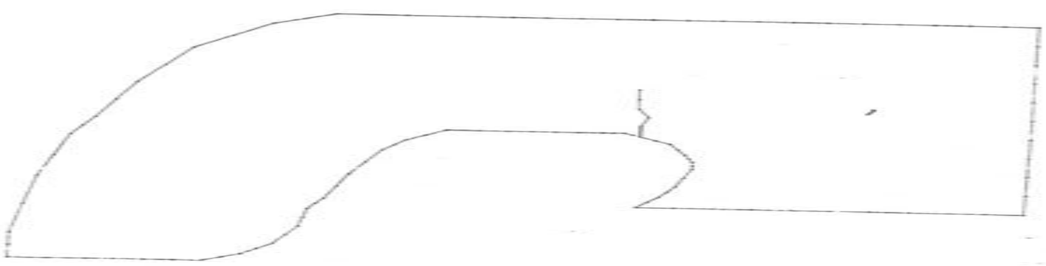


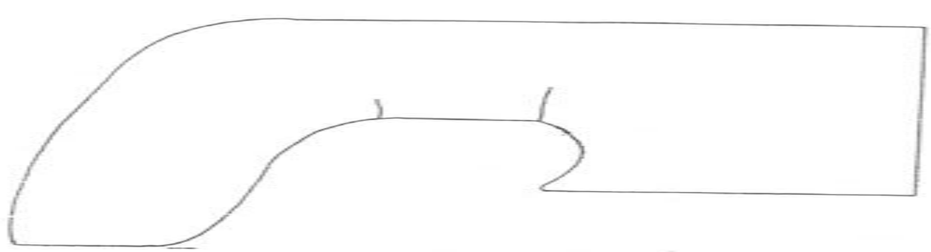
Stress analysis



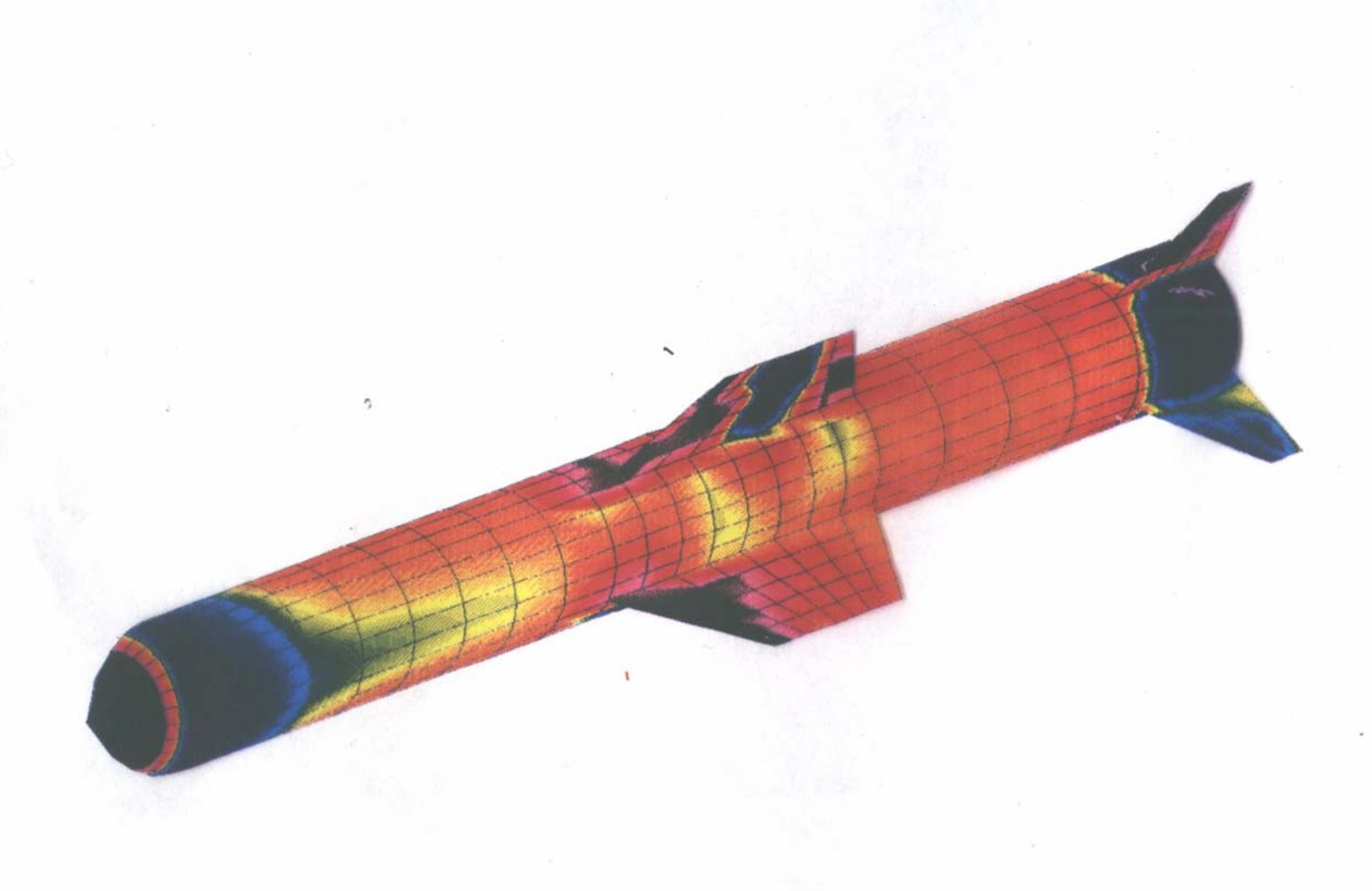
BEM simulation







Shong-Fon II missile



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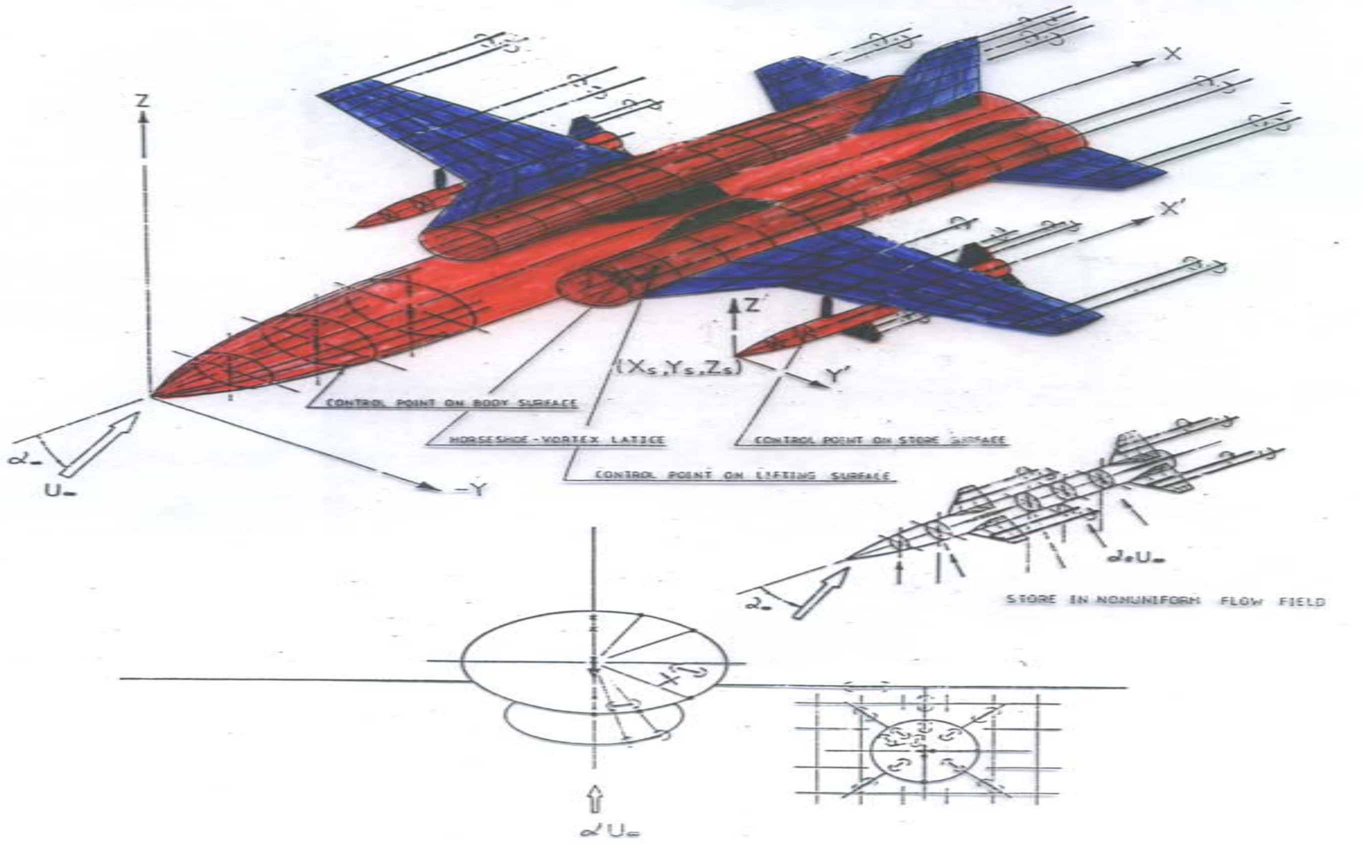


Fig.1 Image system of all the singularities in aircraft/external store configurations.

Flow field

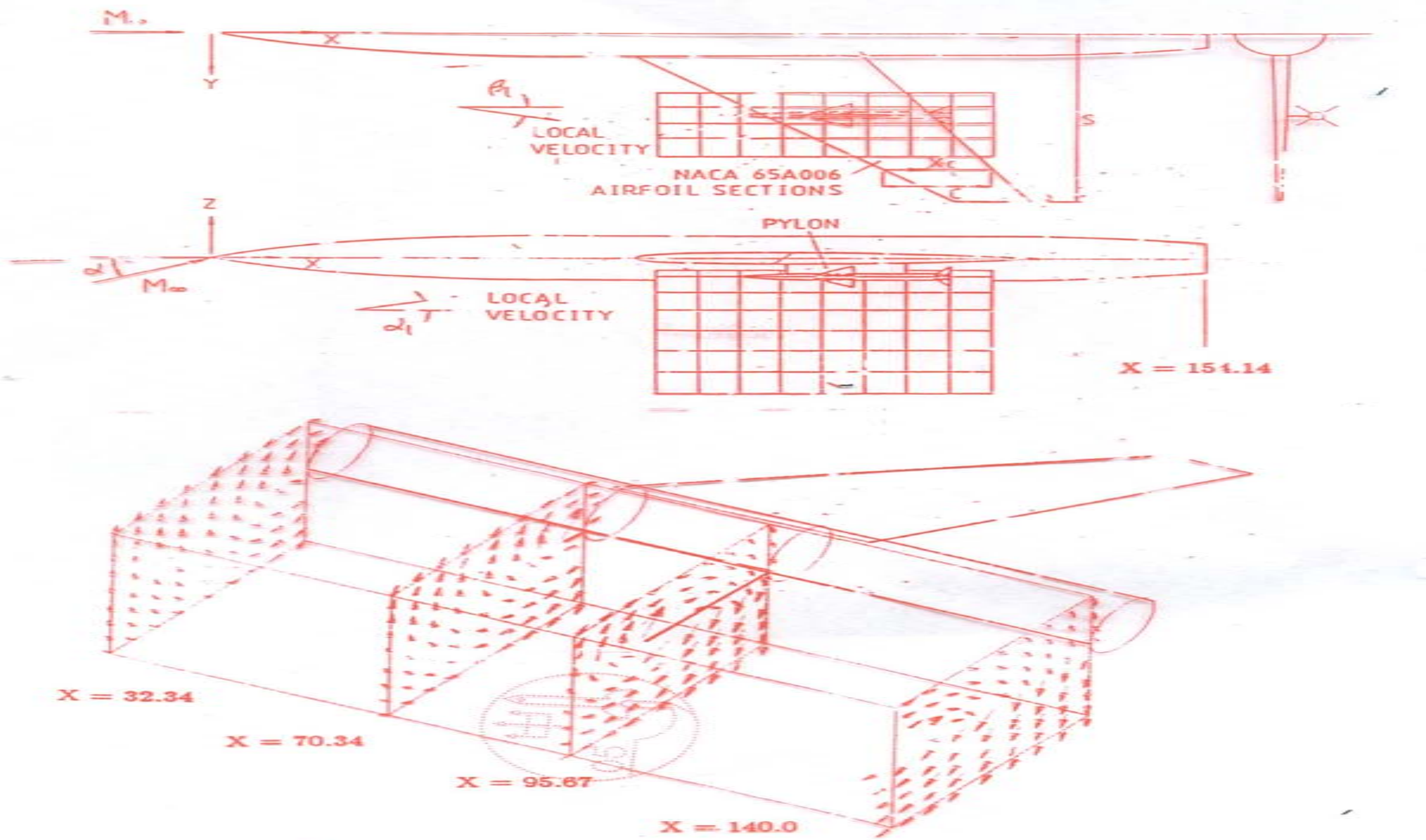
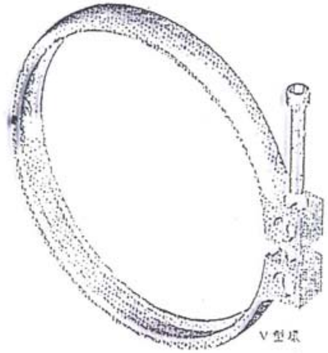


Fig.8 Cross flow velocity field on the reference planes.

V-band structure (Tien-Gen missile)



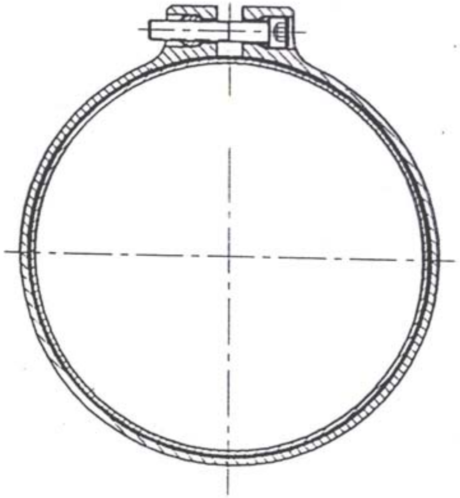


圖 1．V型環的結構示意圖

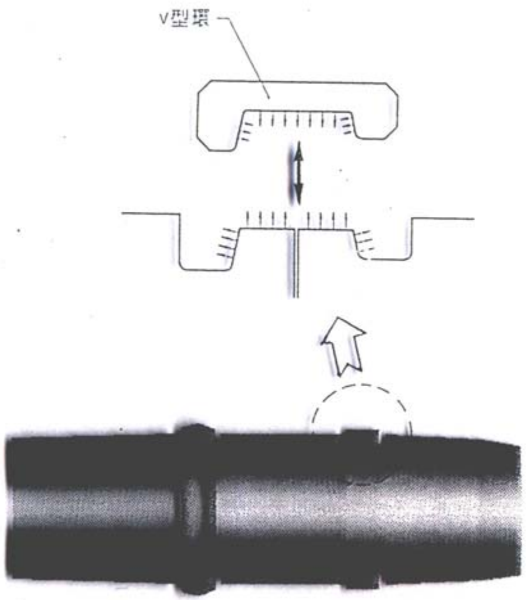
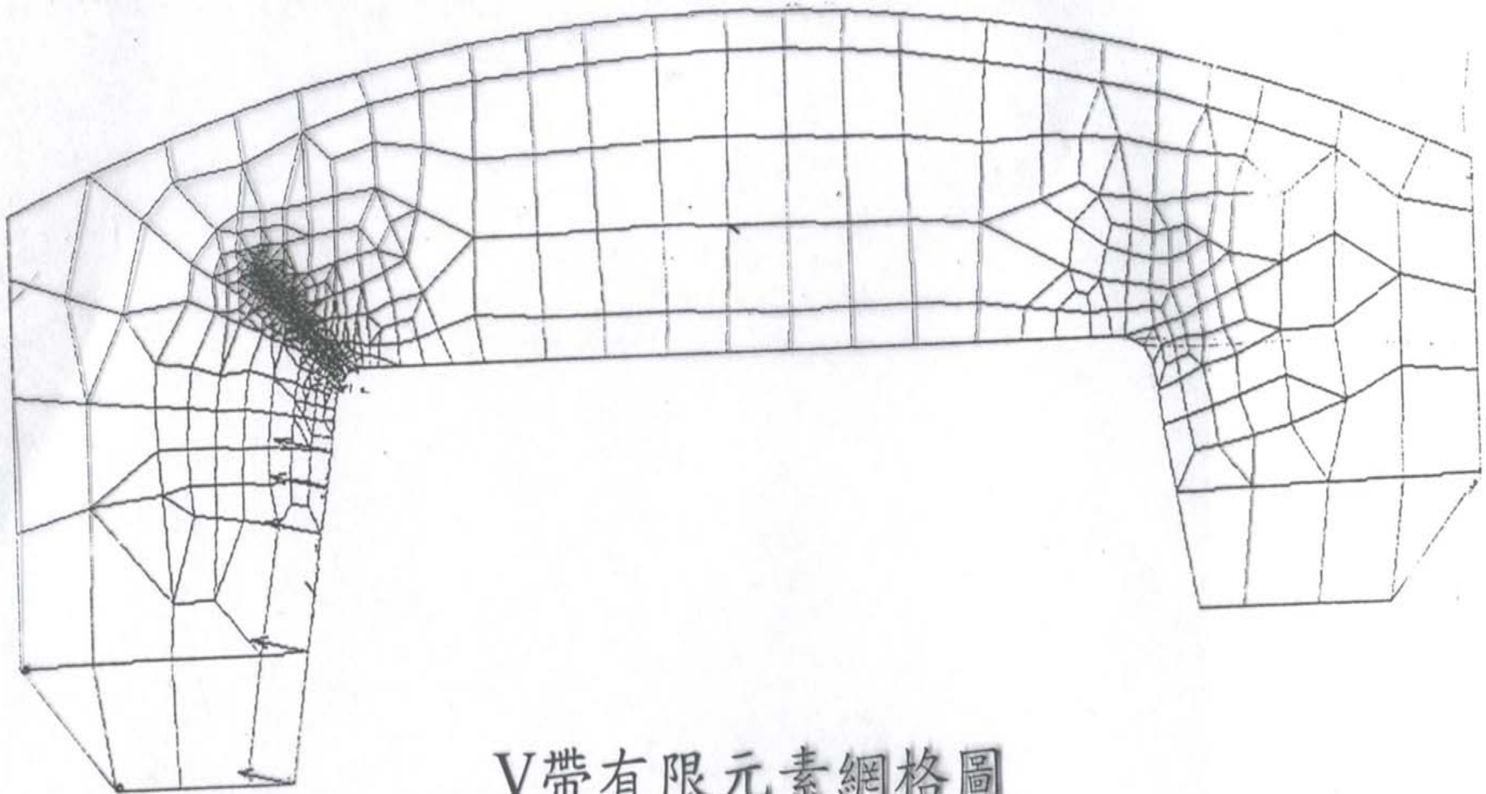


圖 2．V型環的結合功能

V帶結構示意圖

FEM simulation



V帶有限元素網格圖

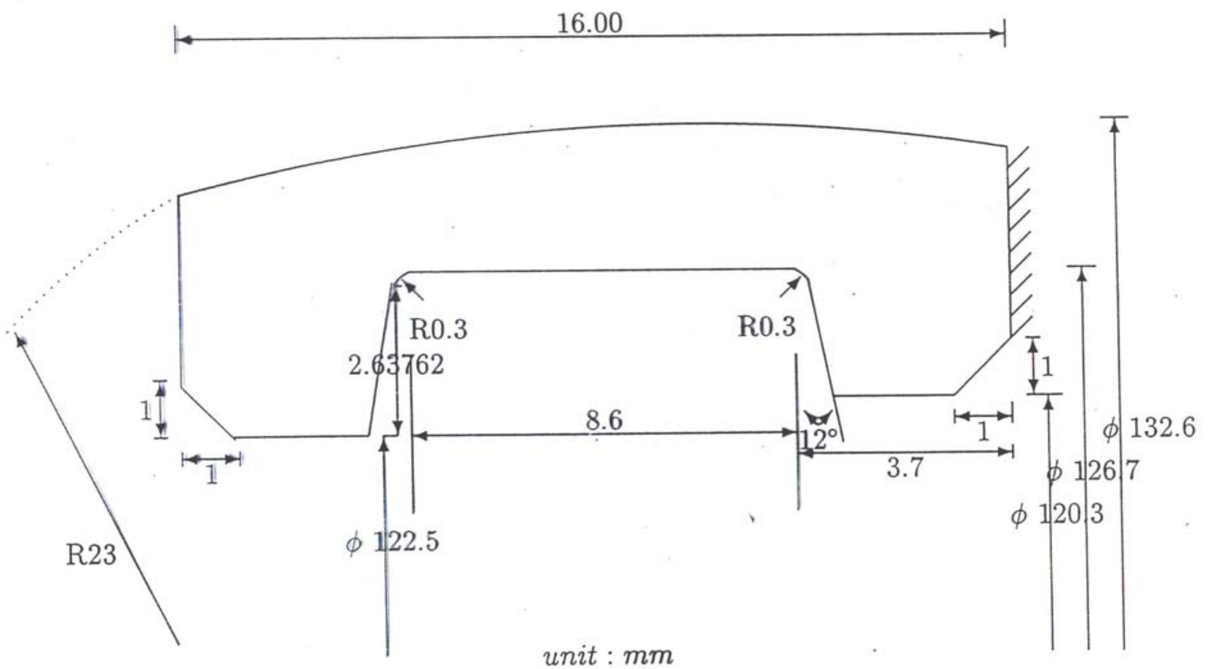
Application to V-band structure:

$E = 19950\ kgf/mm^2$, $\upsilon = 0.27$,

$a = 0.125$ $\sigma = 3.63 kgf / mm^2$

Pari's law: $\frac{da}{dN} = C(\Delta K)^m$

$C = 4.624 \times 10^{-12}$, $m=3.3$, $R = \frac{2}{3}$



Seepage flow

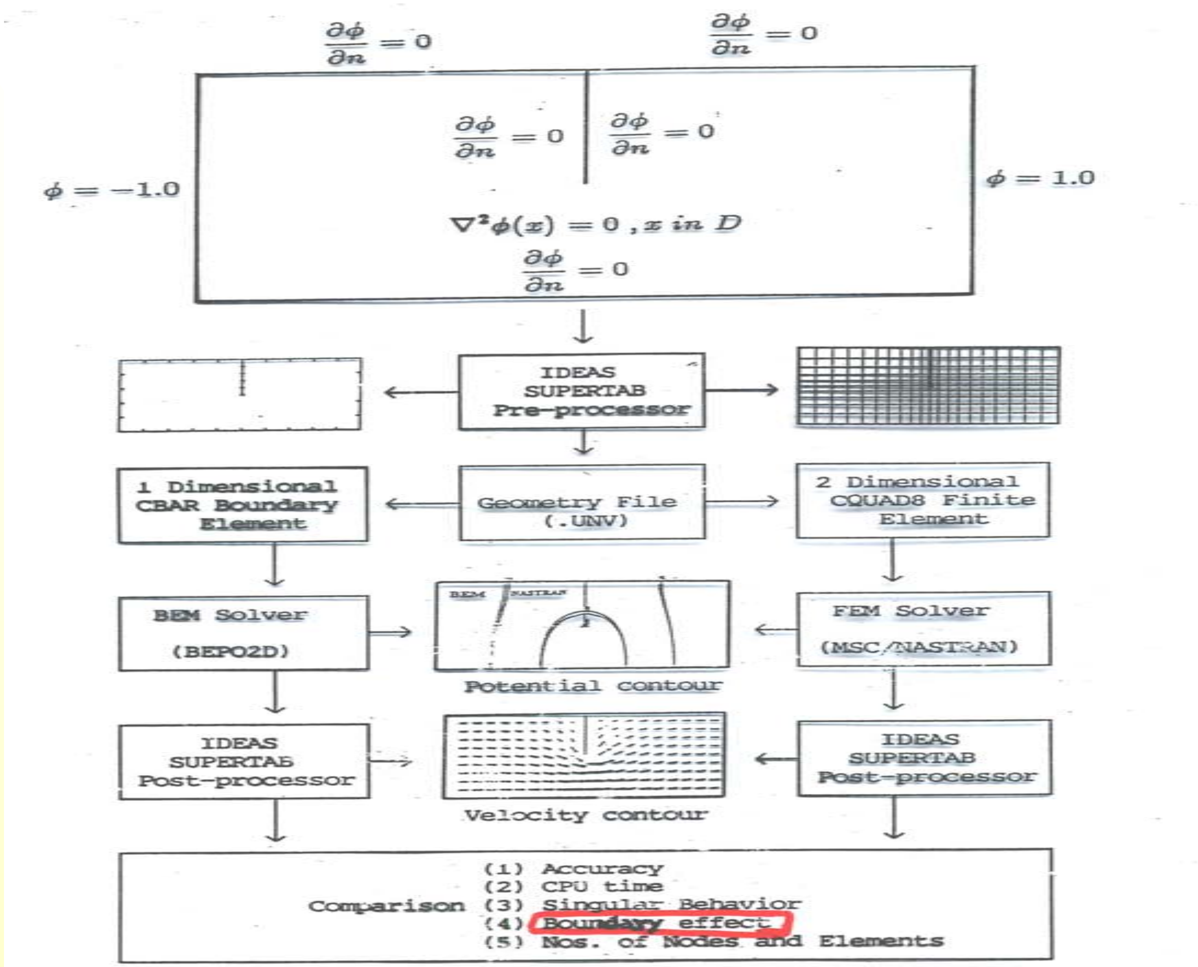
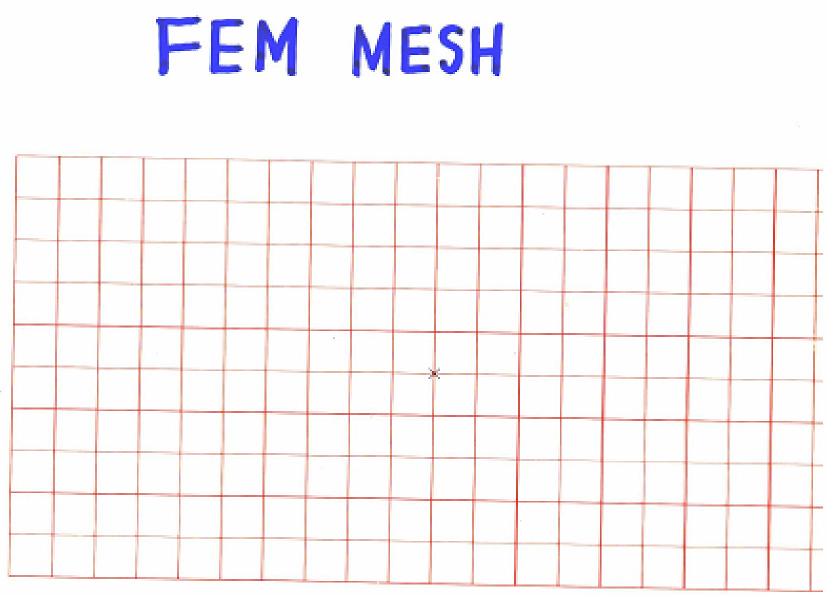
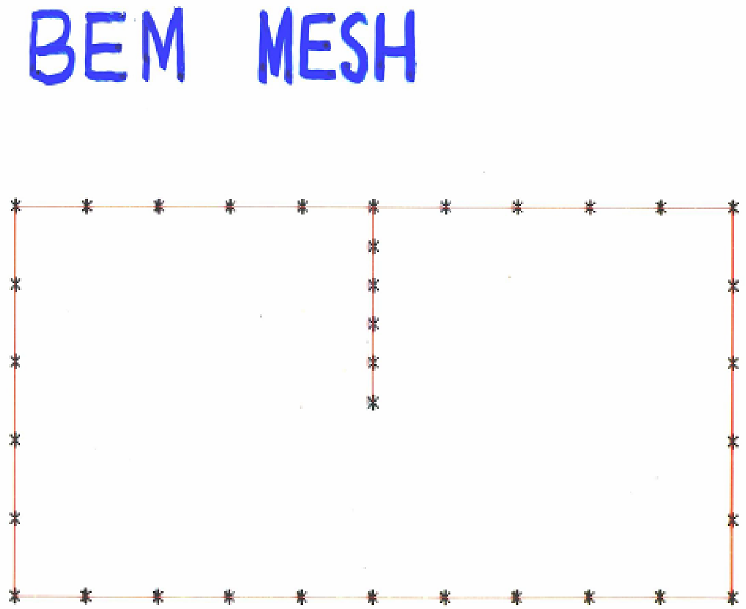


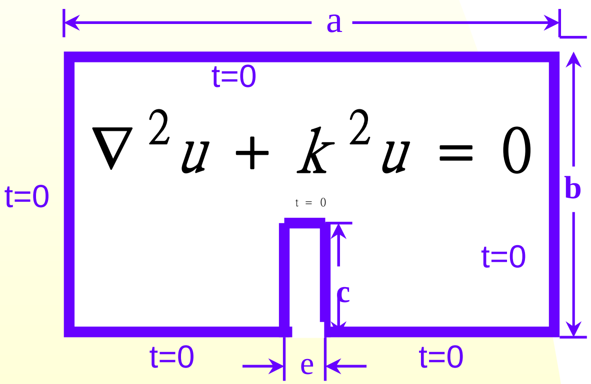
Fig.4 Flowchart of BEM and FEM solver system.

Meshes of FEM and BEM





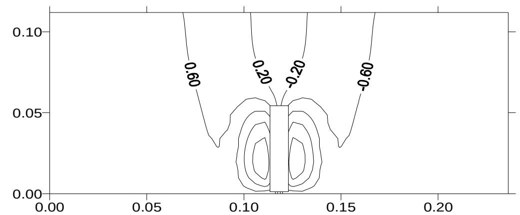
Screen in acoustics



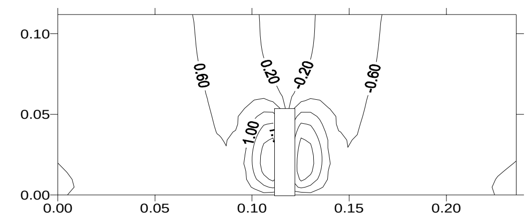
UT *LM*

mode 1.

587.6 Hz

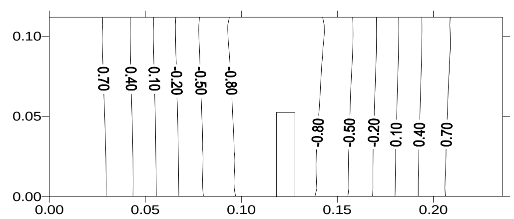


587.2 Hz

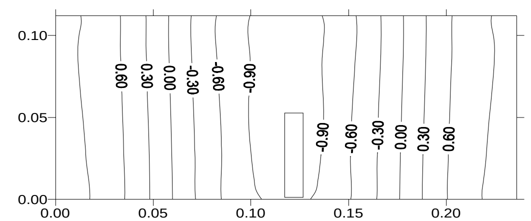


mode 2.

1443.7 Hz

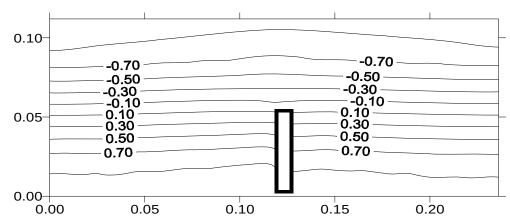


1444.3 Hz

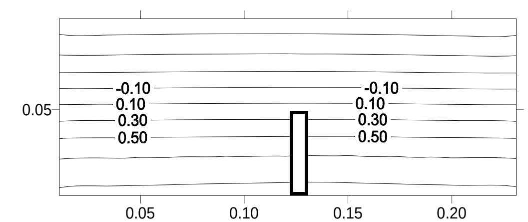


mode 3.

1517.3 Hz



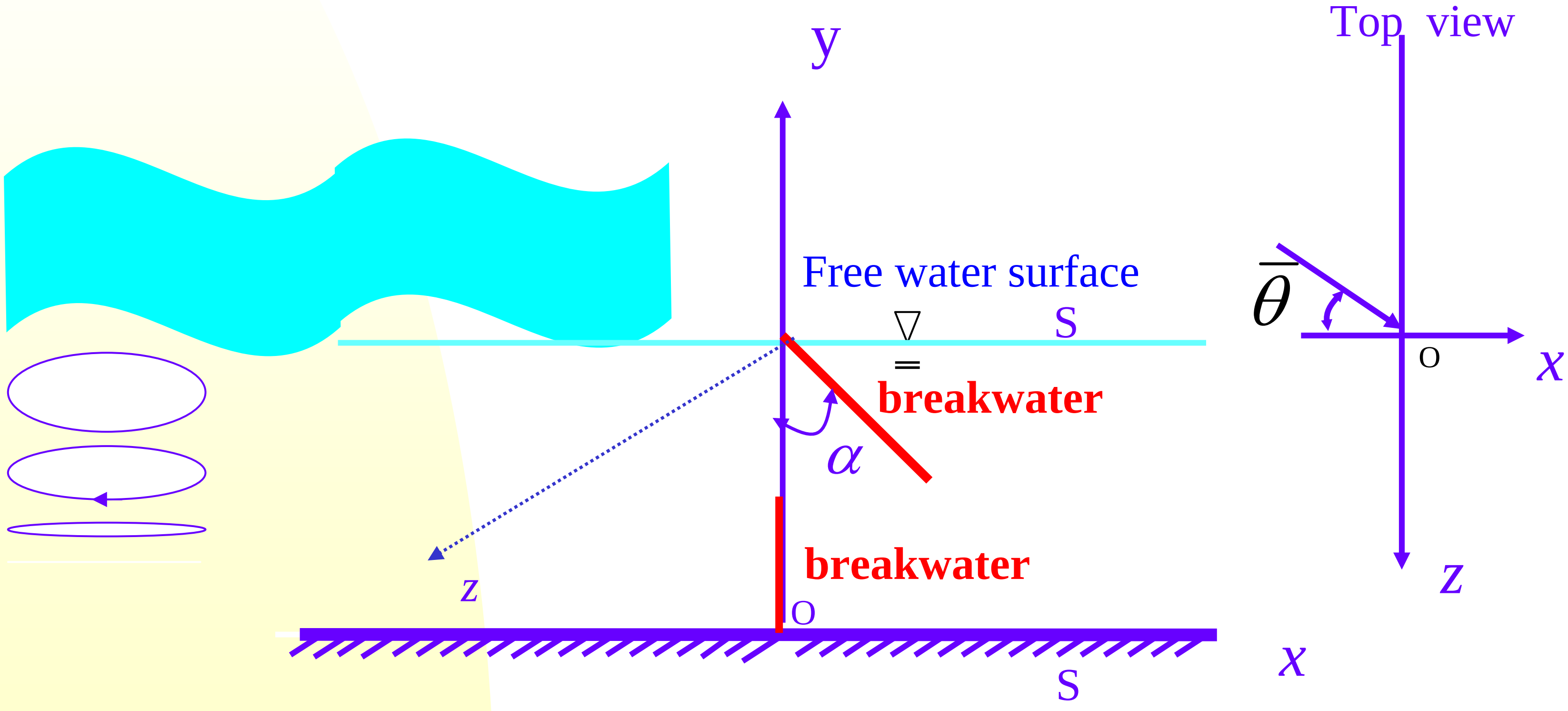
1516.2 Hz



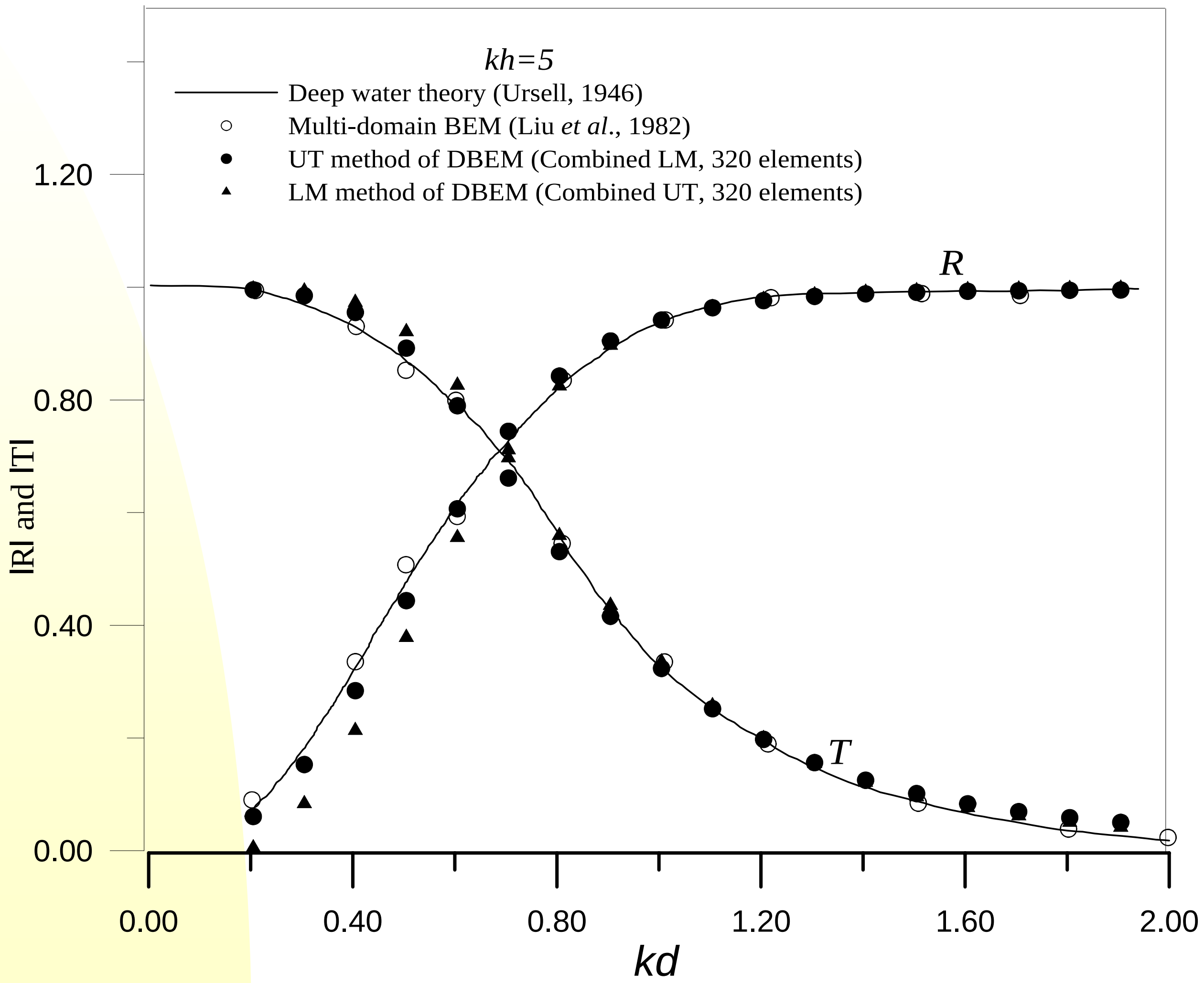
Water wave problem with breakwater

$$\nabla^2 u(\tilde{x}) - \lambda^2 u(\tilde{x}) = 0$$

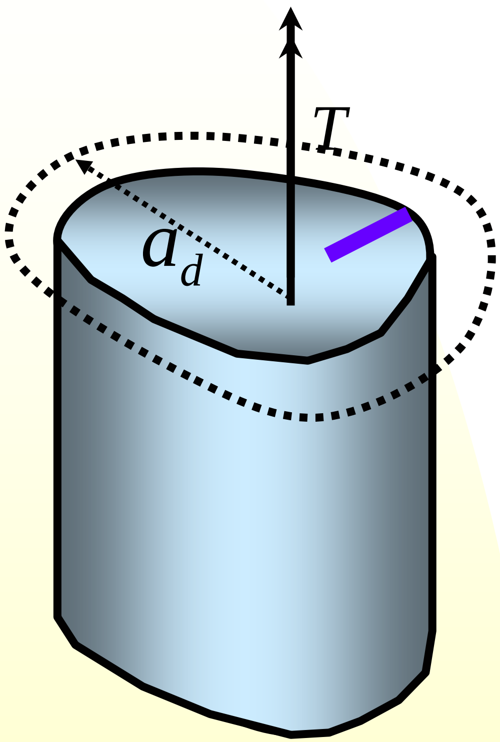
oblique incident water wave

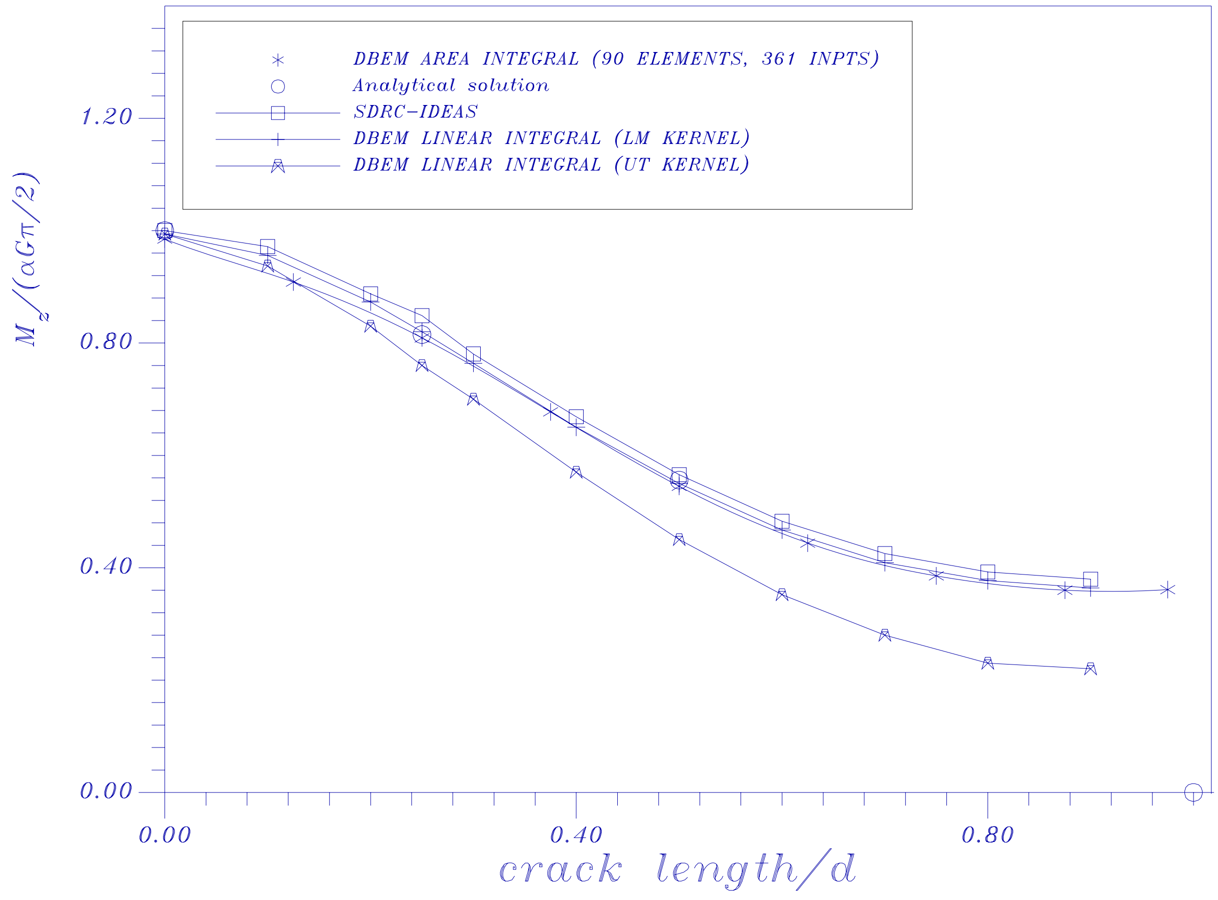


Reflection and Transmission

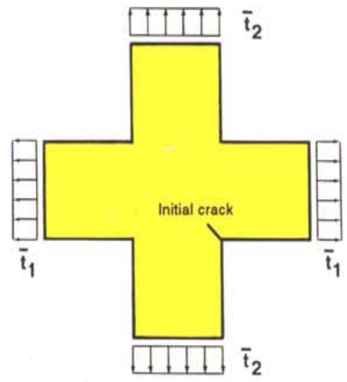


Cracked torsion bar

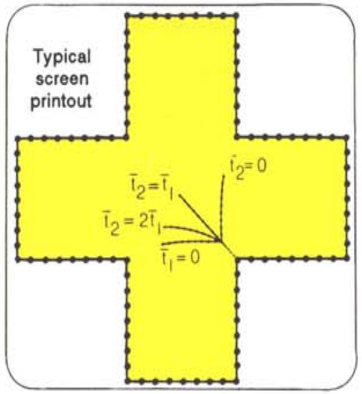




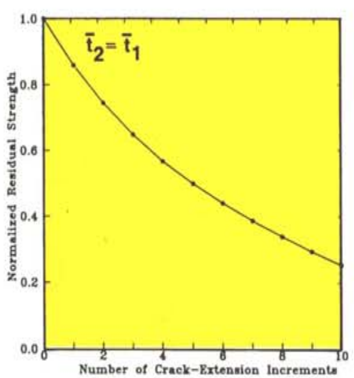
Fatigue life and residual strength calculations



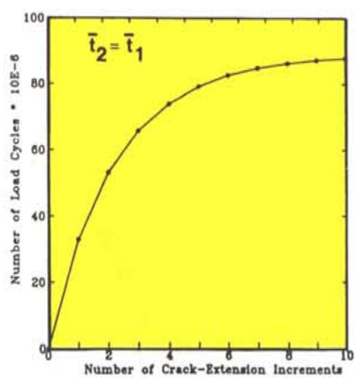
Cruciform cracked plate



Crack paths for the cruciform cracked plate



Residual strength diagram



Fatigue life diagram

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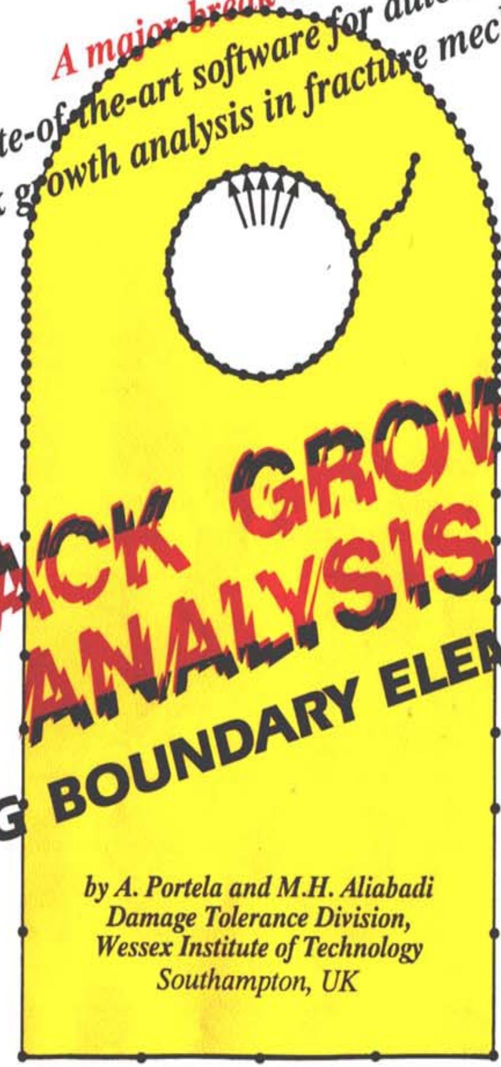
Crack Growth Analysis using Boundary Elements - Software

ISBN: 1 85312 186 X ringbinder/diskette/50 page manual/Topics book. Price: £675/$995

A major break-through

state-of-the-art software for automatic crack growth analysis in fracture mechanics



CRACK GROWTH ANALYSIS USING BOUNDARY ELEMENTS

by A. Portela and M.H. Aliabadi
Damage Tolerance Division,
Wessex Institute of Technology
Southampton, UK

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Crack Growth Analysis

There are many Finite Element software packages for crack growth analysis currently available. However, they all have a common drawback, which is the requirement for remeshing as the crack propagates. This software utilizes the state-of-the-art development in the boundary element method and for the first time removes the difficult and time consuming task of remeshing. Furthermore, it evaluates accurate stress intensity factors for which the Boundary Element Method is renowned. The software uses the established criterion for crack propagation and evaluates the residual strength as well as fatigue life calculations.

MAIN FEATURES:

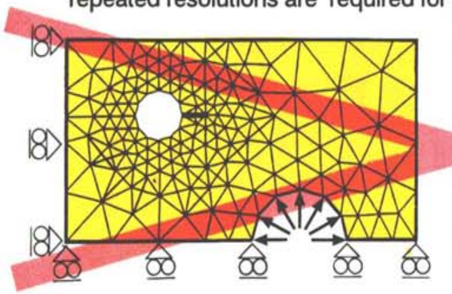
- ☆ **Automatic incremental crack propagation**
- ☆ **Eliminates remeshing for crack growth analysis**
- ☆ **Accurate evaluation of stress intensity factors**
- ☆ **Residual strength and fatigue life computations.**

MODULES IN THE SOFTWARE:

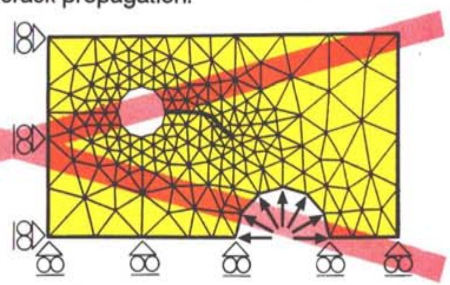
- ☆ **Data generation with a minimum of input**
- ☆ **Plotting of the mesh**
- ☆ **Automatic fatigue crack growth analysis**
- ☆ **Plotting of the deformed configuration and principal stresses**
- ☆ **Plotting of the crack path**

The old approach

The Finite Element approach: continuous remeshing and repeated resolutions are required for crack propagation.



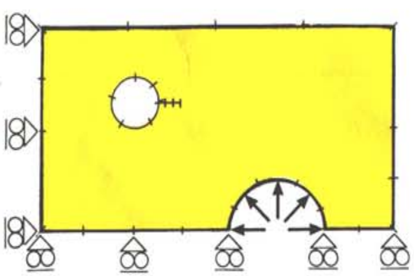
Starting mesh



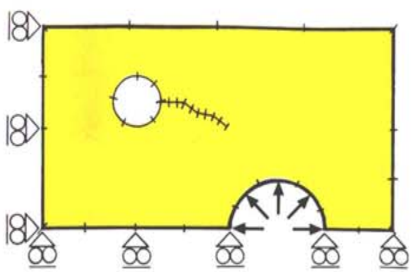
Mesh after three crack propagation increments

The new approach

The Boundary Element approach: No remeshing is required for crack propagation.



Starting Mesh



Mesh after three crack propagation increments

Program Description

The software features include the use of quadratic continuous and discontinuous elements, evaluation of boundary stresses, displacements and tractions, element or point constraint including skew constraints and mixed-mode path independent integrals for the accurate evaluation of stress intensity factors. Automatic crack propagation algorithm is implemented utilizing an incremental crack extension which employs special solver to avoid resolution for each crack extension.

The fracture criterion is based on the maximum principal stress and the fatigue crack growth rates are calculated using established formulae.

The software package is accompanied with a user manual for data generation and the analysis program as well as a book *Boundary Elements in Crack Growth Analysis*- describing the basic theory of the method. The source code in FORTRAN is included along with several example problems to demonstrate the use of the code.

The Boundary Element Method (BEM) is now widely regarded as the most accurate numerical tool for analysis of crack problems in linear elastic fracture mechanics. This software package is based on a new formulation of BEM called Dual Boundary Element Method (DBEM) developed at the Damage Tolerance Division of Wessex Institute of Technology. The Dual Boundary Element Method retains all of the important features of BEM which are: reduced set of equations, simple data preparation, accurate evaluation of stresses, strains and displacements at selected internal points as well as introducing additional improvements which include crack modelling in a single region and accurate stress intensity factors evaluation.

ORDER FORM

Please send me the following software package

Quantity	*Title/Author*	*Price*
	Crack Growth Analysis using Boundary Elements by A. Portela and M.H. Aliabadi	*£675**

**$995 for USA, Canada and Mexico - postage & packing UK £4/$7, USA £5/$9.*

Name ______________________

Organisation ______________________

Position ______________________

Address ______________________

____

Please indicate method of payment ______________________

Cheque number ______________________

From Portela

I wish to pay by Credit card ☐ *Expiry date* ______________________

Name ______________________

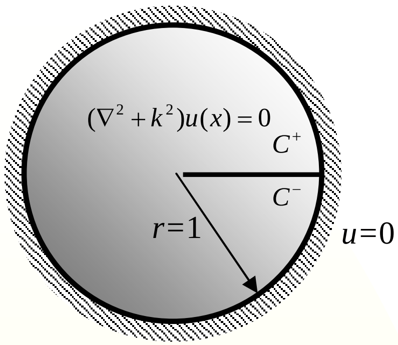
Number ______________________ Nov. 1993

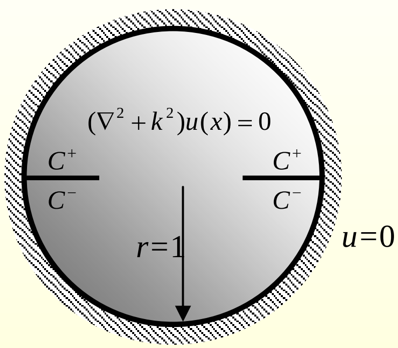
Is it possible !

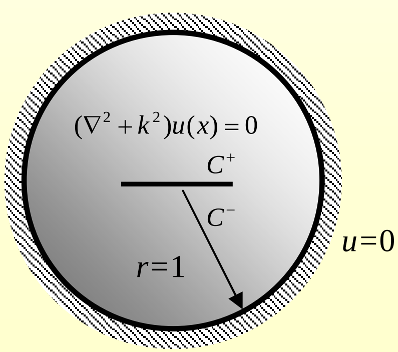
No hypersingularity !

No subdomain !

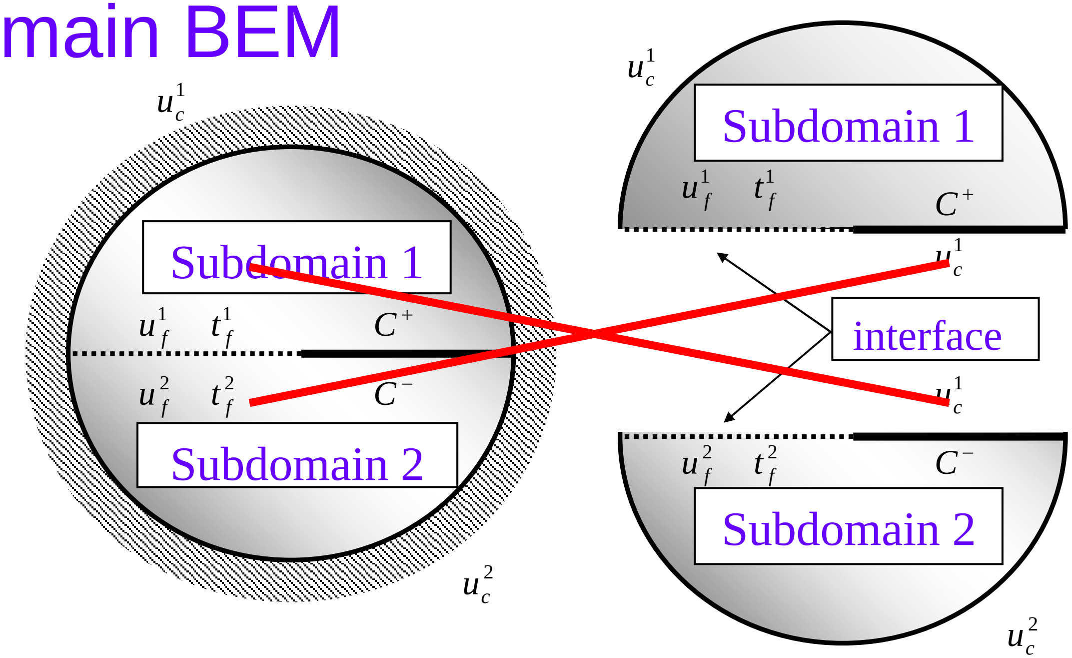
Degenerate boundary problems







- Multi-domain BEM



- Dual BEM

$$[T]\{u\}=[U]\{t\}$$

$$\boxed{\cancel{[M]\{u\}=\{L\}\{t\}}}$$

Conventional BEM in conjunction with SVD

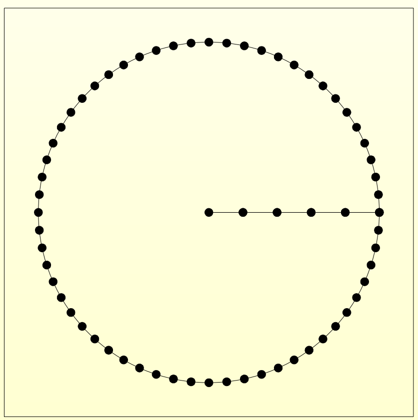
Singular Value Decomposition

$$[U]_{M\times P} = [\Phi]_{M\times M}[\Sigma]_{M\times P}[\Psi]^{H}_{P\times P}$$

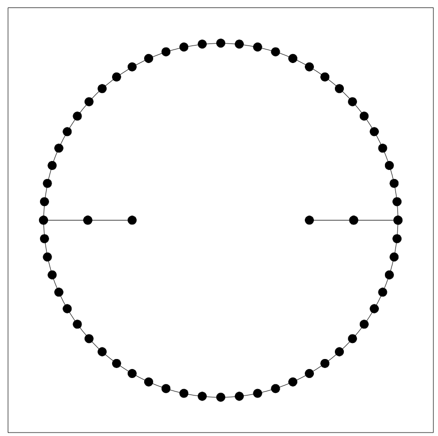
Rank deficiency originates from two sources:

(1). Degenerate boundary

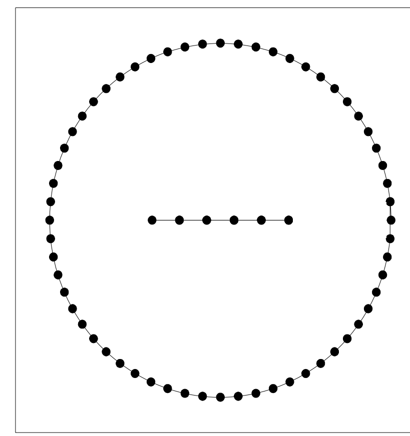
(2). Nontrivial eigensolution



$N_d=5$



$N_d=4$



$N_d=5$

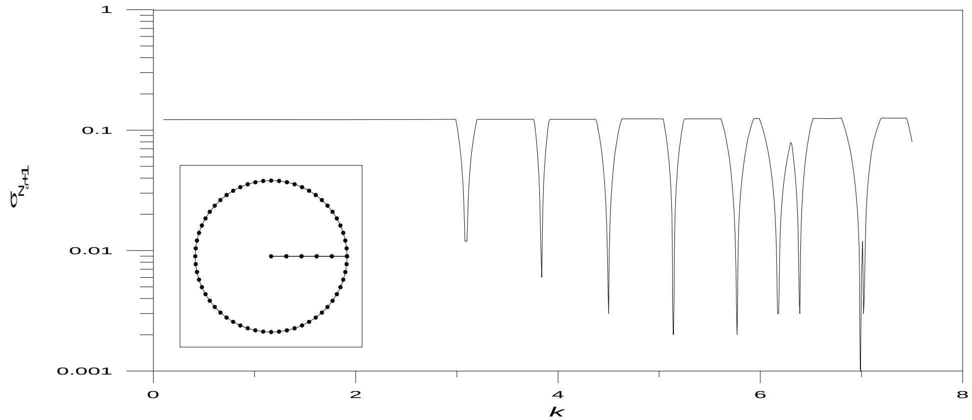
- ***UT* BEM + SVD (Present method)**

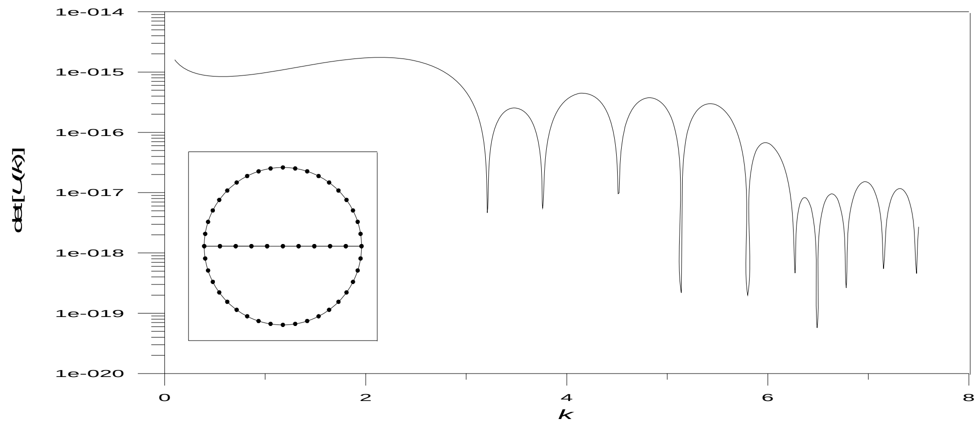
σ_{N_d+1} versus k

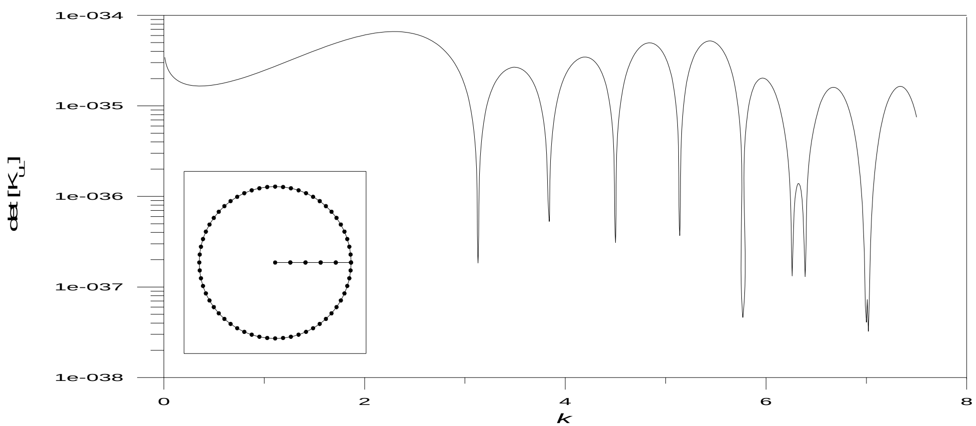
Determinant versus k

- **Dual BEM**

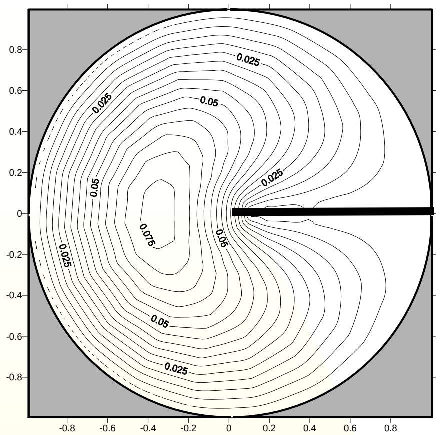
Determinant versus k



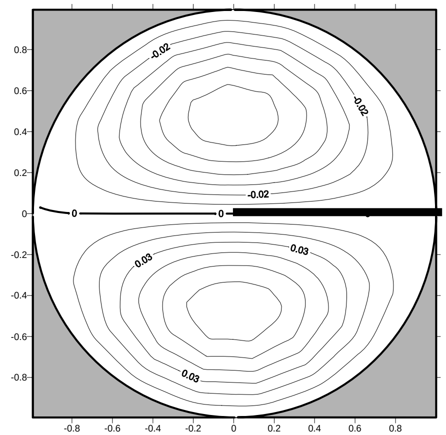




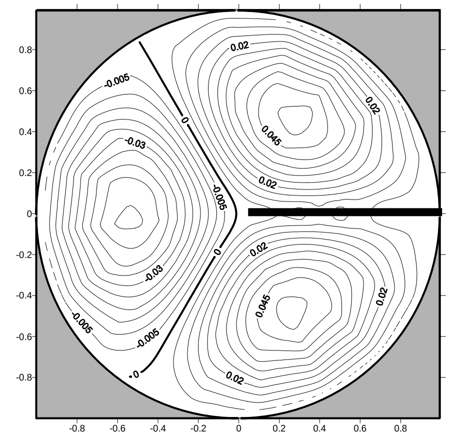
UT BEM+SVD



***k*=3.09**

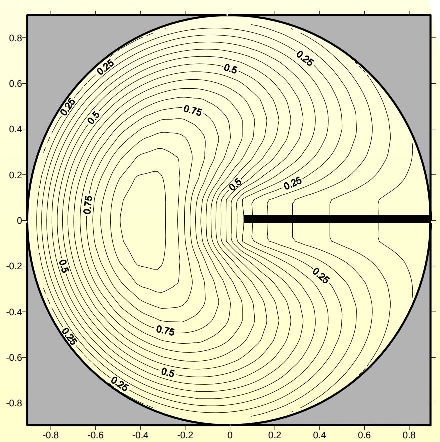


***k*=3.84**

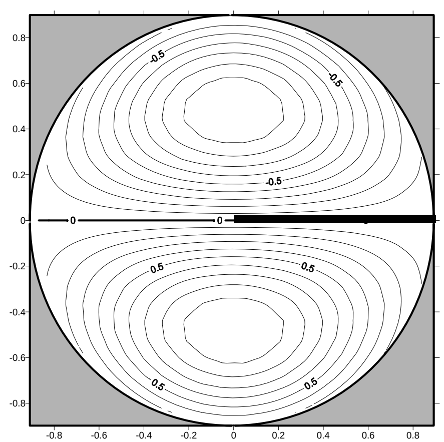


***k*=4.50**

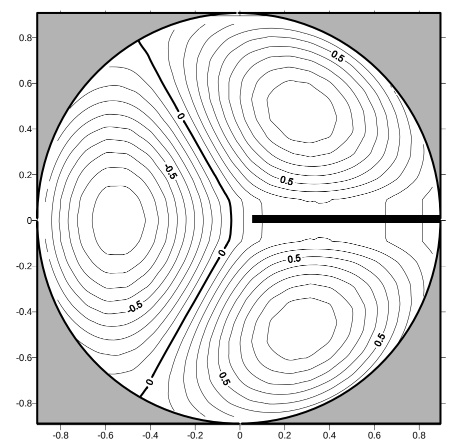
FEM (ABAQUS)



***k*=3.14**



***k*=3.82**



***k*=4.48**

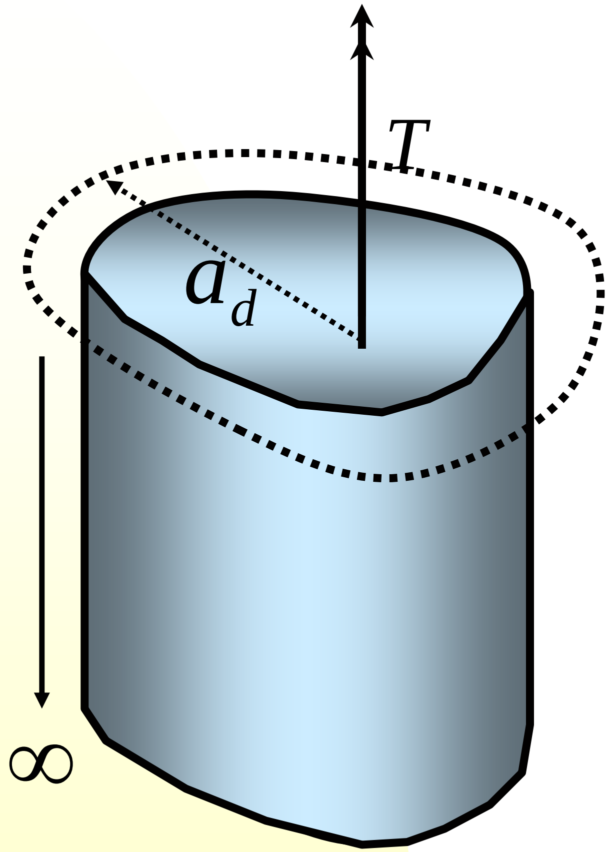
BEM trap ?
Why engineers should learn mathematics ?

- Well-posed ?
- Existence ?
- Unique ?

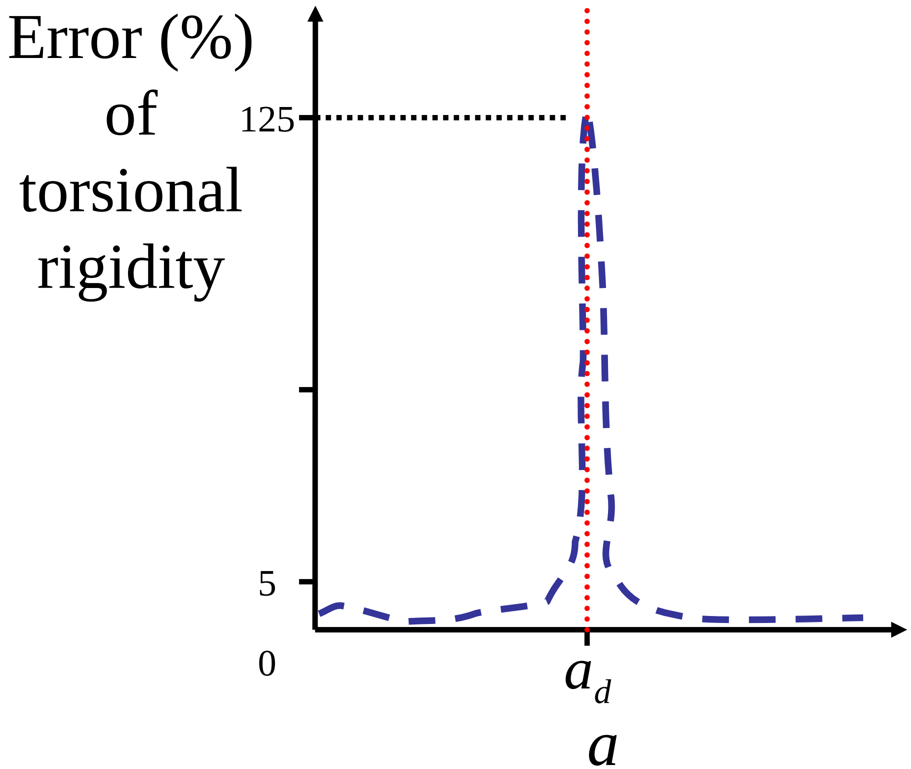
- Mathematics versus Computation

- Some examples

Numerical phenomena
(Degenerate scale)



Commercial ode output ?

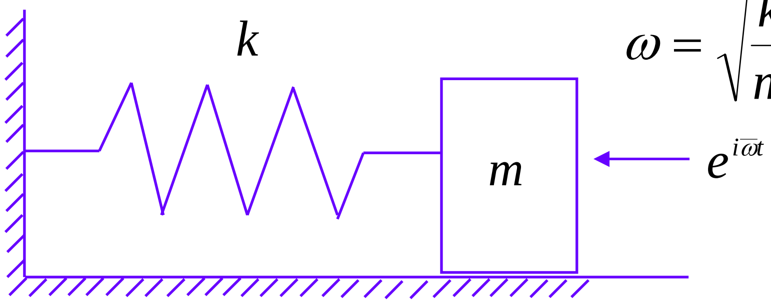


Previous approach : Try and error on a
Present approach : Only one trial

Numerical and physical resonance

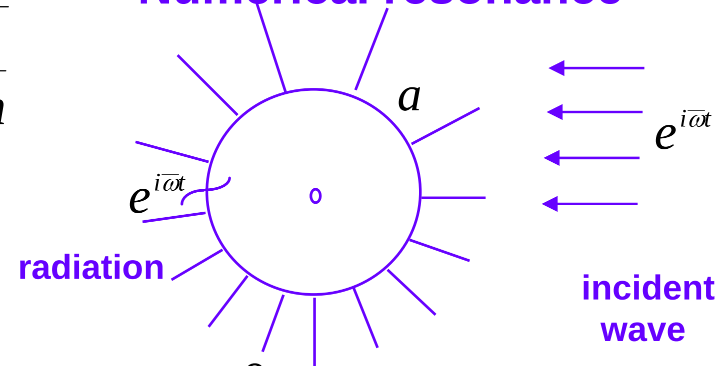
Physical resonance

$$\omega = \sqrt{\frac{k}{m}}$$



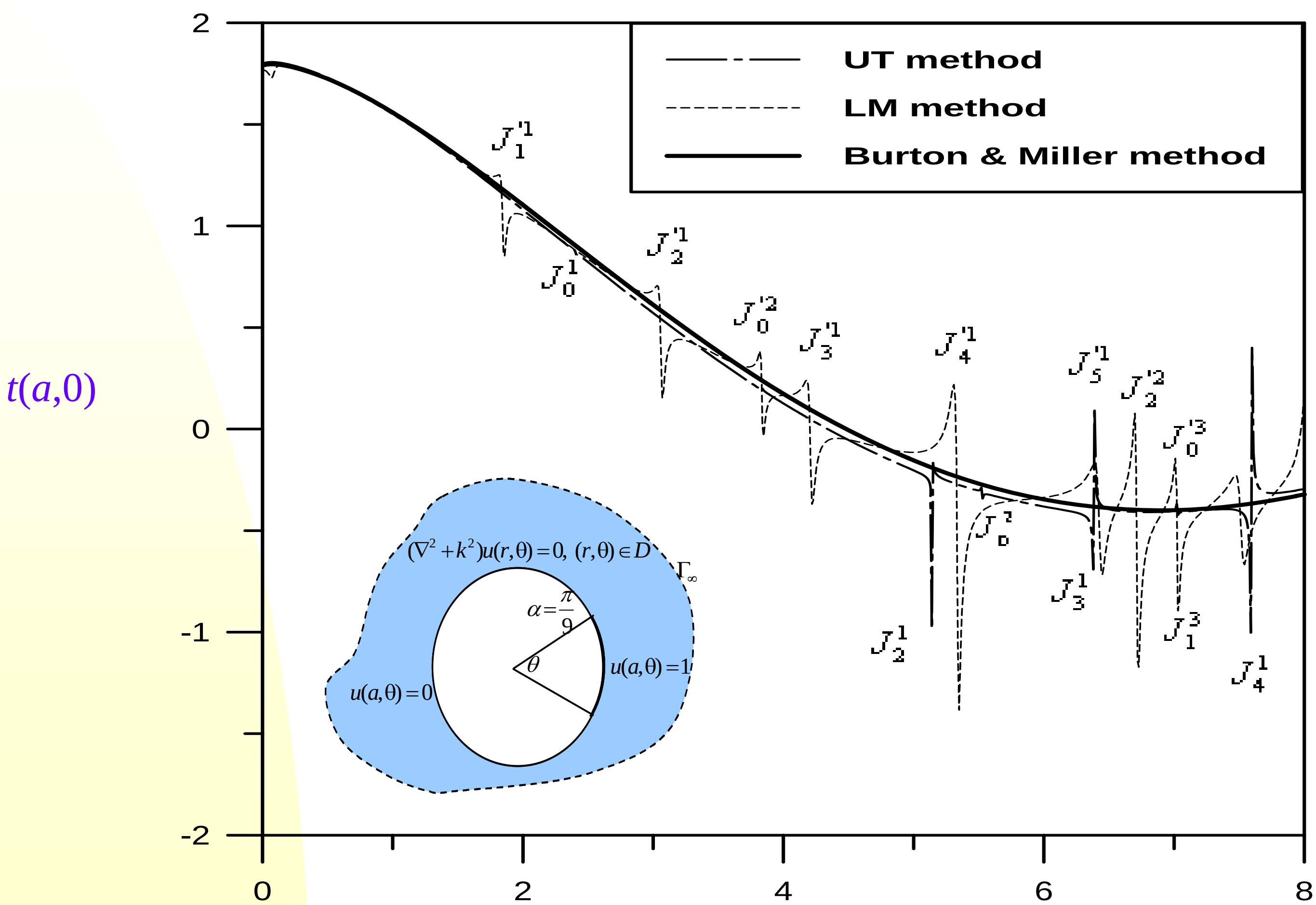
$$u = \frac{finite}{(\omega^2 - \bar{\omega}^2)} \to \infty, \text{ if } \bar{\omega} \to \omega$$

Numerical resonance



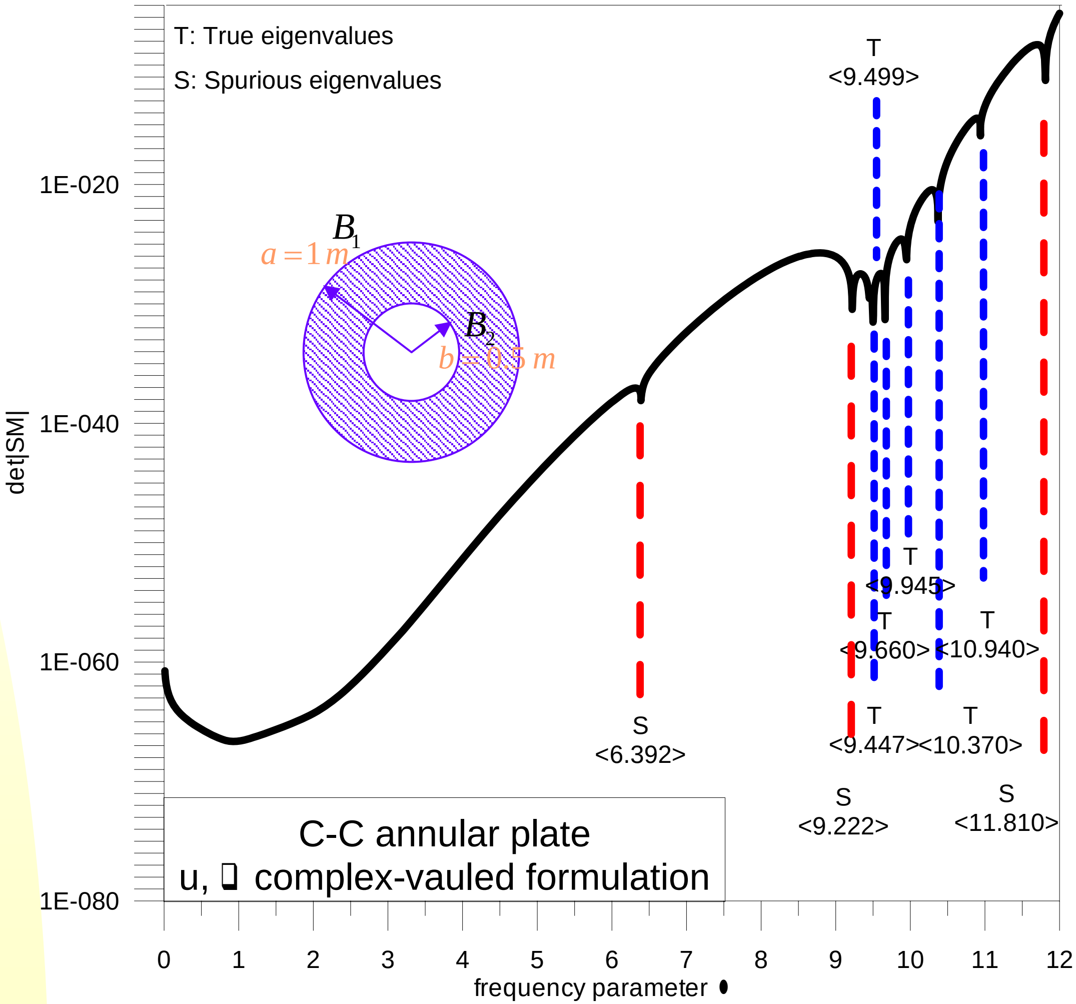
$$u = \lim_{\bar{\omega} \to \omega} \frac{0}{0} \to finite, \text{ if } \bar{\omega} \to \omega$$

Numerical phenomena
(Fictitious frequency)



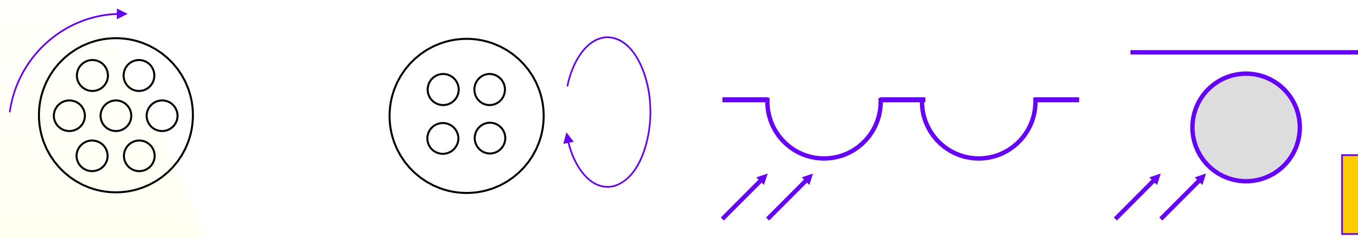
A story of NTU Ph.D. students

Numerical phenomena
(Spurious eigensolution)



Some findings

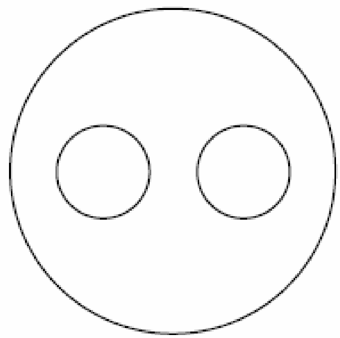
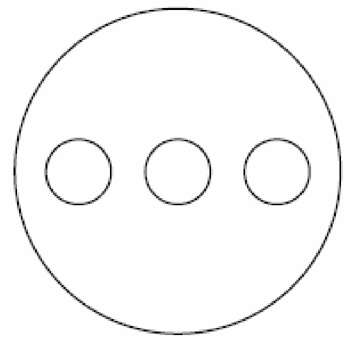
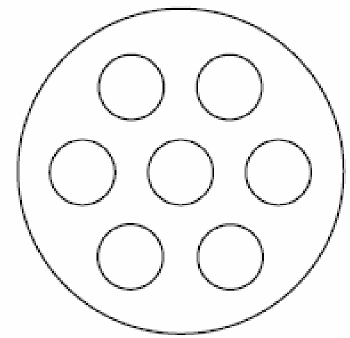
`Laplace



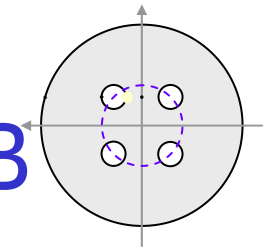
Helmholtz

Ling[1947] *Analytical solution*	Bird & Steele[1991]	房營光[1995] *Analytical solution*	Lee & Manoogian[1992]
Caulk[1983]	Naghdi[1991] *Analytical solution*	Tsaur *et al.*[2004] *Analytical solution*	Present method
Present method			Tsaur *et al.*

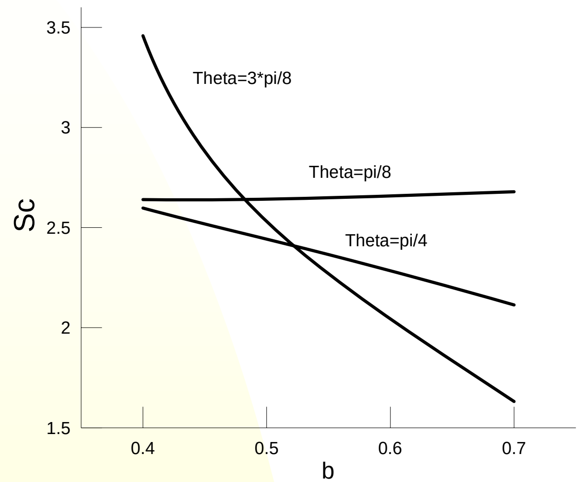
Torsional rigidity

Case				
		$N=2, c/R=0$ $a/R=2/7, b/R=3/7$	$N=2, c/R=1/5$ $a/R=1/5, b/R=3/5$	$N=6, c/R=1/5$ $a/R=1/5, b/R=3/5$
$\frac{2G}{(\mu\pi R^4)}$	Caulk(First-order approximate)	0.8739	0.8741	0.7261
	Exact BIE formulation	0.8713	0.8732	0.7261
	Ling's results	0.8809	0.8093 ?	0.7305
	The present method	0.8712	0.8732	0.7245

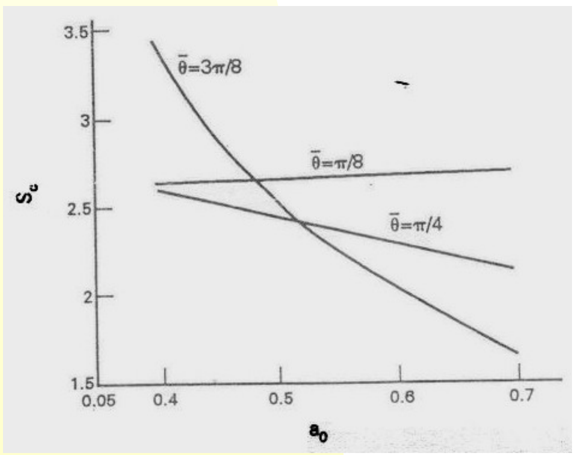
Stress concentration at point B

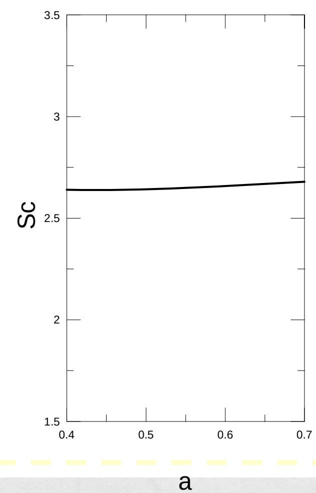


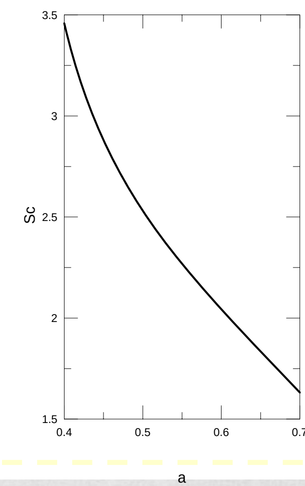
Present method



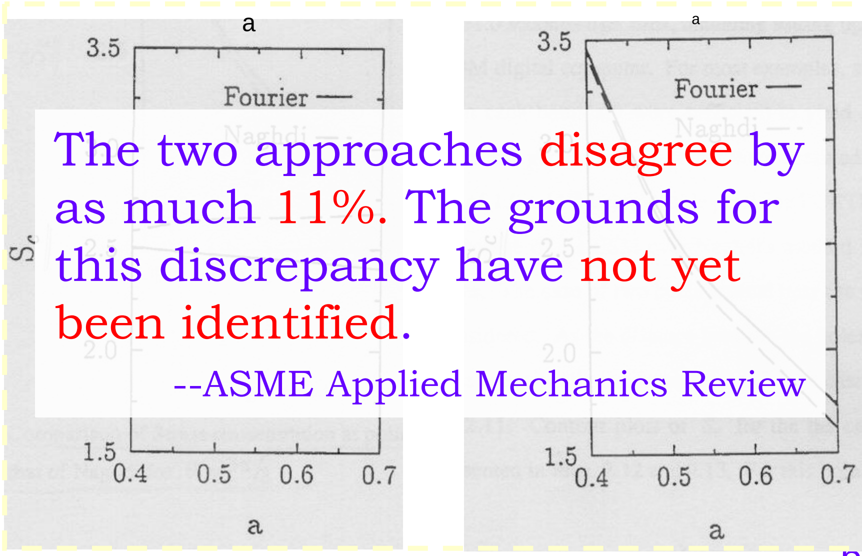
Naghdi's results







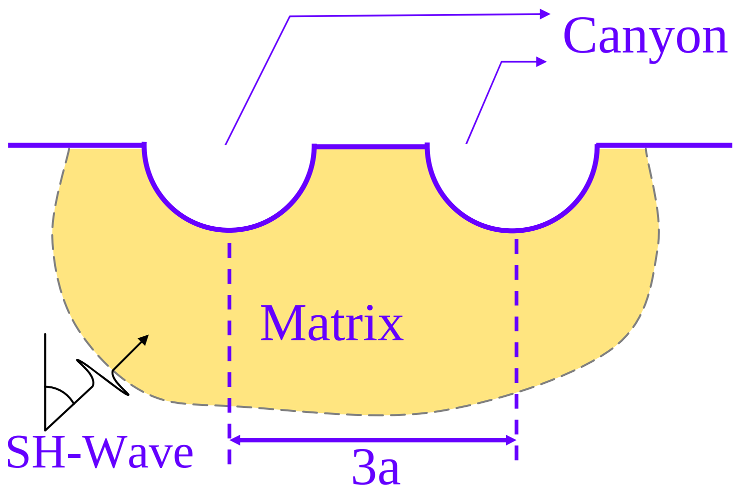
Steele & Bird



The two approaches disagree by as much 11%. The grounds for this discrepancy have not yet been identified.

--ASME Applied Mechanics Review

A half-plane problem with two alluvial valleys subject to the incident SH-wave

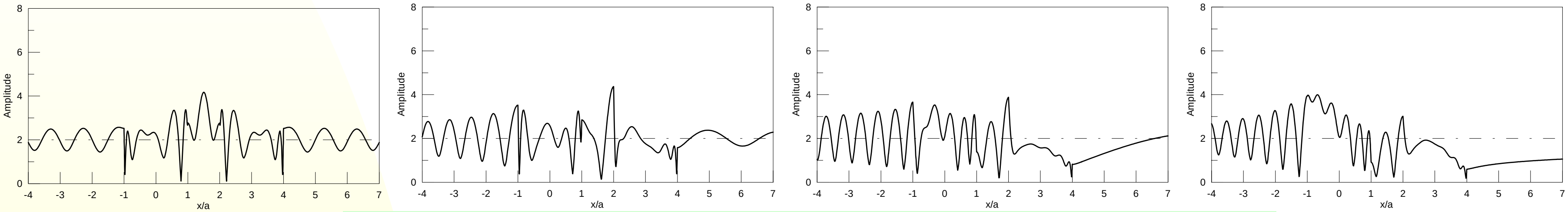


房[93]將正弦和餘弦函數的正交特性使用錯誤，以至於推導出錯誤的聯立方程，求得錯誤的結果。

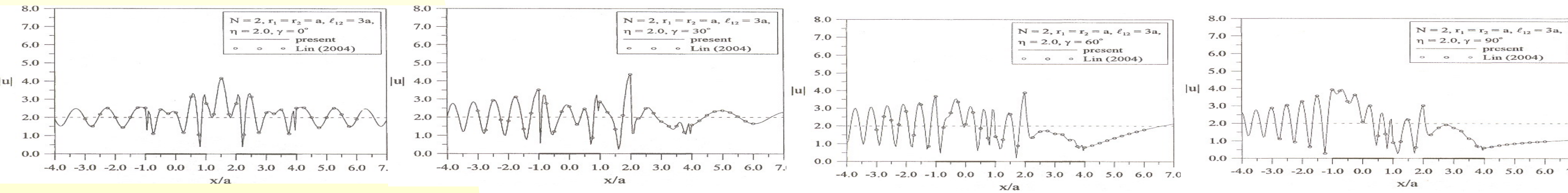
--亞太學報

Limiting case of two canyons

$\gamma = 0^\circ$ $\gamma = 30^\circ$ $\gamma = 60^\circ$ $\gamma = 90^\circ$

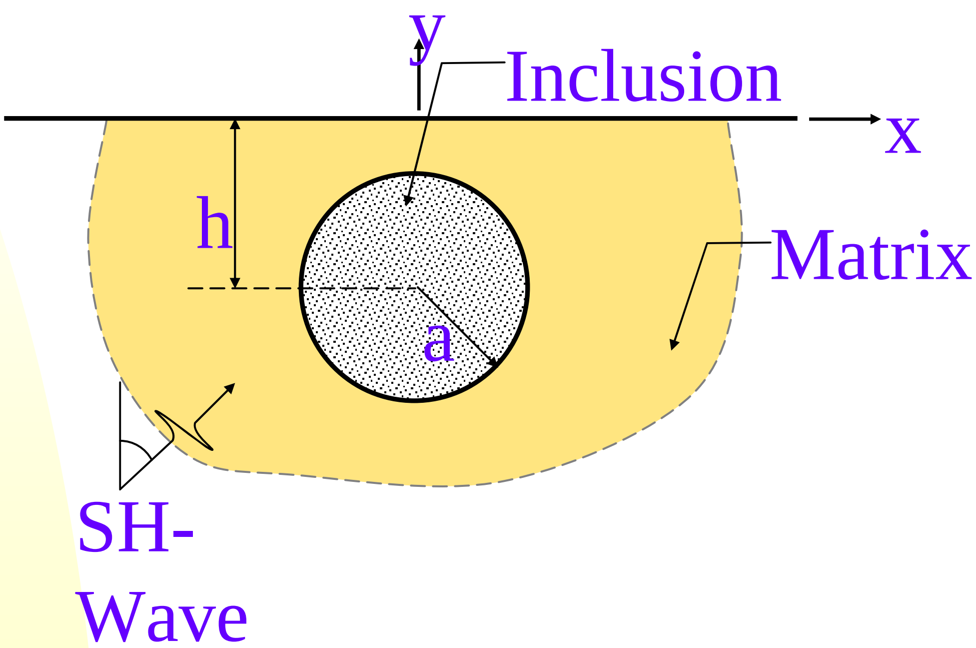


Present method $\eta = 2, \mu^I / \mu^M = 10^{-8}, \rho^I / \rho^M = 2/3$



Tsaur et al.'s results [103]

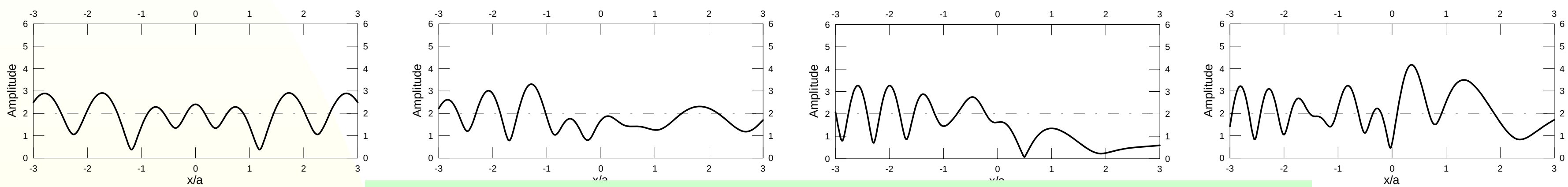
A half-plane problem with a circular inclusion subject to the incident SH-wave



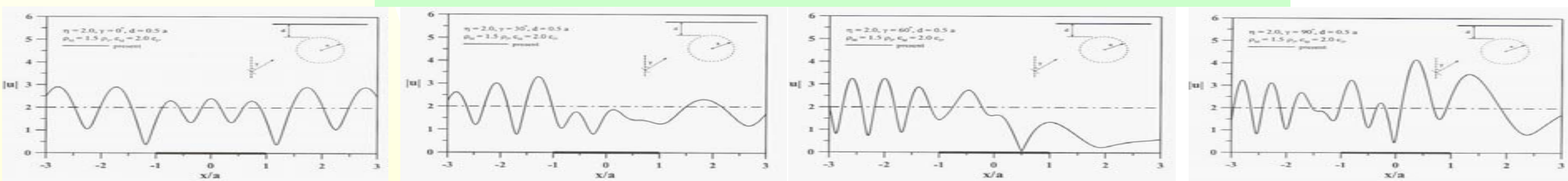
When I solved this problem I could find no published results for comparison. I also verified my results using the limiting cases. I did not have the benefit of published results for comparing the intermediate cases. I would note that due to precision limits in the Fortran compiler that I was using at the time.

--Private communication

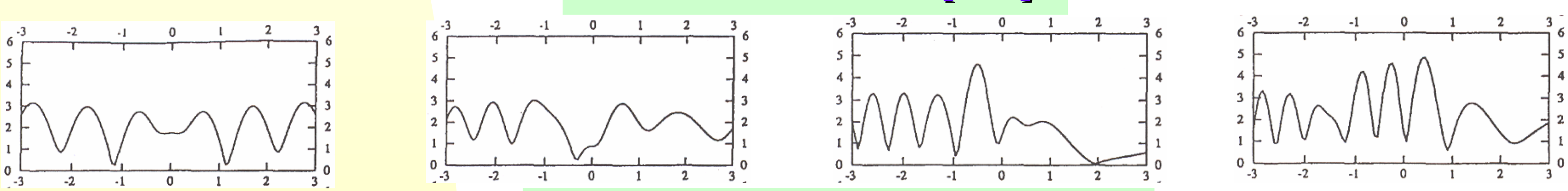
$\gamma = 0^\circ$ $\gamma = 30^\circ$ $\gamma = 60^\circ$ $\gamma = 90^\circ$



Present method $\eta = 2, \mu^I / \mu^M = 1/6, \rho^I / \rho^M = 2/3$



Tsaur et al.'s results [102]



Manoogian and Lee's results [62]

Conclusions

- Introduction of dual BEM
- The role of hypersingular BIE was examined.
- Successful experiences in the engineering applications using BEM were demonstrated.
- The trap of BIEM and BEM were shown
- Previous errors by other investigators were identified

The End

Thanks for your kind attention

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