行政院國家科學委員會專題研究計畫期中報告

計畫編號：NSC 92-2611-E-022-004

無網格法於板的自由振動及外域聲場的分析(2/2)

陳義麟[1]　吳輝在[2]　李應德[3]

1、中文摘要

本研究報告是以基本解法求解外域輻射及散射的聲場問題。藉由兩點函數的基本解，影響函數矩陣的係數很容易便求得。但在求解的過程中也發現與採用邊界元素法求解一樣也會有虛擬頻率的產生，而且亦發現虛擬頻率產生的位置與源點分佈的位置有關。我們採用 Burton & Miller 法來避免此一數值不穩定的現象發生。藉由退化核函數及循環矩陣的特性，對一圓柱的外域聲場問題我們以離散解析來證明虛擬頻率的存在。同時以不均勻輻射及散射問題的數值算例來證明，並同時與邊界元素法的結果作一比較。

關鍵詞：基本解法；兩點函數；不規則頻率；循環矩陣；退化核函數

Abstract

In this report, the applications of the method of fundamental solutions to exterior acoustic radiation and scattering problems are proposed. By using the two-point function of fundamental solution, the coefficients of influence matrices are easily determined. It is found that this method also results in the irregular frequency as well as the boundary element method do. The position of irregular frequency depends on the source location. To avoid this numerical instability, the Burton & Miller technique is employed to deal with the problem. Based on the circulant properties and degenerate kernels, an analytical scheme in the discrete system of a cylinder is achieved to demonstrate the existence of irregular frequency. Two numerical examples of nonuniform radiation and scattering problems of a circular cylinder are examined and are compared with the results by using BEM.

Keywords: Fundamental solutions method; Two-point function; Irregular frequency; Circulant; Degenerate kernels

1、Introduction

In numerical methods, mesh generation of a complicated geometry is always time consuming in the stage of model creation for engineers in dealing with the engineering problems by employing the finite difference method (FDM), finite element method (FEM) and boundary element method (BEM).

In the last decade, researchers have paid attention to the meshless method without employing the concept of element. The initial idea of meshless method dates back to the smooth particle hydrodynamics (SPH) method for modeling astrophysical phenomena (Gingold and Maraghan, 1977). The method of fundamental solutions (Kondapalli *et al.*, 1992; Poullikkas *et al.*, 2002) is a technique for the numerical solution of certain elliptic boundary value problems. MFS may be viewed as an indirect boundary element method with a concentrated source instead of distribution. Like the boundary element method, it is applicable when a fundamental solution of the differential equation in question known. The basic ideal is to approximate the solution by forming a linear combination of fundamental solutions with sources located outside the problem domain. The coefficients of the linear combination are determined so that the approximate solution satisfies the problem boundary conditions. Kondapalli *et al.* (1992) applied the MFS to acoustic scattering in fluids and solids. One can consult the review paper of the MFS approach by Fairweather and Karageorghis (1998). One of the problems frequently addressed in BEM is the problem of irregular (fictitious) frequencies for exterior acoustics. Kondapalli (1992) pointed out that the difficulty of fictitious frequency appearing in the BEM is not present in the MFS. The reason was explained that a discrete set of source points does not define an internal surface uniquely, as quoted from Fairweather *et al.*, 2003. In this report, we will examine this point for the fictitious frequency phenomenon in the MFS. The fictitious frequencies do not represent any kind of physical resonance but are due to the numerical method, which has not a unique solution at some eigenfrequencies for a corresponding interior problem. Following the retracted BEM formulation (Hwang and Chang, 1991), it was found that the position of irregular frequency depends on the source location. The MFS and the retracted BEM can be seen as the similar indirect method instead of the difference of lump source and distributed source. Although the fictitious frequencies can be predicted theoretically (Chen, 1998; Chen and Kuo, 2000), we may not find the positions of numerical instability in the real computation for some cases.

This research will focus on the study of the occurring mechanism of fictitious-frequency for exterior acoustics by using the MFS. An analytical study of the fictitious frequency in a discrete system for a circular cylinder is conducted by using the degenerate kernel and circulants. Three numerical examples of uniform radiation, nonuniform radiation and scattering problems of a circular cylinder will also be examined and will be compared with the analytical solution and the results by using BEM.

2、The MFS formulation for Helmholtz

1 國立高雄海洋科技大學造船工程系 副教授
ilchen@mail.nkmu.edu.tw
高雄市海專路 142 號、07-3617141 轉 3402
2 高雄海洋科技大學輪機工程技術系 研究生
3 台灣海洋大學河海工程系 研究生

equation

The boundary value problem one wish to solve can be stated as follows: The acoustic pressure u(x) must satisfy the Helmholtz equation,

$$(\nabla^2 + k^2)u(x) = 0,\ x \in D, \tag{1}$$

in which $k = \omega / c$ is the wave number and ω is the angular frequency and D is the domain of interest. By using the MFS, the acoustic field and flux can be described by linear combinations of fundamental solutions:

Single-layer potential approach

$$u(x) = \sum_{j=1}^{2N} U(s_j, x)\Gamma(s_j), \tag{2}$$

$$t(x) = \sum_{j=1}^{2N} L(s_j, x)\Gamma(s_j), \tag{3}$$

Double-layer potential approach

$$u(x) = \sum_{j=1}^{2N} T(s_j, x)\Omega(s_j), \tag{4}$$

$$t(x) = \sum_{j=1}^{2N} M(s_j, x)\Omega(s_j), \tag{5}$$

where x and s are the collocation and source points, respectively, as shown in Fig.1, $L(s,x) = \frac{\partial U(s,x)}{\partial n_x}$, $T(s,x) = \frac{\partial U(s,x)}{\partial n_s}$, $M(s,x) = \frac{\partial^2 U(s,x)}{\partial n_x \partial n_s}$, n is the normal vector, $\Gamma(s_j)$ and $\Omega(s_j)$ are the generalized unknowns for single and double densities, respectively, at s_j, $2N$ is the number of collocation points and U(s, x) is the fundamental solution. The fundamental solution satisfies

$$\nabla^2 U(s,x) + k^2 u(s,x) = 2\pi\delta(x-s), \tag{6}$$

where δ is the Dirac delta function. The U kernel is,

$$U(s,x) = \frac{-i\pi}{2} H_0^{(1)}(kr), \tag{7}$$

in which r ≡ |s-x| is the distance between the source and collocation points; $H_0^{(1)}$ denotes the first kind of the *0*th order Hankel functions.

We consider an infinite circular cylinder with the Dirichlet boundary conditions

$$u(x) = \bar{u},\ x \in B, \tag{8}$$

where B is the boundary. By matching the boundary conditions for x on the *2N* boundary points into Eq.(2) we have

$$\{\bar{u}\} = [U]\{\Gamma\} \tag{9}$$

where $\{\Gamma\}$ is the vectors of undetermined coefficients.

Eq.(9) can be rearranged to

$$\{t\} = [L]\{\Gamma\} \tag{11}$$

$$\{\Gamma\} = [U]^{-1}\{\bar{u}\} \tag{10}$$

We obtain the unknown boundary density $\{t\}$ as follow:

$$\{t\} = [L][U]^{-1}\{\bar{u}\} = [SD]\{\bar{u}\} \tag{12}$$

By substituting Eq.(10) into Eq.(2), we obtained the field pressure

$$u(x) = \langle w \rangle [U]^{-1}\{\bar{u}\}, \tag{13}$$

where <*w*> is the influence row vector of field point obtained by using the $U(s,x)$ kernel.

2、Analytical study of the irregular frequency for the circular radiator using circulants

For the circular case, we can express $x = (\rho, \phi)$ and $s = (R, \theta)$ in terms of polar coordinate. The U kernel can be expressed in terms of degenerate kernels as shown below:

$$U(s,x) = \begin{cases} U^i(R,\theta;\rho,\phi) = \sum_{n=-\infty}^{\infty} \frac{-i\pi}{2} H_n^{(1)}(kR) J_n(k\rho)\cos n(\theta-\phi), R > \rho \\ U^e(R,\theta;\rho,\phi) = \sum_{n=-\infty}^{\infty} H_n^{(1)}(k\rho) J_n(kR)\cos n(\theta-\phi), R < \rho \end{cases} \tag{14}$$

where the superscripts "*i*" and "*e*" denote the interior $(R > \rho)$ and exterior domains $(R < \rho)$, respectively. Since the rotation symmetry is preserved for a circular boundary, the four influence matrices are denoted by [U], [L], [T] and [M] of the circulants. Based on the circulant theory, the eigenvalues for the four influence matrices are found as follows:

$$\lambda_l = -i\pi^2 \rho H_l^{(1)}(k\rho) J_l(kR), l = 0, \pm1, \pm2 \cdots \pm(N-1), N \tag{15}$$

$$\mu_l = -i\pi^2 \rho H_l^{'(1)}(k\rho) J_l(kR), l = 0, \pm1, \pm2 \cdots \pm(N-1), N \tag{16}$$

$$\upsilon_l = -i\pi^2 \rho H_l^{(1)}(k\rho) J_l^{'}(kR), l = 0, \pm1, \pm2 \cdots \pm(N-1), N \tag{17}$$

$$\kappa_l = -i\pi^2 \rho H_l^{'(1)}(k\rho) J_l^{'}(kR), l = 0, \pm1, \pm2 \cdots \pm(N-1), N \tag{18}$$

where λ, μ, υ and κ are the eigenvalues of [U], [L], [T] and [M] matrix, respectively. The determinants for the four matrices are obtained by multiplying all the eigenvalues.

4. Derivation of fictitious frequency by using the single-layer potential approach

For the Dirichlet problem, we have

$$[SD] = \Phi \begin{bmatrix} \sigma_0^{SD} & 0 & \cdots & 0 & 0 \\ 0 & \sigma_1^{SD} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \sigma_{-(N-1)}^{SD} & 0 \\ 0 & 0 & \cdots & 0 & \sigma_N^{SD} \end{bmatrix} \Phi^{-1} \tag{19}$$

where the superscript "SD" denotes by using the single-layer potential approach for the Dirichlet problem and

$$\sigma_l^{SD} = \frac{H_l^{'(1)}(ka) J_l(kR)}{H_l^{(1)}(ka) J(kR)}, l = 0, \pm1, \cdots, \pm(N-1), N \tag{20}$$

where a is the radius of the circular cylinder. According to Eqs.(19) and (20), we have

$$\begin{aligned} \det|SD| &= \det|\Phi| \sigma_0^{(SD)} (\sigma_1^{SD}\sigma_2^{SD} \cdots \sigma_{N-1}^{SD})^2 \sigma_N^{SD} \det|\Phi^{-1}| \\ &= \sigma_0^{(SD)} (\sigma_1^{SD}\sigma_2^{SD} \cdots \sigma_{N-1}^{SD})^2 \sigma_N^{SD} \end{aligned} \tag{21}$$

since $\det|\Phi| = \det|\Phi^{-1} = 1.|$

Based on the Eq.(21), the numerical instability of zero divided by zero occurs at the denominator where k satisfies

$$H_l^{(1)}(ka) J_l(kR) = 0. l = 0, \pm1, \pm2, \cdots, \pm N-1, N \tag{22}$$

Since the term of $H_l^{(1)}(ka)$ is never zero for any value of k, the k value satisfying Eq.(22), implies

$$J_l(ka) = 0.. \tag{23}$$

For the Neumann problem, we have

$$[SN] = \Phi \begin{bmatrix} \sigma_0^{SN} & 0 & \cdots & 0 & 0 \\ 0 & \sigma_1^{SN} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \sigma_{-(N-1)}^{SN} & 0 \\ 0 & 0 & \cdots & 0 & \sigma_N^{SN} \end{bmatrix} \Phi^{-1} \tag{24}$$

where the superscript "SN" denotes by using the single-layer potential approach for the Neumann problem and

$$\sigma_l^{SN} = \frac{H_l^{(1)}(ka) J_l(kR)}{H_l^{'(1)}(ka) J(kR)}, l = 0, \pm1, \cdots, \pm(N-1), N \tag{25}$$

According to Eqs.(24) and (25), we have

$$\begin{aligned} \det|SN| &= \det|\Phi| \sigma_0^{SN} (\sigma_1^{SN}\sigma_2^{SN} \cdots \sigma_{N-1}^{SN})^2 \sigma_N^{SN} \det|\Phi^{-1}| \\ &= \sigma_0^{SN} (\sigma_1^{SN}\sigma_2^{SN} \cdots \sigma_{N-1}^{SN})^2 \sigma_N^{SN} \end{aligned} \tag{26}$$

Based on the Eq.(26), the numerical instability of zero divided by zero occurs at the denominator where k satisfies

$$H_l^{'(1)}(ka)J_l(kR)=0. l=0,\pm1,\pm2,\cdots,\pm N-1,N \tag{27}$$

Since the term of $H_l^{'(1)}(ka)$ is never zero for any value of k, the k value satisfying Eq.(27), implies

$$J_l(ka)=0. \tag{28}$$

5. Burton & Miller method

In the exterior acoustics of Helmholtz equation by using dual BEM, the Burton & Miller utilized the product of hypersingular equation with an imaginary constant to the singular equation to deal with fictitious frequency which is the non-uniqueness solution problem. We will extend this concept to MFS approach.

$$u(x_i)=\sum_j (U(s_j,x_i)+\frac{i}{k}T(s_j,x_i))\varphi(s_j) \tag{29}$$

$$u(x_i)=\sum_j (U(s_j,x_i)+\frac{i}{k}T(s_j,x_i))\varphi(s_j)\text{,} \tag{30}$$

where φ is the mixed potential.

6. Numerical examples

Case 1: Nonuniform radiation from an infinite circular cylinder

This problem was chosen because the exact solution is known (Harari {\it et al.}, 1998). The boundary condition is shown in Fig.3. In this example we computed the nonuniform radiation from an infinite circular cylinder. The Neumann boundary condition is applied to the cylinder surface.

The portion $(-\alpha<\theta<\alpha)$ is assigned a unit value, while the remaining portion is assigned a homogeneous value. The analytical solution to this cylinder problem with a radius $a=1.0$ *m* is given by

$$u(\rho,\phi)=\frac{2}{\pi}\sum_{n=0}^{\infty}{}' \frac{-\sin(n\alpha)H_n^{(1)}(k\rho)}{knH_n^{'(1)}(ka)}\cos(n\phi),\ \rho>a,\ 0<\phi<2\pi \tag{31}$$

where the symbol "`" denotes that the first term (n=0) is halved. We select $\alpha=\pi/9$, ka=1. Figure 4 show the contour plots for the real part of the numerical solutions. Sixty-four nodes are adopted in the MFS and $\alpha=5\pi/32$ for this case. The source points are located at $R=0.9$ *m*. The positions where the irregular values occur can be found in Fig.5 for the solution $u(a,0;k)$ versus k. It is found that by using the *UL* formulation the irregular values occur at the positions of $J_{n,m}$, which is the mth zero of $J_n(kR)$. The fictitious frequencies occurred at the position are described in Eq.(28). By using the *TM* formulation the irregular values occur at the positions of $J_n^{'}(kR)$, which is the nth zero of $J_n^{'}(kR)$. The results of the MFS, the Burton & Miller approach and analytical solution are shown in Fig.5.

Case 2: Plane wave scattering for a rigid infinite circular cylinder (Neumann boundary condition)

In order to check the validity of the program for the scattering problem, example 2 is considered (Harari *et al.*, 1997). The incident wave is a plane wave and the scatter is a rigid cylinder, as shown in Fig.6. Sixty nodes are adopted for this case. The analytical solution for the scattering field is

$$u(\rho,\theta)=\frac{-J_0^{'}(ka)}{H_0^{(1)}(ka)}H_0^{(1)}(k\rho)-2\sum_{n=1}^{\infty}i^n\frac{J_n^{'}(ka)}{H_n^{(1)}(ka)}H_n^{(1)}(k\rho)\cos(n\theta) \tag{32}$$

Figure 7 shows the contour plot for the real-part solution $ka=4\pi$. The positions where the irregular values occur can be found in Fig.8 for the solution $u(a,0;k)$ versus k. Sixty-four nodes are adopted in the MFS. The source points are located at $R=0.9$ *m*. It is found that by using the *UL* formulation the irregular values also occur at the zeros of $J_{n,m}$. The results among the MFS, analytical solution and the Burton & Miller solution agree well as shown in Fig.8.

7、Conclusions

In this research, the mechanism why fictitious frequencies occur in the MFS has been examined by considering the radiation and scattering problems of a cylinder. Based on the circulant properties and degenerate kernels, an analytical scheme in discrete system of a cylinder was achieved. The occurrence of fictitious frequency only depends on the formulation instead of the specified boundary condition. The numerical results from this study indicated that the irregular frequency also appears at the eigenvalue of interior problem where the boundary is connected by the source locations instead of the real boundary in the direct BEM. In a circular cylinder case, the position of irregular frequency depends on the source location *R*.The Burton & Miller technique was demonstrated successfully to filter out the fictitious frequency analytically and numerically.

8、REFERENCES

[1] Gingold R.A. and Maraghan J.J.,1977, "Smoothed particle hydrodynamics: theory and applications to non-spherical stars," *Monthly Notices of the Royal Astronomical Society*, Vol.181, pp.375-389.

[2]. Kondapalli P.S., Shippy D.J. and Fairweather G., 1992, "Analysis of acoustic scattering in fluids and solids by the method of fundamental solutions,"
The Journal of Acoustical Society of America, Vol.91, No.4, pp-1844-1854.

[3]. Poullikkas A., Karageorghis A. and Georgiou G., 2002, ``The method of fundamental solutions for three-dimensional elastostatics problems,"
Computers and Structures, Vol.80, pp.365-370.

[4]. Fairweather G. and Karageorghis A., 1998, "The method of fundamental solutions for elliptic boundary value problems,"*Advances in Computational Mathematics*, Vol.9, pp.69-95.

[5]. Fairweather G., Karageorghis A. and Martin P.A., 2003,"The method of fundamental solutions for scattering and radiation problems,"*Engineering Analysis with Boundary Elements*, (Accepted)

[6]. Hwang J.Y. and Chang S.C., 1991,"A retracted boundary integral equation for exterior acoustic problem with unique solution for all wave numbers," *Journal of the Acoustical Society of America*, Vol.90, pp.1167-1180.

[7]. *Chen J.T., 1998," On fictitious frequencies using dual series representation, "Mechanics Research Communications, Vol.25, No.5,pp.529-534.*

[8]. Chen J.T., 1998,"On fictitious frequencies using dual series representation, "*Mechanics Research Communications*, Vol.25, No.5,pp.529-534.

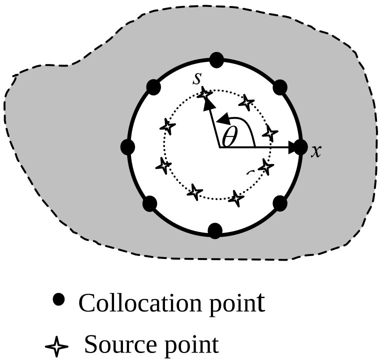


Fig.1 The located position of source and collocation points and the definitions of ρ, θ, R and r..

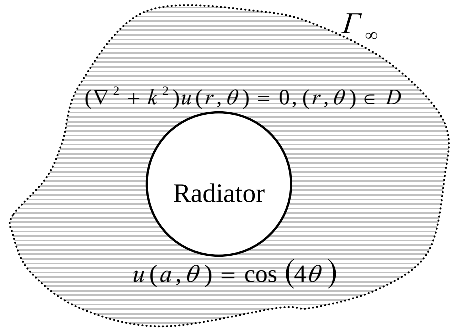


Fig.2 The radiation problem (Dirichlet type) for a cylinder.

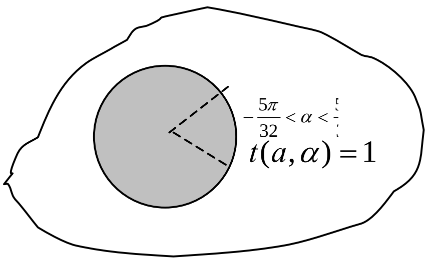


Fig.3 The nonuniform radiation problem (Neumann type) for a cylinder.

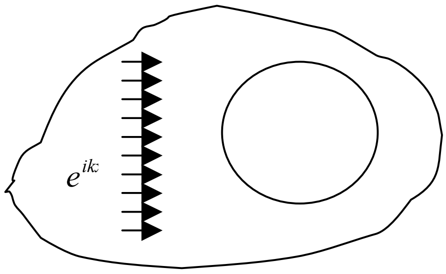


Fig.6 The problem of a plane wave scattered by a rigid infinite circular cylinder

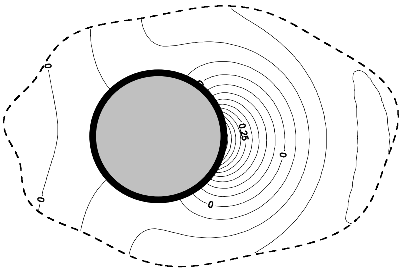


Fig.4 The numerical solution for the nonuniform radiation problem (ka=1, $\alpha = \frac{\pi}{9}$).

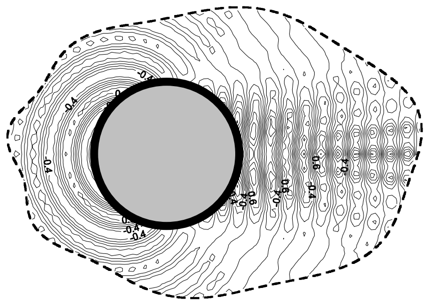
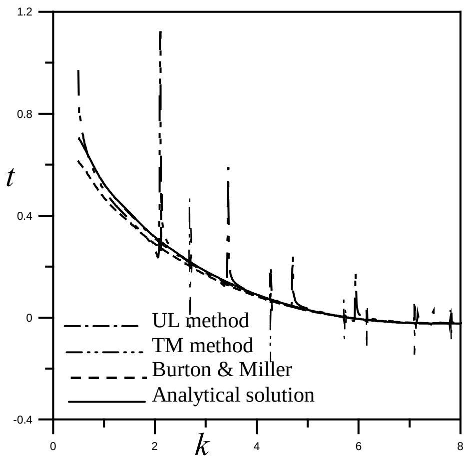


Fig.7 The contour plot for the real-part numerical solution for a plane wave scattered by an infinite circular cylinder ($ka = 4\pi$).



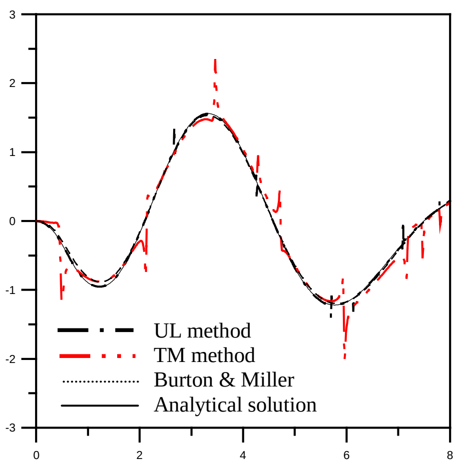


Fig.5 The $u(a,0;k)$ versus k using the MFS for the nonuniform radiation by a circular cylinder.

Fig.8 The $u(a,0;k)$ versus *k* using the MFS for plane wavescattered by an infinite circular cylinde