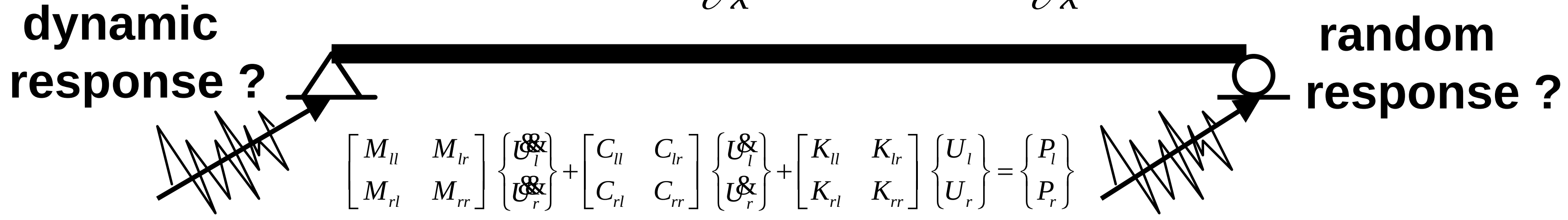
On the Stokes' Transformation and Its Application to Support Motion Problems

$$\rho \ddot{u}(x,t) + (2\alpha\rho - \beta G \frac{\partial^2}{\partial x^2}) \dot{u}(x,t) - G \frac{\partial^2 u(x,t)}{\partial x^2} = 0$$



$$\begin{bmatrix} M_{ll} & M_{lr} \\ M_{rl} & M_{rr} \end{bmatrix} \begin{Bmatrix} \ddot{U}_l \\ \ddot{U}_r \end{Bmatrix} + \begin{bmatrix} C_{ll} & C_{lr} \\ C_{rl} & C_{rr} \end{bmatrix} \begin{Bmatrix} \dot{U}_l \\ \dot{U}_r \end{Bmatrix} + \begin{bmatrix} K_{ll} & K_{lr} \\ K_{rl} & K_{rr} \end{bmatrix} \begin{Bmatrix} U_l \\ U_r \end{Bmatrix} = \begin{Bmatrix} P_l \\ P_r \end{Bmatrix}$$

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The Series Representation Solutions of the Three Analytical Formulations

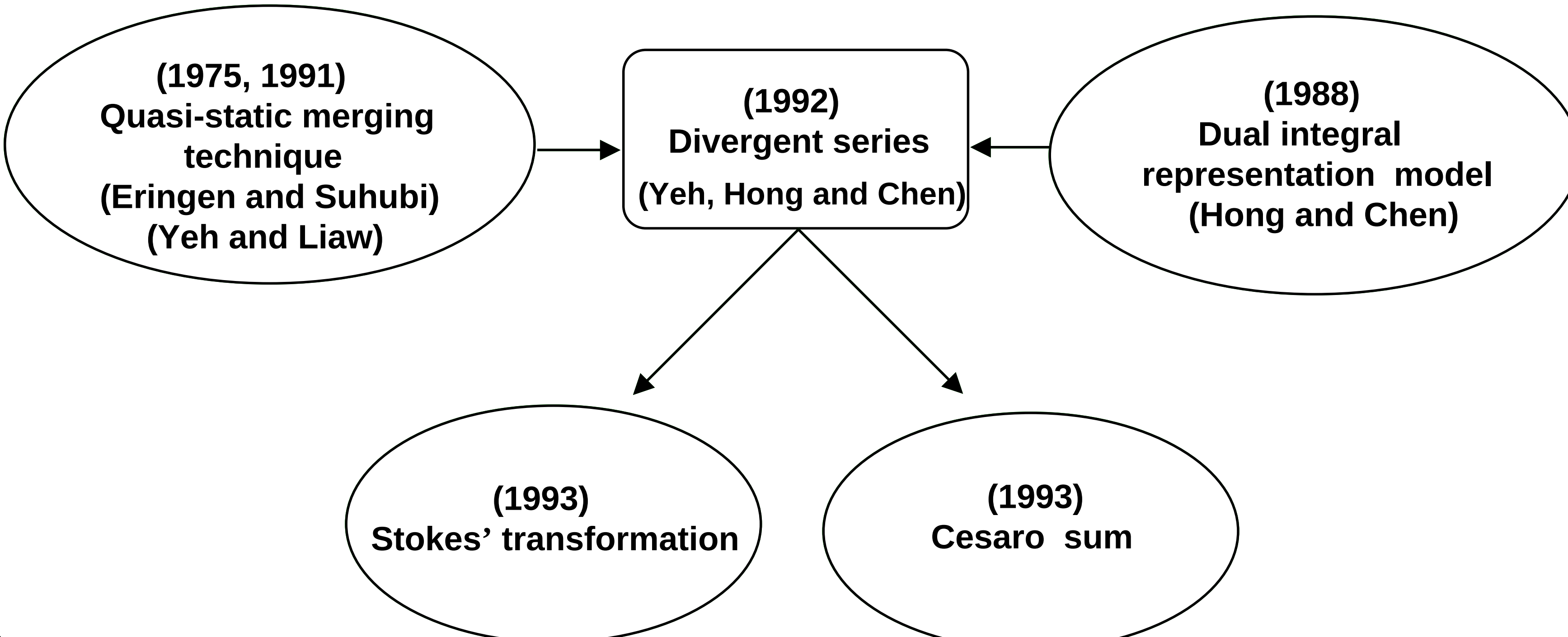
Methods of Solution for Multi-Support Motions

- **Mode Superposition (Series Solution)**
- **Large Mass Technique (Including Rigid Body Modes)**
- **Large Stiffness Technique (Including High Frequency Modes)**
- **Cesaro sum (Regularize the infinite value to finite part)**
- **Quasi-static Decomposition (Decompose the quasi-static part)**
- **Stokes' Transformation (Legal way to differentiate series)**

Methods of Solution for Multi-Support Motions

- **Large Mass Technique (1983) → Multi-support Motions(1992)**
- **Large Stiffness Technique (1983) → Multi-support Motions(1992)**
- **Cesaro Sum (1890) → Base Shear Force (1992)**
- **Quasi-static Decomposition (1950) → Discrete System (1975)**
- **Stokes' Transformation (1880) → Base Shear Force (1993)**

History of This Research



Motivations of Quasi-static Decomposition and Stokes' Transformation

Quasi-static decomposition

$$u(x,t) = \underline{\underline{U(x,t)}} + \sum_{n=0}^{\infty} q_n(t)\, u_n(x)$$

(Physical meaning)

Differentiation

$$u'(x,t) = U'(x,t) + \sum_{n=0}^{\infty} q_n(t)\, u_n'(x)$$

Stokes' transformation

$$u(x,t) = \sum_{n=0}^{\infty} \overline{q}_n(t)\, u_n(x)$$

Asymptotic analysis

$$= U(x,t) + \sum_{n=0}^{\infty} c_n(t)\, u_n(x) + \sum_{n=0}^{\infty} \overline{q}_n(t)\, u_n(x)$$

Integration

$$u'(x,t) = \sum_{n=0}^{\infty} \underline{\underline{b_n(t)\, u_n'(x)}} + \sum_{n=0}^{\infty} \overline{q}_n(t)\, u_n'(x)$$

$-\infty$ $\quad$ $\infty + F.P.$

(Mathematical way)

$$= U'(x,t) + \sum_{n=0}^{\infty} c_n(t)\, u_n'(x) + \sum_{n=0}^{\infty} \overline{q}_n(t)\, u_n'(x)$$

$-\infty$ $\quad$ $\infty + F.P.$

$$q_n(t) = c_n(t) + \overline{q}_n(t)$$

$$U(x,t) = -\sum_{n=0}^{\infty} c_n(t)\, u_n(x)$$

$$\sum_{n=0}^{\infty} b_n(t)\, u_n'(x) =$$ **Series representation for distribution on boundary**

Three Analytical Ways and Two Simulation Techniques to Introduce the Quasi-static Part

- **By Solving Boundary Value Problem Directly**

  Quasi-static decomposition method (Mindlin and Goodman)

- **By Integrating the Secondary Field Derived from Stokes' Transformation**

  Boundary terms are available

- **By Adding-and-Subtracting Technique Using Asymptotic Analysis**

  Series representation (Eringen and Suhubi, Yeh and Liaw)

- **Large Mass Technique (MSC/NASTRAN) --- Rigid Body Modes**
- **Large Stiffness Technique (MSC/NASTRAN) --- High Frequency Modes**

Cesaro Regularization Technique

- **Series Solution(Partial Sum)**

$$s_0 = a_0$$

$$s_1 = a_0 + a_1$$

$$s_2 = a_0 + a_1 + a_2$$

$$\vdots \quad \vdots$$

$$s_{N-1} = a_0 + a_1 + a_2 + \cdots + a_{N-1}$$

$$(\textit{partial} \;\; \textit{sum}) \quad s_N = a_0 + a_1 + a_2 + \cdots + a_{N-1} + a_N \quad (\textit{divergent}, \;\; N \to \infty)$$

$$\frac{s_0 + s_1 + \cdots + s_{N-1} + s_N}{N+1} = a_0 + \frac{N}{N+1} a_1 + \frac{N-1}{N+1} a_2 + \cdots + \frac{2}{N+1} a_{N-1} + \frac{1}{N+1} a_N \quad (\textit{convergent}, \;\; N \to \infty)$$

$$(\textit{Cesaro} \;\; \textit{sum}) \quad S_N = \frac{1}{N+1} \sum_{k=0}^{N} (N - k + 1) \;\; a_k \quad (\textit{moving} \;\; \textit{average})$$

Stokes' Transformation

- **Term by Term Differentiation Is Not Always Legal**
- **Boundary Term Is Present for Some Cases**

$$f^{'}(x) = \frac{d}{dx}\{f(x)\} = \frac{d}{dx}\left\{\sum_{k=0}^{n} c_k\ u_k(x)\right\} = \sum_{k=0}^{n} c_k\ u_k^{'}(x) + \underline{\underline{\sum_{k=0}^{n} b_k\ u_k^{'}(x)}}$$

Boundary term

if $\sum_{k=0}^{n} b_k\ u_k^{'}(x) \neq 0$

- **Term by Term Differentiation Is Legal**

if $\sum_{k=0}^{n} b_k\ u_k^{'}(x) = 0$

Why Cesaro sum can Extract the Finite Part of Divergent Series

$$u'(x,t) = \sum_{l=0}^{N} \bar{q}_l^{'}(t)\ u_l^{'}(x) = \sum_{l=0}^{N} \frac{1}{N_l \lambda_l} \{u(y,t)\ u_l^{'}(y)\}\Big|_{y=0}^{y=L}\ u_l^{'}(x) + \sum_{l=0}^{N} \bar{q}_l(\mathrm{t})\ u_l^{'}(x)$$

(convergent) **(divergent)** **(divergent)**

$C(N,2)$ operator ↓ $C(N,2)$ operator ↓ $C(N,2)$ operator ↓

finite part = **zero** + **finite part**

(Stokes' transformation) ← **(Cesaro sum)**

Literature Review of Stokes' Transformation

- **Single Fourier Series :**
  - **Oscillating waves (Stokes, 1880)**
  - **Stability of viscous fluid (Goldstein, 1936)**
  - **Free vibration**
    - **twisted beam (Budiansky and Diprima, 1960)**
    - **shell (Chung, 1981)**
    - **beam on viscoelastic foundation (Chuang and Wang, 1991)**
  - **Support motion (Chen, Hong and Yeh, 1993)**
  - **Heat conduction (Chen and Hong, 1993)**
- **Double Fourier Series :**
  - **Static analysis of doubly curved shells (Chaudhuri and Kabir, 1993)**

Transient Responses at t=1 second

Displacement profile

Shear force profile

Random Responses of Mean Square Spectra

Displacement spectra

Shear force spectra

Random Responses of Mean Square Profile

Displacement

Shear force

Comparisons of the Three Formulations

Relations of Series Representation, Large Stiffness Technique, Cesaro Sum, Quasi-static Decomposition and Stokes' Transformation

Conclusions

- **New Method for Multi-support Motion --- Stokes' Transformation**
  - **Free from calculating quasi-static solution**
  - **Accelerate convergence rate**
- **Why Cesaro sum can extract finite part is proved by Stokes' transformation**
- **The transient and random responses of multi-support motion problems have been solved.**