Regularized meshless method for solving Laplace problems with holes

J. H. Kao, K. H. Chen and J. T. Chen

National Taiwan Ocean University

National Tsing Hua University EB I 210

12, 16, 2005   16:00~17:45

Outlines

- Literature review
- Introduction of MFS
- Introduction of RMM
- Formulation
- Numerical examples
- Conclusions

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Literature review

- Kupradze and Aleksidze, 1964.

  MFS, RBF.
- Fairweather and Karageorghis, 1998.

  MFS, image part of RBF.
- Chen and Tanaka, 2002.

  BKM, image part of RBF.
- Young and Chen, 2005.

  RMM, no off-boundary.

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Introduction of MFS

Single layer

$$\phi(x_i) = \sum_{j=1}^{N} U(s_j, x_i)\alpha_j, \quad x \in B, \quad s \in B'$$

$$\psi(x_i) = \sum_{j=1}^{N} T(s_j, x_i)\alpha_j, \quad x \in B, \quad s \in B'$$

Kernel functions

$$U(s_j, x_i) = \ln(r_{ij})$$

$$T(s_j, x_i) = \frac{\partial}{\partial n_s} U(s_j, x_i) = \frac{-y_i n_i}{r_{ij}^2}$$

Double layer

$$\phi(x_i) = \sum_{j=1}^{N} L(s_j, x_i)\alpha_j, \quad x \in B, \quad s \in B'$$

$$\psi(x_i) = \sum_{j=1}^{N} M(s_j, x_i)\alpha_j, \quad x \in B, \quad s \in B'$$

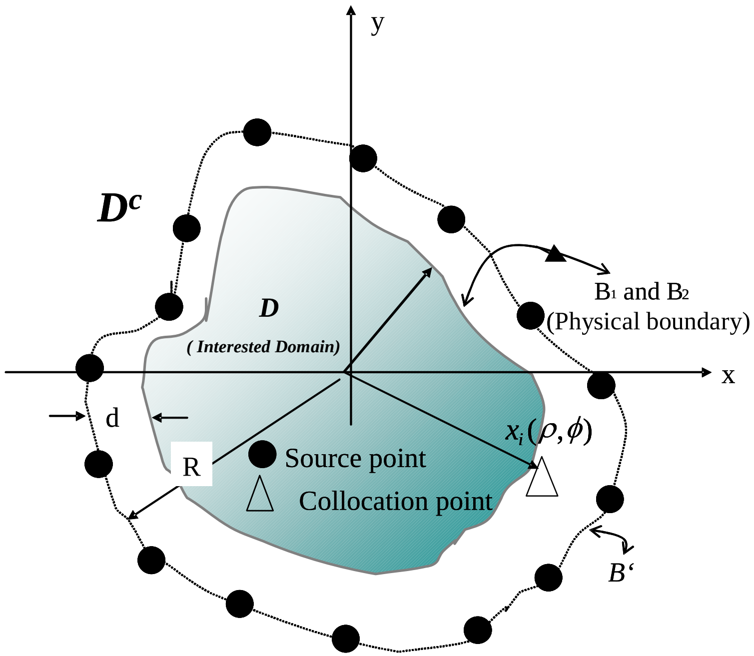
Kernel functions

$$L(s_j, x_i) = \frac{\partial}{\partial n_x} U(s_j, x_i) = \frac{y_i \overline{n_i}}{r_{ij}^2}$$

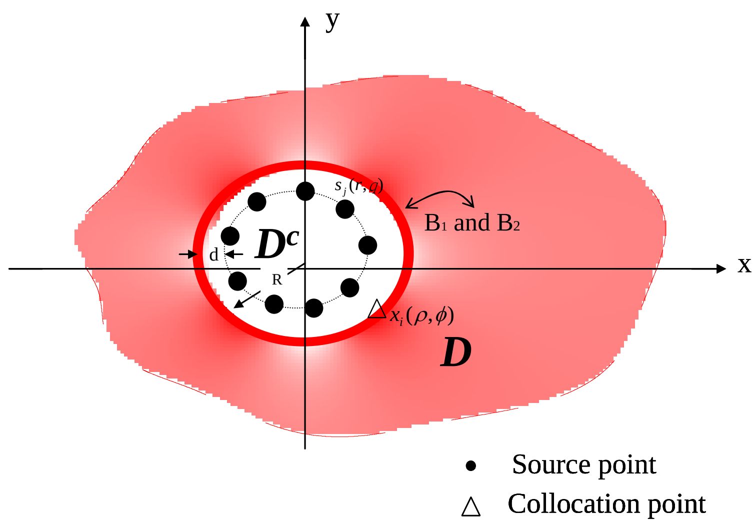
$$M(s_j, x_i) = \frac{\partial^2}{\partial n_s \partial n_x} U(s_j, x_i) = \frac{2 y_i n_i y_j \overline{n_j}}{r_{ij}^2} - \frac{n_i \overline{n_i}}{r_{ij}^2}$$

Introduction of MFS

- Interior Problem



- Exterior Problem



Introduction of MFS

- Linear systems

Single layer

$$\{\phi_i(x_i)\} = \lfloor U(s_j, x_i) \rfloor \{\alpha_j\}$$

$$\{\psi_i(x_i)\} = \lfloor T(s_j, x_i) \rfloor \{\alpha_j\}$$

Double layer

$$\{\phi_i(x_i)\} = \lfloor L(s_j, x_i) \rfloor \{\alpha_j\}$$

$$\{\psi_i(x_i)\} = \lfloor M(s_j, x_i) \rfloor \{\alpha_j\}$$

$$\phi(x_i) = \sum_{j=1}^{N} U(s_j, x_i)\alpha_j, \quad x \in B, \quad s \in D$$

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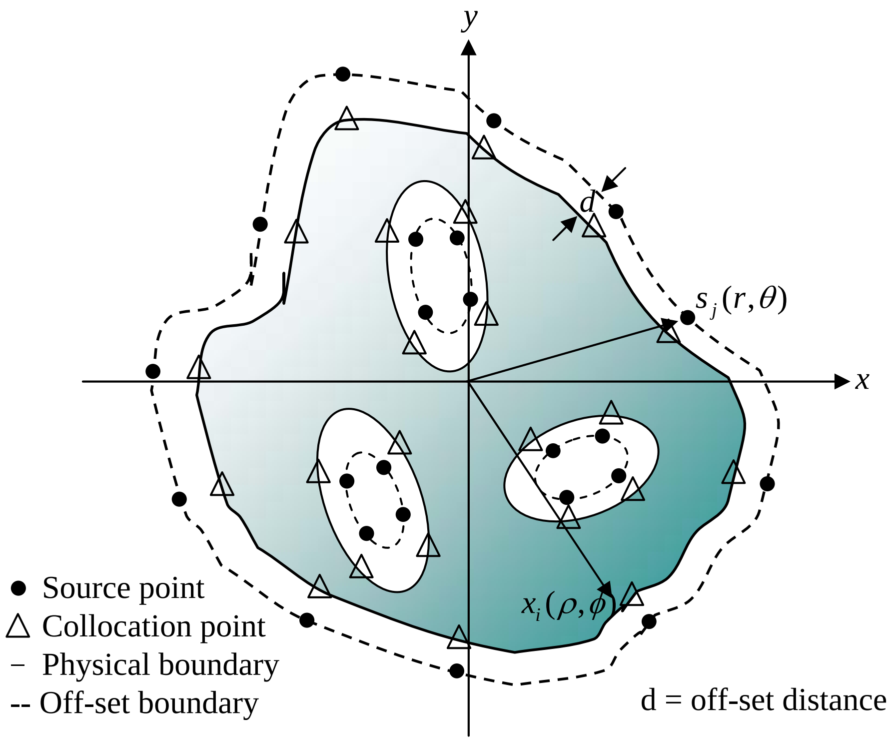
Introduction of RMM

- We propose a improved method for MFS after eliminating singularity and hypersingularity that double layer potential will encounter.

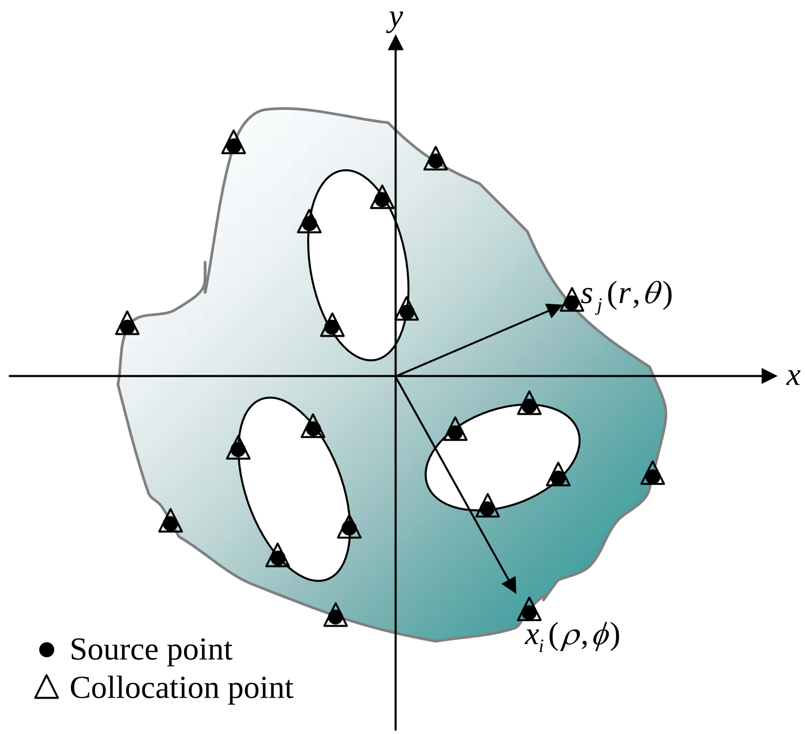
- The controversial test for determining off-boundary distance is no more needed.

Introduction of RMM

MFS



RMM



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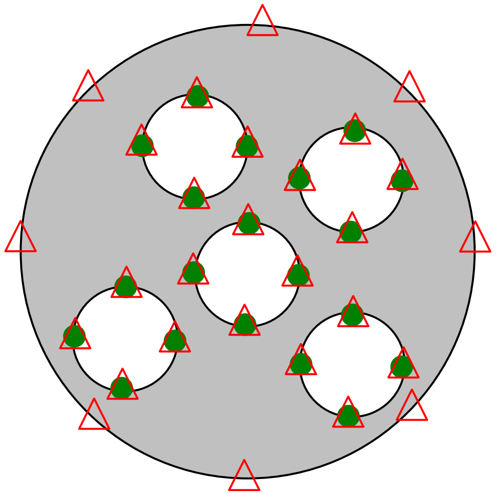
Formulation



$$u(x_i^I)=\sum_{j=1}^{N_1}T(s_j^I,x_i^I)\alpha_j+\cdots+\sum_{j=N_1+\cdots+N_{P-1}+1}^{N_1+\cdots+N_P}T(s_j^I,x_i^I)\alpha_j+\cdots$$

$$+\sum_{j=N_1+\cdots+N_{m-2}+1}^{N_1+\cdots+N_{m-1}}T(s_j^I,x_i^I)\alpha_j+\sum_{j=N_1+\cdots+N_{m-1}+1}^{N}T(s_j^O,x_i^I)\alpha_j$$

$$-\sum_{j=N_1+\cdots+N_{P-1}+1}^{N_1+\cdots+N_p}T(s_j^I,x_i^I)\alpha_i \quad p=1,\ 2,\ 3,\cdots,\ m-1$$

$$u(x_i^I)=\sum_{j=1}^{N_1}T(s_j^I,x_i^I)\alpha_j+\cdots+\sum_{j=N_1+\cdots+N_{p-1}+1}^{i-1}T(s_j^I,x_i^I)\alpha_j+\sum_{j=i+1}^{N_1+\cdots+N_p}T(s_j^I,x_i^I)\alpha_j+\cdots+\sum_{j=N_1+\cdots+N_{m-2}+1}^{N_1+\cdots+N_{m-1}}T(s_j^I,x_i^I)\alpha_j+\sum_{j=N_1+\cdots+N_{m-1}+1}^{N}T(s_j^O,x_i^I)\alpha_j$$

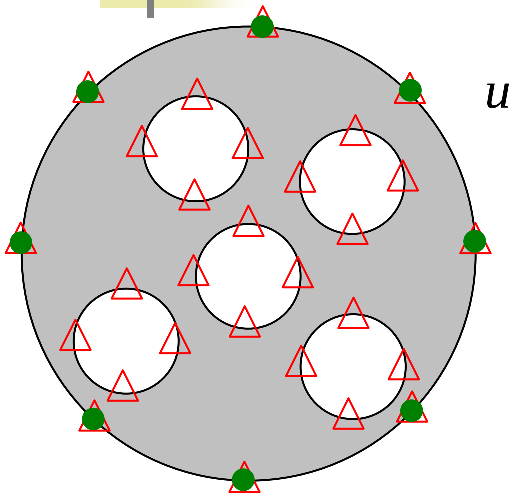
$$-\left[\sum_{j=N_1+\cdots+N_{p-1}+1}^{N_1+\cdots+N_p}T(s_j^I,x_i^I)-T(s_j^I,x_i^I)\right]\alpha_i$$

Formulation

$$\begin{bmatrix} -\left[\sum_{j=1}^{N_1} T(s_j^I, x_1^I) - T(s_1^I, x_1^I)\right] & T(s_2^I, x_1^I) & \cdots & T(s_{N_1}^I, x_1^I) \\ T(s_1^I, x_2^I) & -\left[\sum_{j=1}^{N_1} T(s_j^I, x_2^I) - T(s_2^I, x_2^I)\right] & \cdots & T(s_{N_1}^I, x_2^I) \\ \vdots & \vdots & \ddots & \vdots \\ T(s_1^I, x_{N_1}^I) & T(s_2^I, x_{N_1}^I) & \cdots & -\left[\sum_{j=1}^{N_1} T(s_j^I, x_{N_1}^I) - T(s_{N_1}^I, x_{N_1}^I)\right] \end{bmatrix}$$

where $p = 1$

Formulation



$$u(x_i^O) = \sum_{j=1}^{N_1} T(s_j^I, x_i^O)\alpha_j + \cdots + \sum_{j=N_1+\cdots+N_{p-1}+1}^{N_1+\cdots+N_p} T(s_j^I, x_i^O)\alpha_j + \sum_{j=N_1+\cdots+N_{m-2}+1}^{N_1+\cdots+N_{m-1}} T(s_j^I, x_i^O)\alpha_j$$

$$+ \sum_{j=N_1+\cdots+N_{m-1}+1}^{N} T(s_j^O, x_i^O)\alpha_j - \sum_{j=N_1+\cdots+N_{p-1}+1}^{N} T(s_j^I, x_i^I)\alpha_i \qquad p = m$$

$$u(x_i^O) = \sum_{j=1}^{N_1} T(s_j^I, x_i^O)\alpha_j + \sum_{j=N_1+1}^{N_1+N_2} T(s_j^I, x_i^O)\alpha_j + \cdots + \sum_{j=N_1+\cdots+N_{m-2}+1}^{N_1+\cdots+N_{m-1}} T(s_j^I, x_i^O)\alpha_j + \sum_{j=N_1+\cdots+N_{m-2}+1}^{i-1} T(s_j^O, x_i^O)\alpha_j + \sum_{j=i+1}^{N} T(s_j^O, x_i^O)\alpha_j$$

$$-\left[\sum_{j=N_1+\cdots+N_{m-1}+1}^{N} T(s_j^I, x_i^I) - T(s_i^O, x_i^O)\right]\alpha_i$$

Formulation

$$\begin{bmatrix} -\left[\sum_{j=N_1+\cdots N_{m-1}+1}^{N} T(s_j^I, x_{N_1+\cdots N_{m-1}+1}^I) - T(s_{N_1+\cdots N_{m-1}+1}^O, x_{N_1+\cdots N_{m-1}+1}^O)\right] & \cdots & T(s_N^O, x_{N_1+\cdots N_{m-1}+1}^O) \\ \vdots & \ddots & \vdots \\ T(s_{N_1+\cdots N_{m-1}+1}^O, x_N^O) & \cdots & -\left[\sum_{j=N_1+\cdots N_{m-1}+1}^{N} T(s_j^I, x_N^I) - T(s_{N+\cdots N_{m-1}+1}^O, x_N^O)\right] \end{bmatrix}$$

$$where \quad p = m$$

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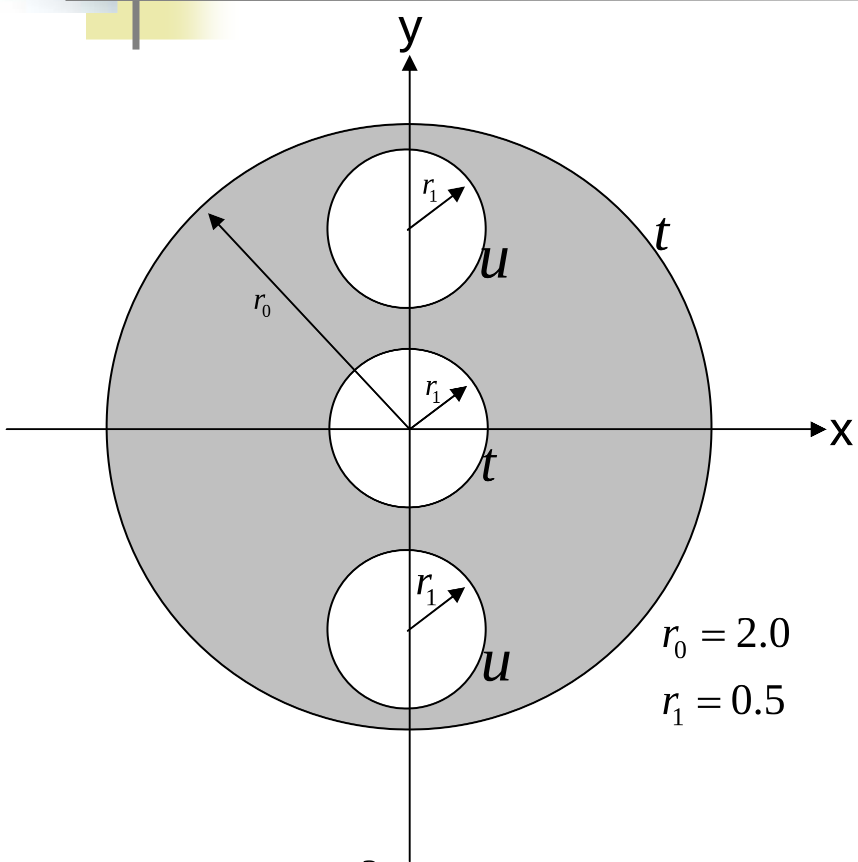
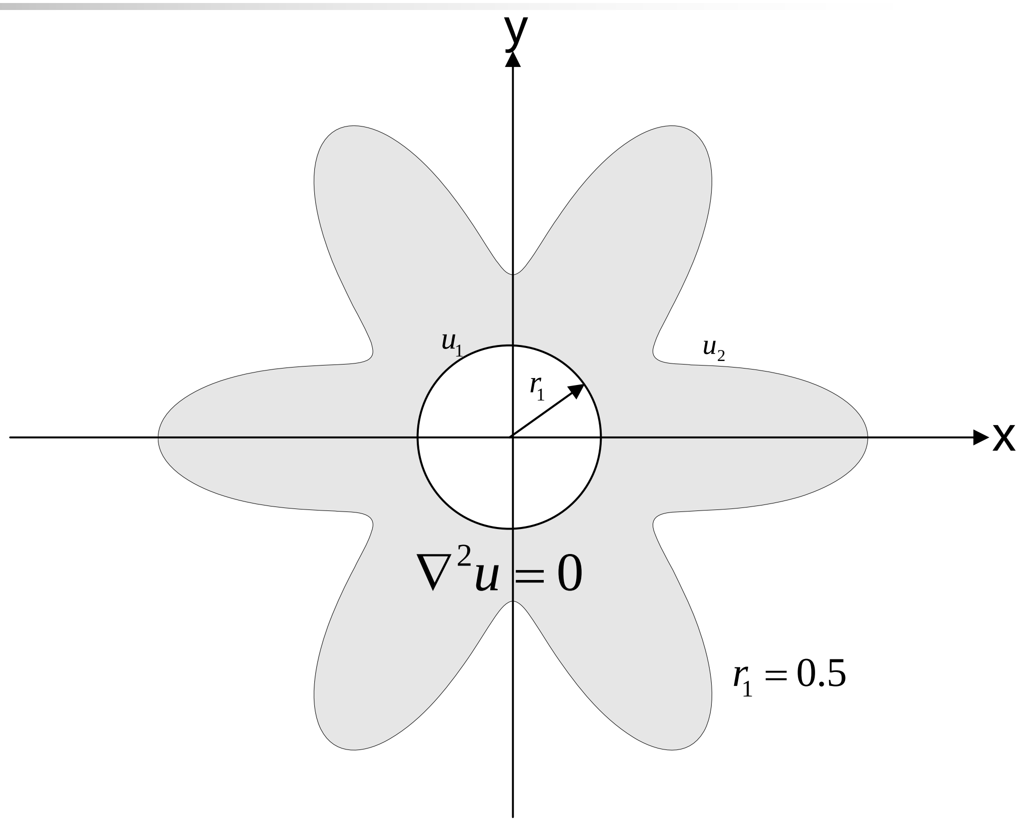
Numerical examples

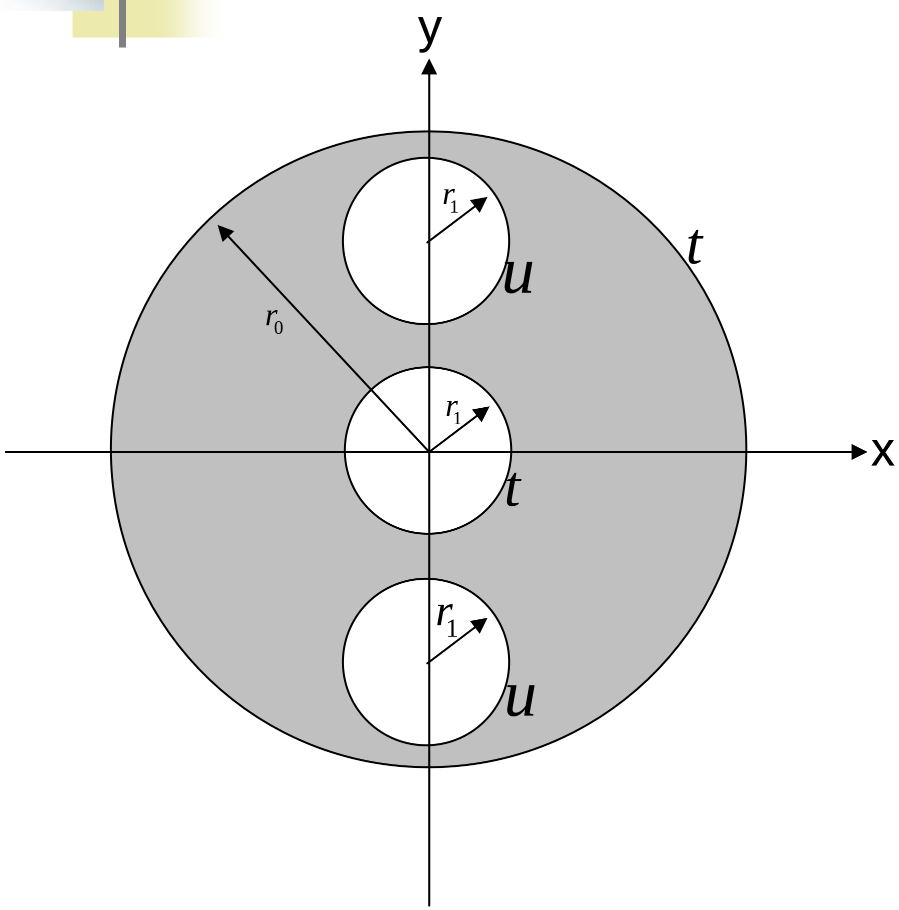


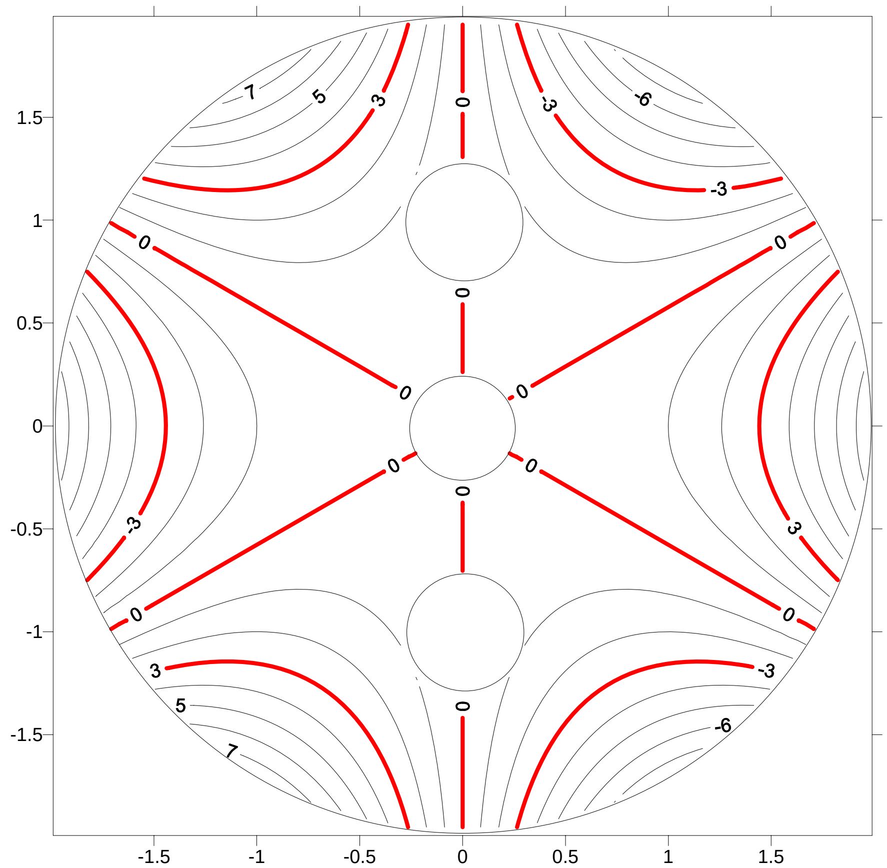
$$u = r^3 \cos(3\theta)$$



$$u = e^x \cos(y)$$

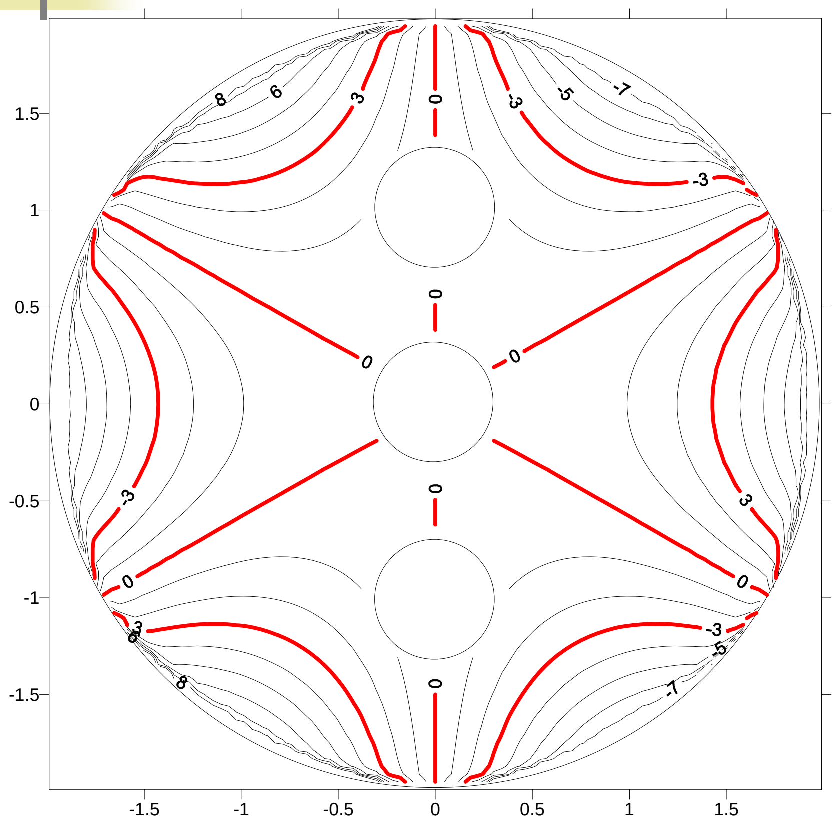
Numerical example 1



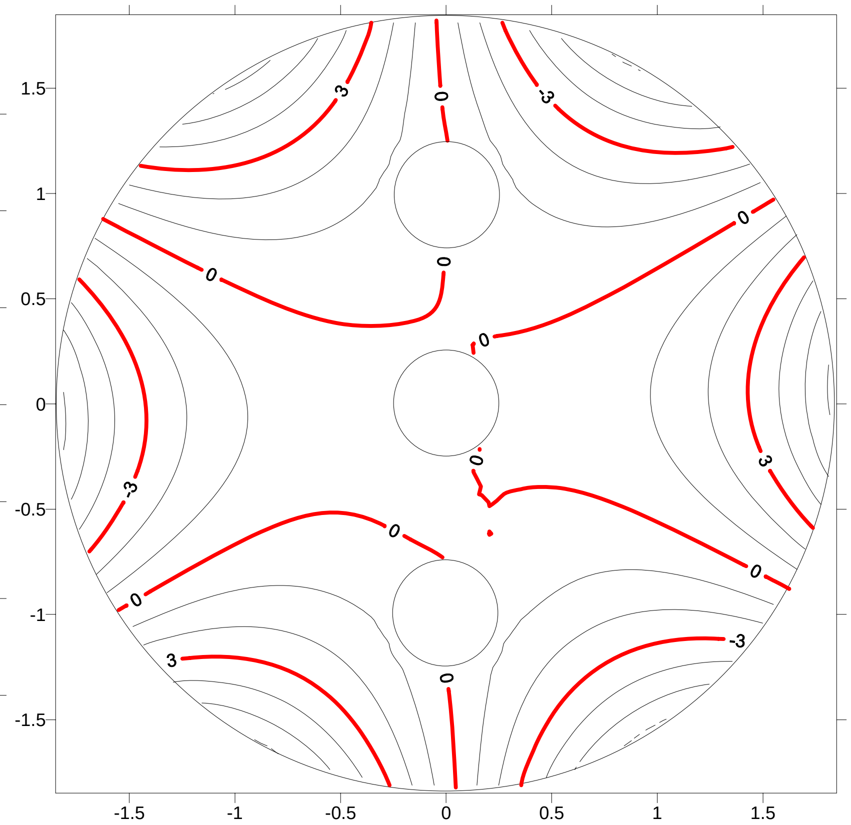


Exact solution

Numerical example 1



RMM (400 nodes)



BEM (400 elements)

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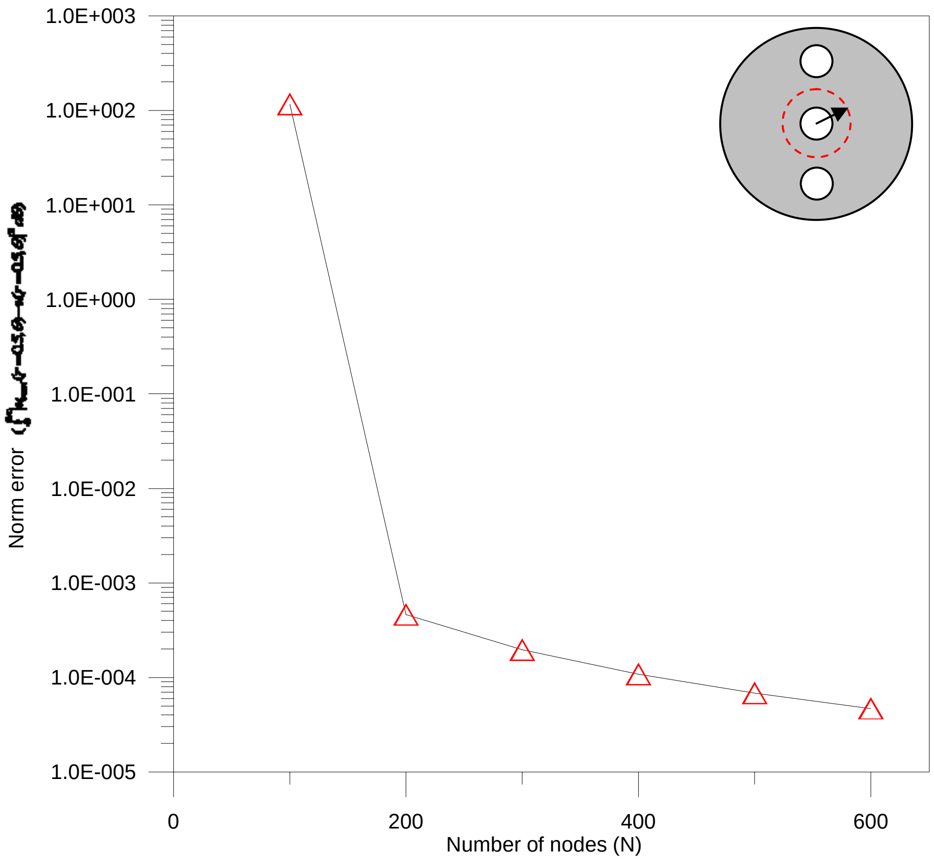
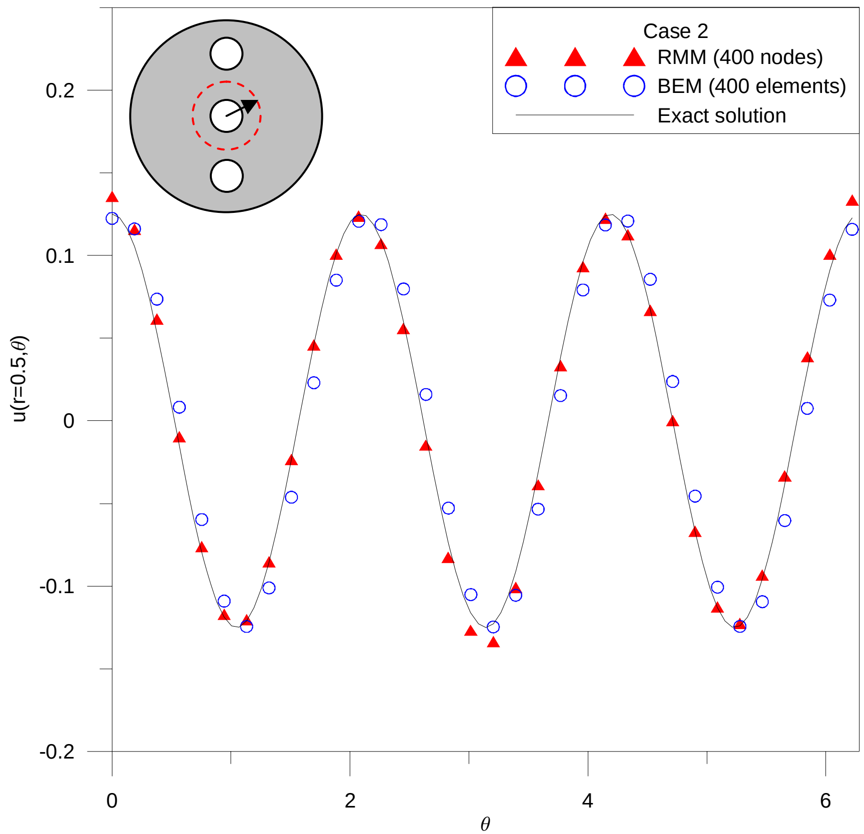
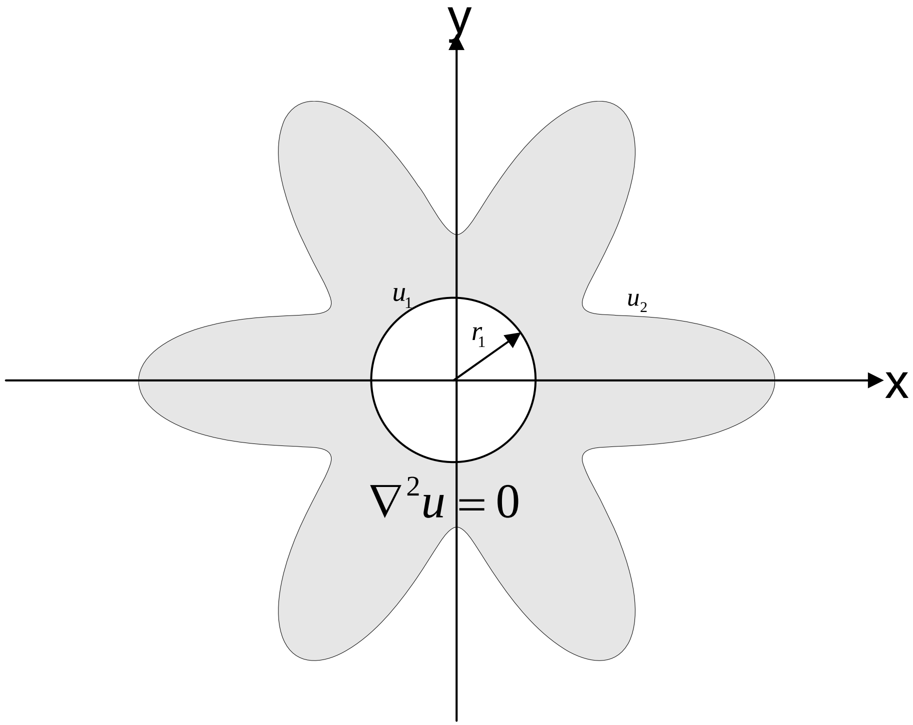
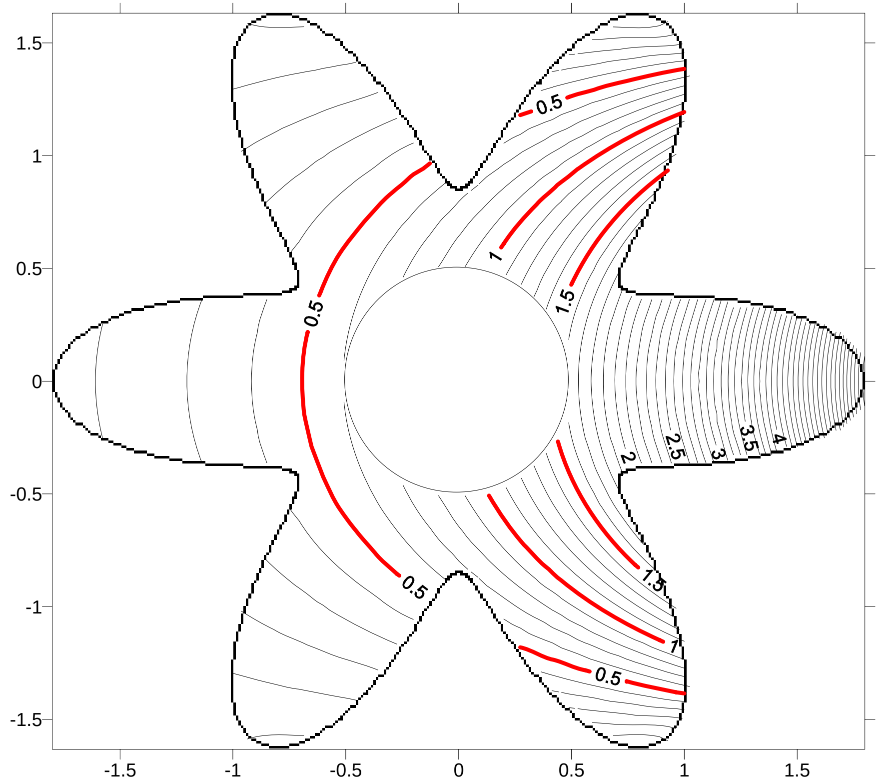
Numerical example 1





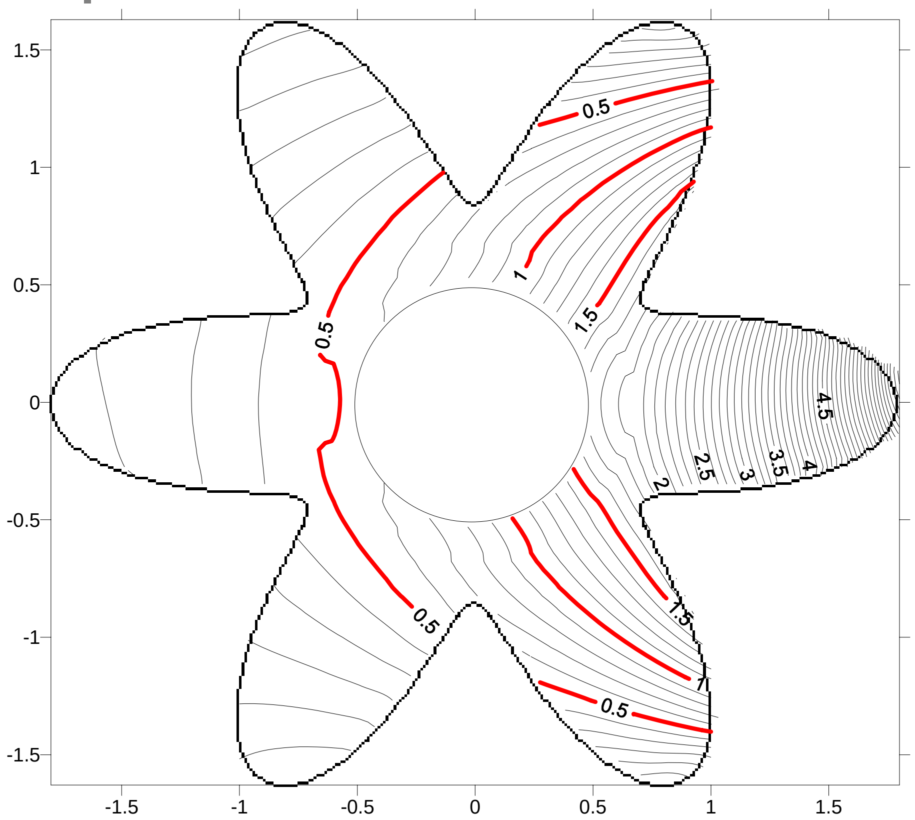
Numerical example 2



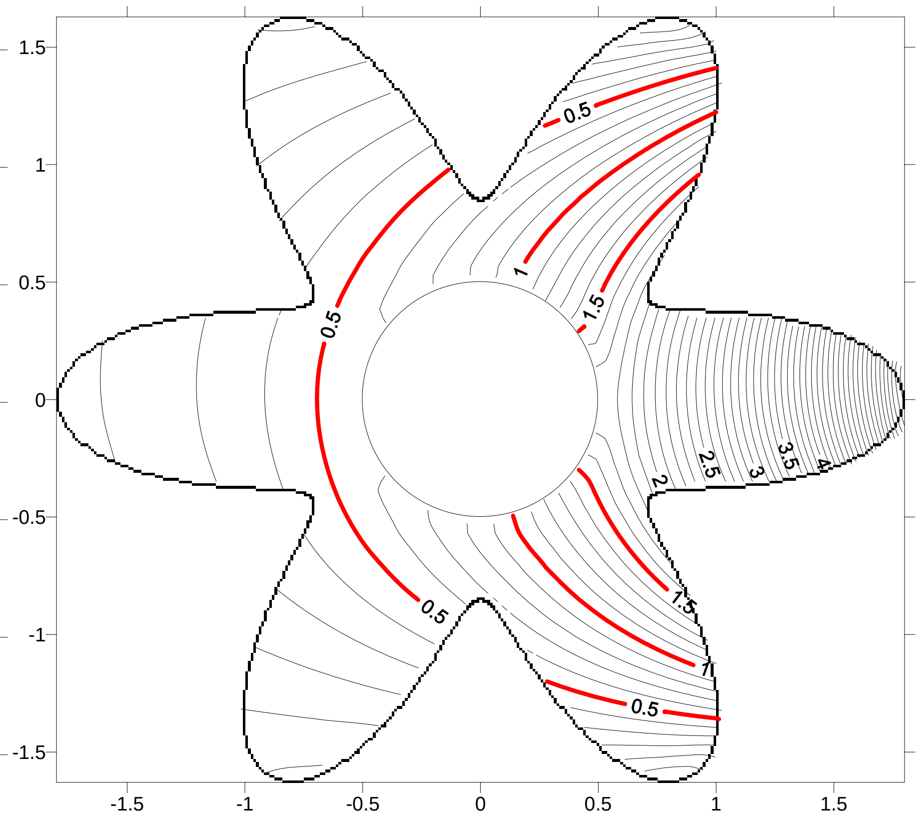


Exact solution

Numerical example 2

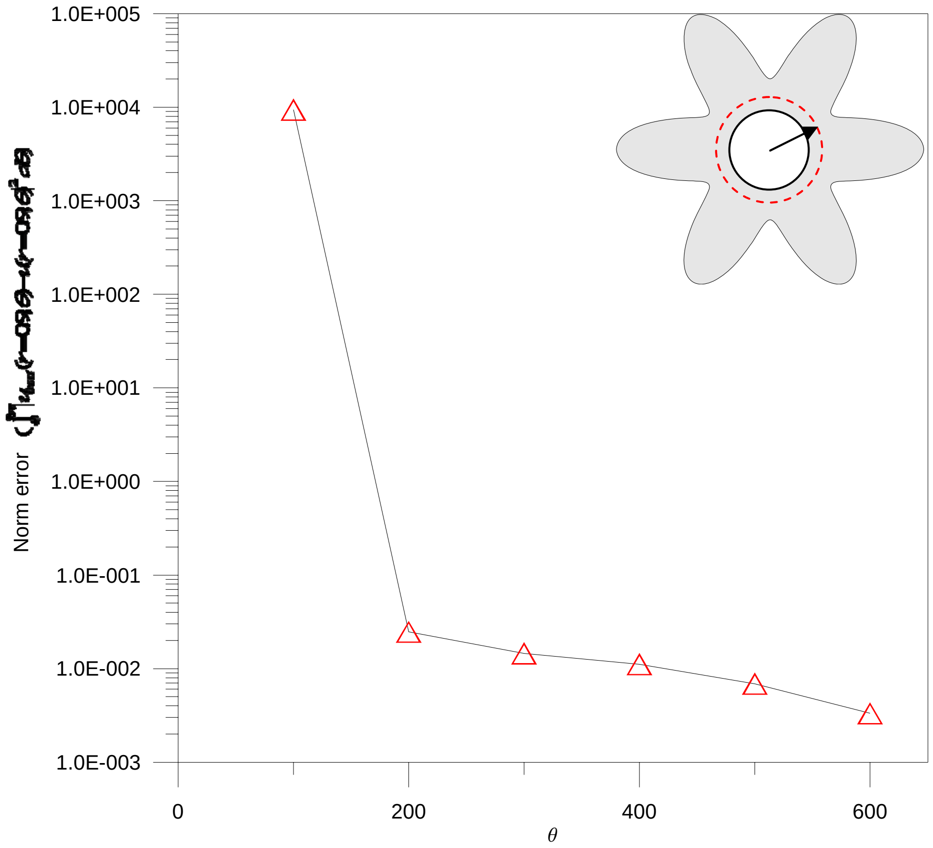


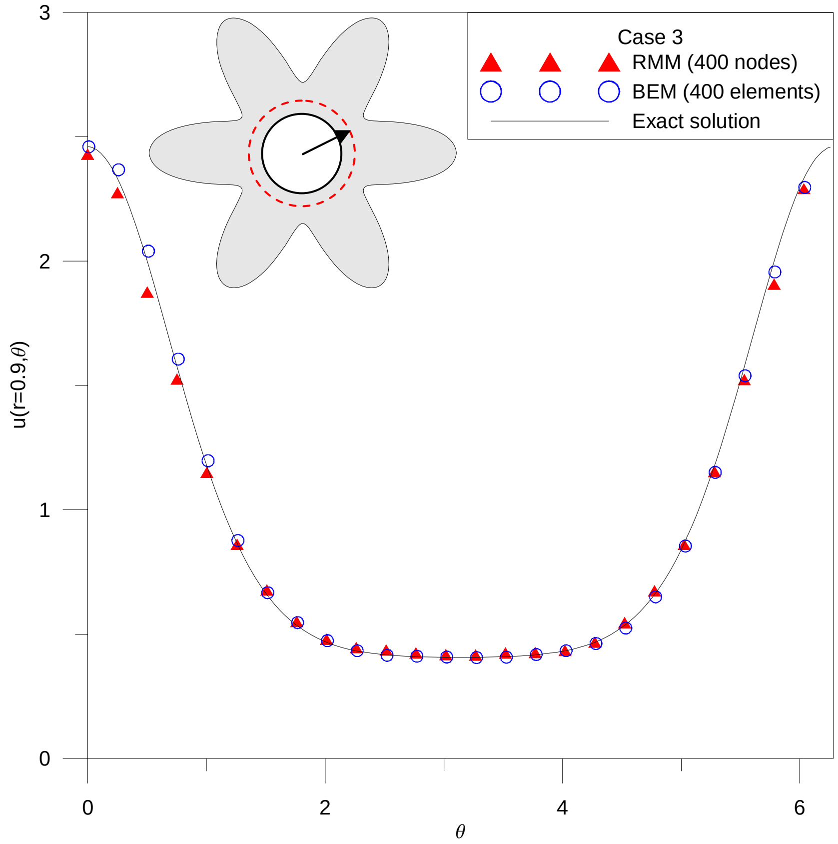
RMM (400 nodes)



BEM (400 elements)

Numerical example 2





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Conclusions

- Because of the boundary nodes on the real boundary, therefore determining off-boundary distance is no more needed.
- We succeed to solve the Laplace problem for multi-arbitrary domain by using the RMM.
- Solutions were compared very well with the analytical solutions and the BEM.

The end

Thanks for your attentions.

Your comment is much appreciated.

You can get more information on our website.

http://msvlab.hre.ntou.edu.tw