



潛堤對入射波消波影響之研究

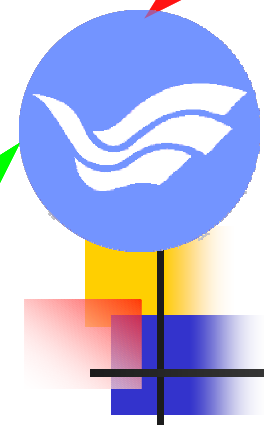
二十五屆力學會議報告

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海洋大學河海工程研究所

日期：九十年十二月十五日

地點：逢甲大學



報告流程圖

問題描述

研究目的

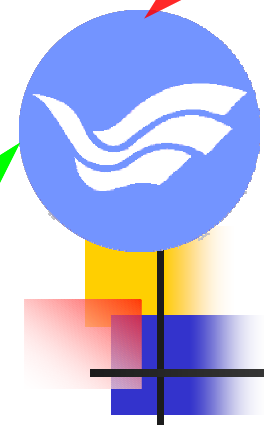
研究動機

分析方法
Dual BEM

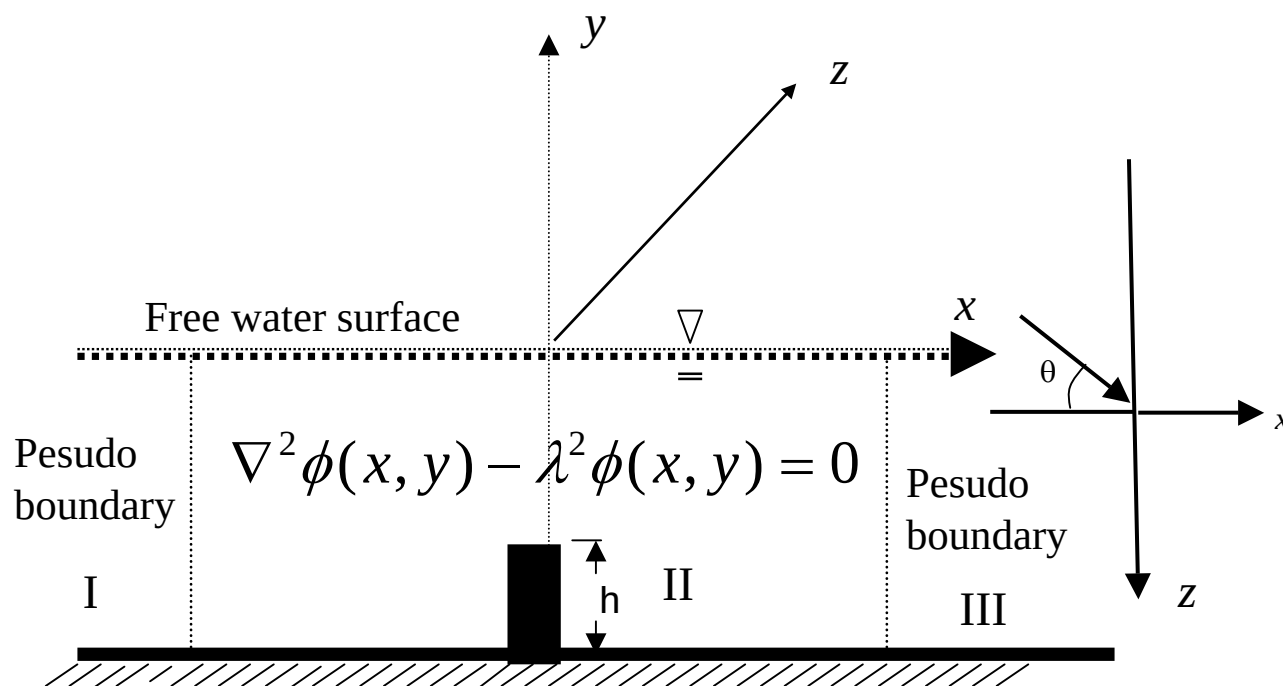
奇異積分的獲得

數值分析

結論

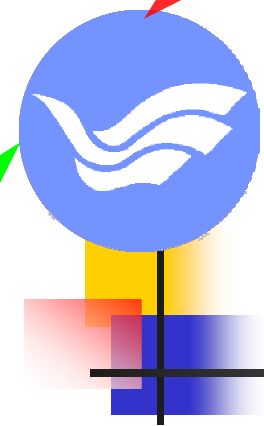


問題描述



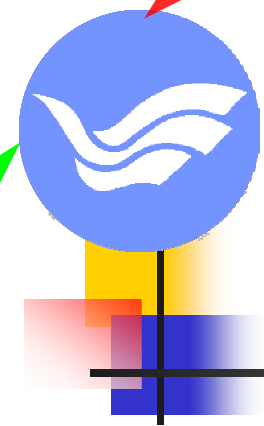
$$\nabla^2 \Phi(x, y, z, t) = 0 \Rightarrow \phi(x, y) e^{i(\lambda z - \sigma t)} \quad \text{Periodic time}$$

$$\lambda = k \sin \theta \quad \text{uniform water depth in z-axis}$$

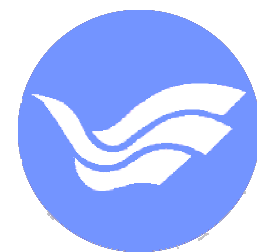


研究目的

- 建立一套Dual BEM程式，分析極薄潛堤（退化邊界）承受入射波的消能分析。

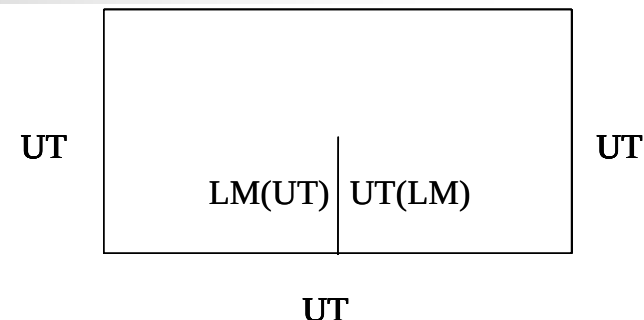


研究動機



Method UT (normal boundary)+LM

UT



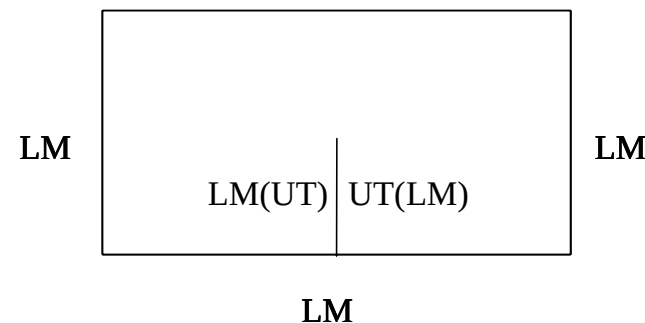
傳統邊界元素法 UT equation



對偶邊界元素法 LM equation

Method LM (normal boundary)+UT

LM





以多項式方程式為例

$$f(x) = (x-a)^2 Q(x) + px + q$$

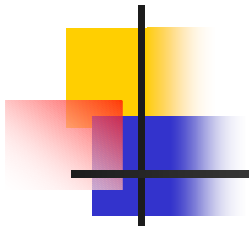
$$f(a) = pa + q, \quad \text{when } x = a$$

The constraint equation is not enough to determine the coefficients p and q ,

another constraint equation is required

$$f'(x) = 2(x-a)\overline{Q}(x) + p$$

$$f'(a) = p, \quad \text{when } x = a$$



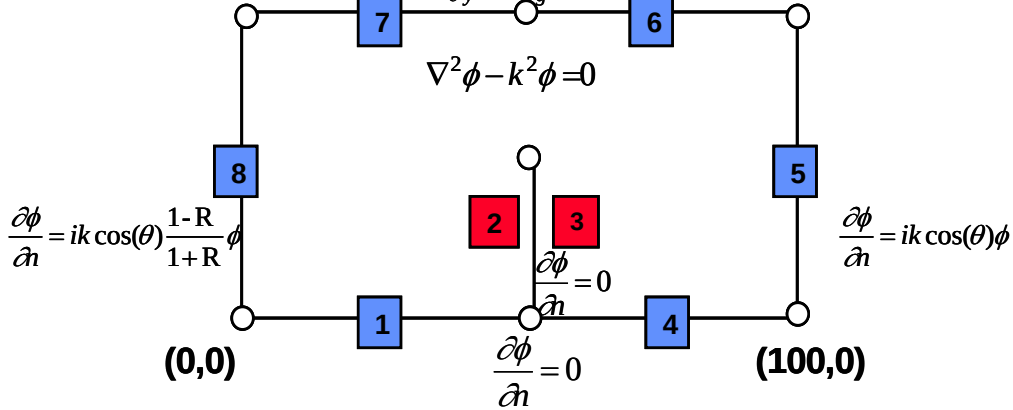
分析方法

$$2\pi \ u(x) = \int_B \{T(s, x)u(s) - U(s, x)t(s)\}dB(s)$$

$$2\pi \ t(x) = \int_B \{M(s, x)u(s) - L(s, x)t(s)\}dB(s)$$

$$U(\mathbf{s}, \mathbf{x}) = \frac{-i\pi D_0^{(1)}(\lambda r)}{2}, \quad T(\mathbf{s}, \mathbf{x}) = \frac{-i\lambda\pi}{2} D_1^{(1)}(\lambda r) \frac{y_i n_i}{r}$$

$$L(\mathbf{s}, \mathbf{x}) = \frac{i\lambda\pi}{2} D_1^{(1)}(\lambda r) \frac{y_i \bar{n}_i}{r}, \quad M(\mathbf{s}, \mathbf{x}) = \frac{-i\lambda\pi}{2} \left\{ -\lambda \frac{D_2^{(1)}(\lambda r)}{r^2} y_i y_j n_i \bar{n}_j \right. \\ \left. + \frac{D_1^{(1)}(\lambda r)}{r} n_i \bar{n}_i \right\}$$



- geometry node
- ▣ the Nth constant or linear element

$$[U]\{t\} = [T]\{u\}$$

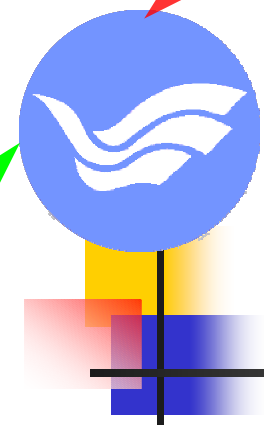
$$[L]\{t\} = [M]\{u\}$$

$$J: \begin{bmatrix} -3.96 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\ -.19 & -4.3 & -4.3 & -.19 & .00 & -.02 & -.02 & .00 \\ -.19 & -4.3 & -4.3 & -.19 & .00 & -.02 & -.02 & .00 \\ .00 & .00 & .00 & -3.96 & .00 & .00 & .00 & .00 \\ .00 & .00 & .00 & -.06 & -4.39 & -.06 & .00 & .00 \\ .00 & .00 & .00 & .00 & .00 & -3.96 & .00 & .00 \\ .00 & .00 & .00 & .00 & .00 & .00 & -3.96 & .00 \\ -.06 & .00 & .00 & .00 & .00 & .00 & -.06 & -4.39 \end{bmatrix}$$

$$2(+), 3(+), n(s) \begin{bmatrix} 1E+08 & 4E+08 & 4E+08 & .8E+22 & .7E+23 & .1E+23 & .5E+08 & .8E+08 \\ .2E+15 & .1E+02 & .1E+02 & .2E+15 & .2E+16 & .3E+15 & .3E+15 & .2E+16 \\ .2E+15 & .1E+02 & .1E+02 & .2E+15 & .2E+16 & .3E+15 & .3E+15 & .2E+16 \\ .8E+22 & .4E+08 & .4E+08 & .1E+08 & .8E+08 & .5E+08 & .1E+23 & .7E+23 \\ .4E+30 & .1E+16 & .1E+16 & .3E+15 & .3E+02 & .3E+15 & .4E+30 & .3E+31 \\ .1E+23 & .6E+08 & .6E+08 & .5E+08 & .8E+08 & .1E+08 & .8E+22 & .7E+23 \\ .5E+08 & .6E+08 & .6E+08 & .1E+23 & .7E+23 & .8E+22 & .1E+08 & .8E+08 \\ .3E+15 & .1E+16 & .1E+16 & .4E+30 & .3E+31 & .4E+30 & .3E+15 & .3E+02 \end{bmatrix} 2(+), 3(+)$$

$$T: \begin{bmatrix} -3.14 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\ .13 & -3.14 & -3.14 & .13 & .00 & .02 & .02 & .00 \\ .13 & -3.14 & -3.14 & .13 & .00 & .02 & .02 & .00 \\ .00 & .00 & .00 & -3.14 & .00 & .00 & .00 & .00 \\ .00 & .00 & .00 & .05 & -3.14 & .05 & .00 & .00 \\ .00 & .00 & .00 & .00 & .00 & -3.14 & .00 & .00 \\ .00 & .00 & .00 & .00 & .00 & .00 & -3.14 & .00 \\ .05 & .00 & .00 & .00 & .00 & .00 & .05 & -3.14 \end{bmatrix}$$

$$+ i \begin{bmatrix} .0E+00 & 3E+08 & -3E+08 & .0E+00 & .5E+23 & .1E+22 & .1E+08 & .5E+08 \\ .1E+14 & .0E+00 & .0E+00 & .1E+14 & .1E+16 & .3E+14 & .3E+14 & .1E+16 \\ .1E+14 & .0E+00 & .0E+00 & .1E+14 & .1E+16 & .3E+14 & .3E+14 & .1E+16 \\ .0E+00 & -3E+08 & 3E+08 & .0E+00 & .5E+08 & .1E+08 & .1E+22 & .5E+23 \\ .1E+29 & .8E+15 & .8E+15 & .2E+14 & .0E+00 & .2E+14 & .1E+29 & .2E+31 \\ .1E+22 & .4E+08 & .4E+08 & .1E+08 & .5E+08 & .0E+00 & .2E+13 & .5E+23 \\ .1E+08 & .4E+08 & -.4E+08 & .1E+22 & .5E+23 & -.2E+13 & .0E+00 & .5E+08 \\ .2E+14 & .8E+15 & -.8E+15 & .1E+29 & .2E+31 & .1E+29 & .2E+14 & .0E+00 \end{bmatrix}$$



奇異積分的獲得



(採常數元素離散)

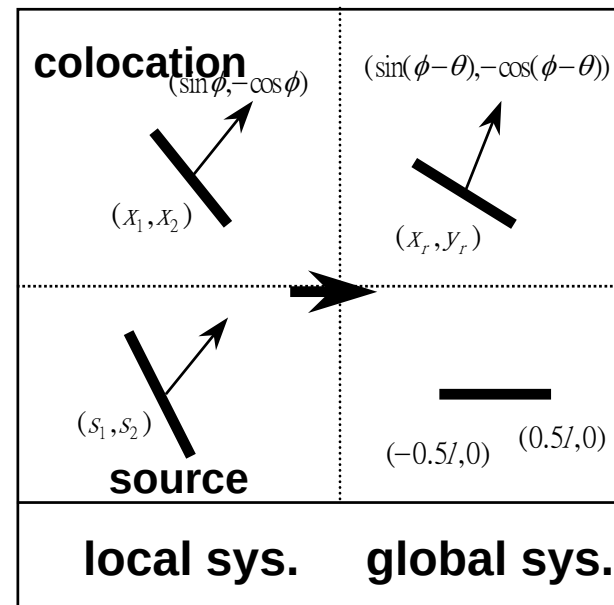
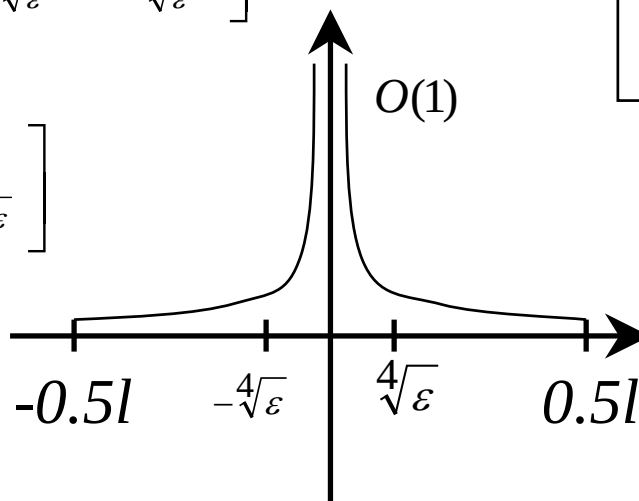
- 奇異積分轉成有限值 $= \pi$
- (1) $L(s, x)$ kernel: $p=q$ (singular integral)

$$L_{pq} = ik \lim_{\varepsilon \rightarrow 0} \int_{-0.5l}^{0.5l} D_1^2(k\sqrt{s^2 + \varepsilon^2}) \frac{-\varepsilon}{\sqrt{s^2 + \varepsilon^2}} ds$$

$$= -ik \lim_{\varepsilon \rightarrow 0} \left[\int_{-0.5l}^{-\sqrt[4]{\varepsilon}} + \int_{-\sqrt[4]{\varepsilon}}^{\sqrt[4]{\varepsilon}} + \int_{\sqrt[4]{\varepsilon}}^{0.5l} \right] \text{極限方法}$$

$$= -\lim_{\varepsilon \rightarrow 0} \left[\tan^{-1} \frac{s}{\varepsilon} \right]_{-\sqrt[4]{\varepsilon}}^{\sqrt[4]{\varepsilon}}$$

$$= -\pi, \text{ (i no sum)}$$



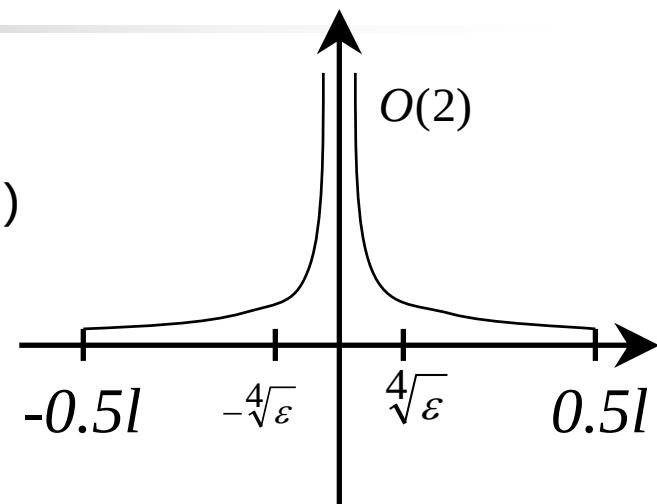
座標系統轉換



• 超強奇異積分轉成正規積分

(1) $M(s, x)$ kernel: $p=q$ (hypersingular integral)

$$\begin{aligned}
 M_{pq} &= ik \lim_{\varepsilon \rightarrow 0} \int_{-0.5l}^{0.5l} k \frac{D_2^1(k\sqrt{s^2 + \varepsilon^2})}{s^2 + \varepsilon^2} \varepsilon^2 + \frac{D_1^2(k\sqrt{s^2 + \varepsilon^2})}{\sqrt{s^2 + \varepsilon^2}} ds \\
 &= ik^2 \lim_{\varepsilon \rightarrow 0} \left[\int_{-0.5l}^{-\sqrt[4]{\varepsilon}} + \int_{-\sqrt[4]{\varepsilon}}^{\sqrt[4]{\varepsilon}} + \int_{\sqrt[4]{\varepsilon}}^{0.5l} \right] ds \quad (\text{極限方法，部分積分}) \\
 &= ik \left\{ -2D_1^2\left(\frac{kl}{2}\right) + k \left[D_0^1\left(\frac{kl}{2}\right) - k \int_{-0.5l}^{0.5l} D_1^2(k|s|)|s| ds \right] \right\}. \text{ (ino sum) } \neq \text{H.P.V}
 \end{aligned}$$





• 解題流程

對偶積分方程式的推導

對偶積分方程式的離散化

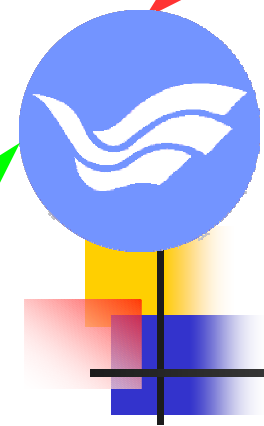
水面邊界條件

潛堤邊界條件(剛性、柔性、
不透水、可透水)

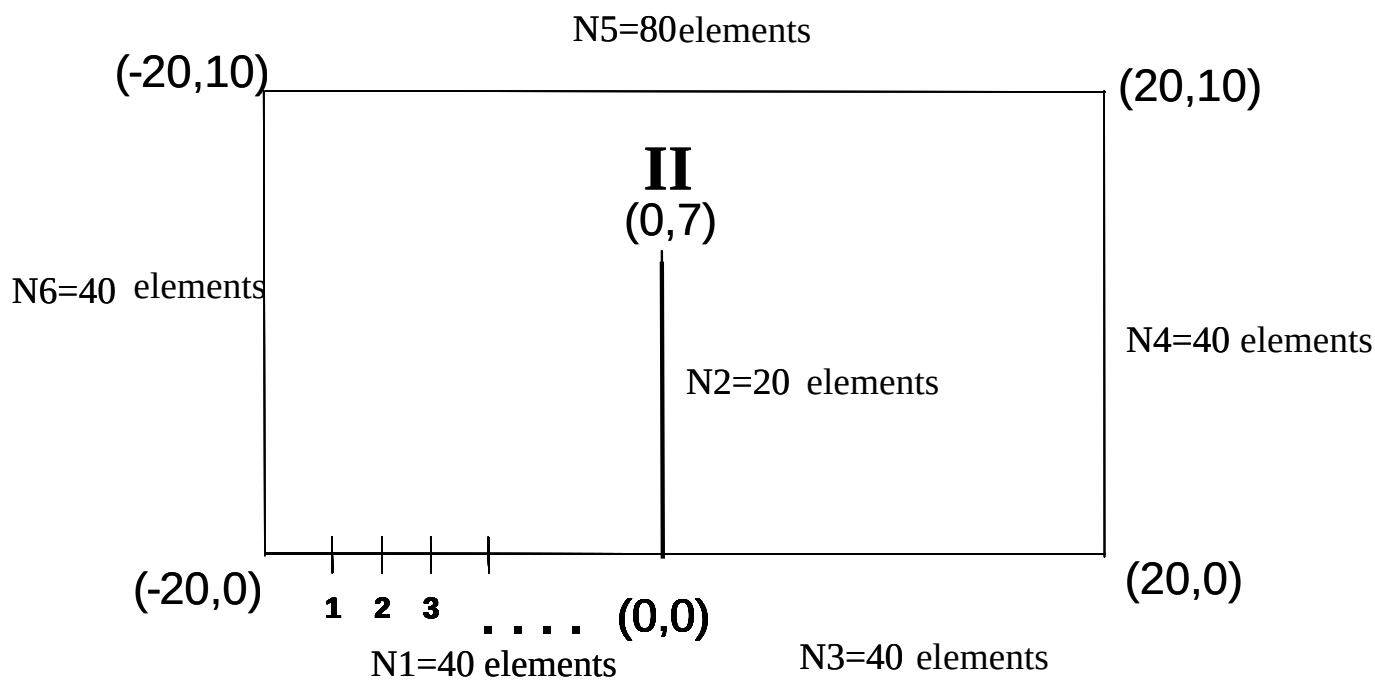
人工邊界

解線性代數方程

計算 Reflection (R) , Transmission (T)



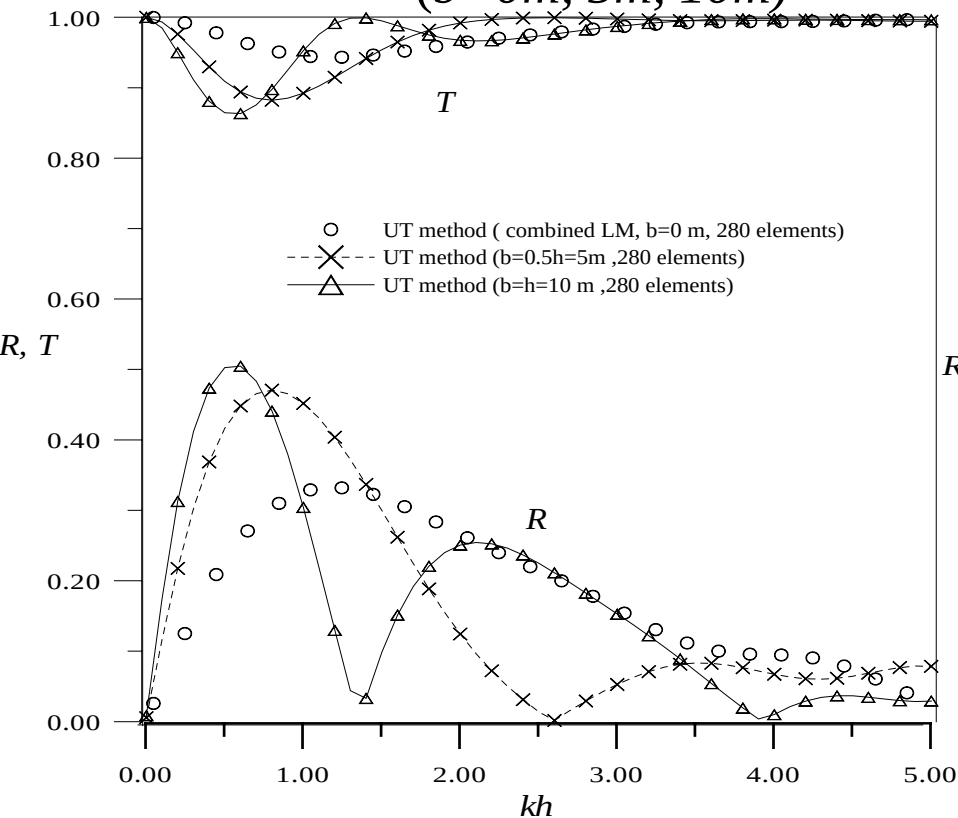
數值分析



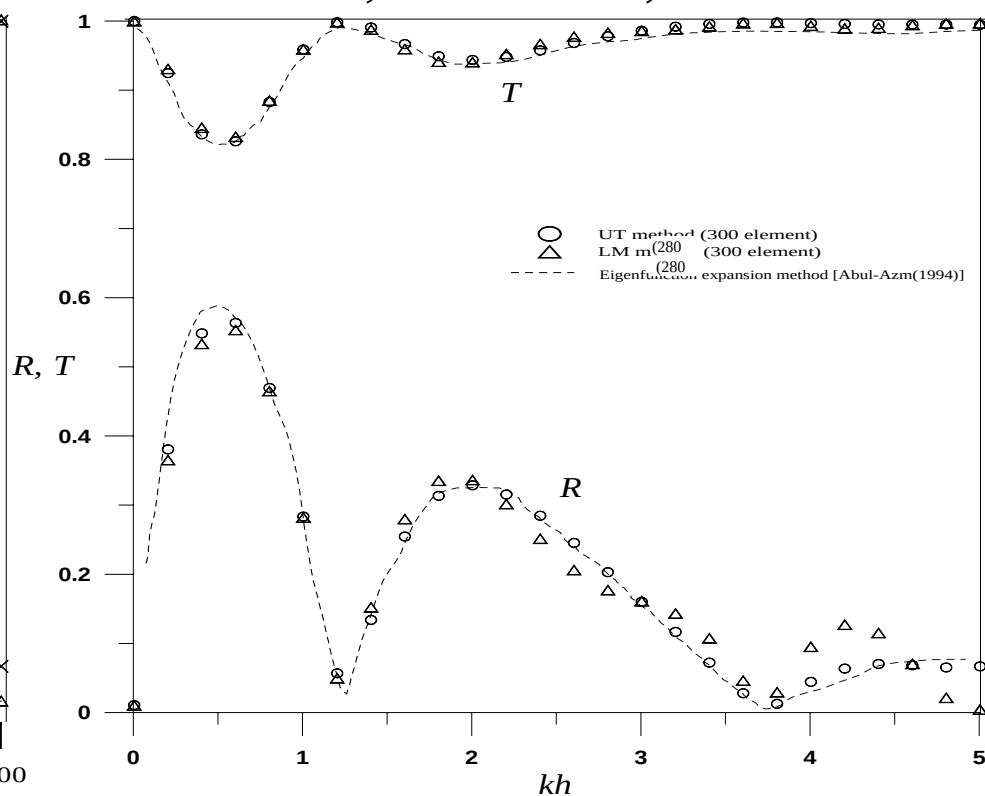
$$N=N1+2N2+N3+N4+N5+N6=280 \text{ elements}$$

•不同週期水波的消波效果（正向入射， $\theta=0^\circ$ ）

不同潛堤厚度
($b=0\text{m}, 5\text{m}, 10\text{m}$)

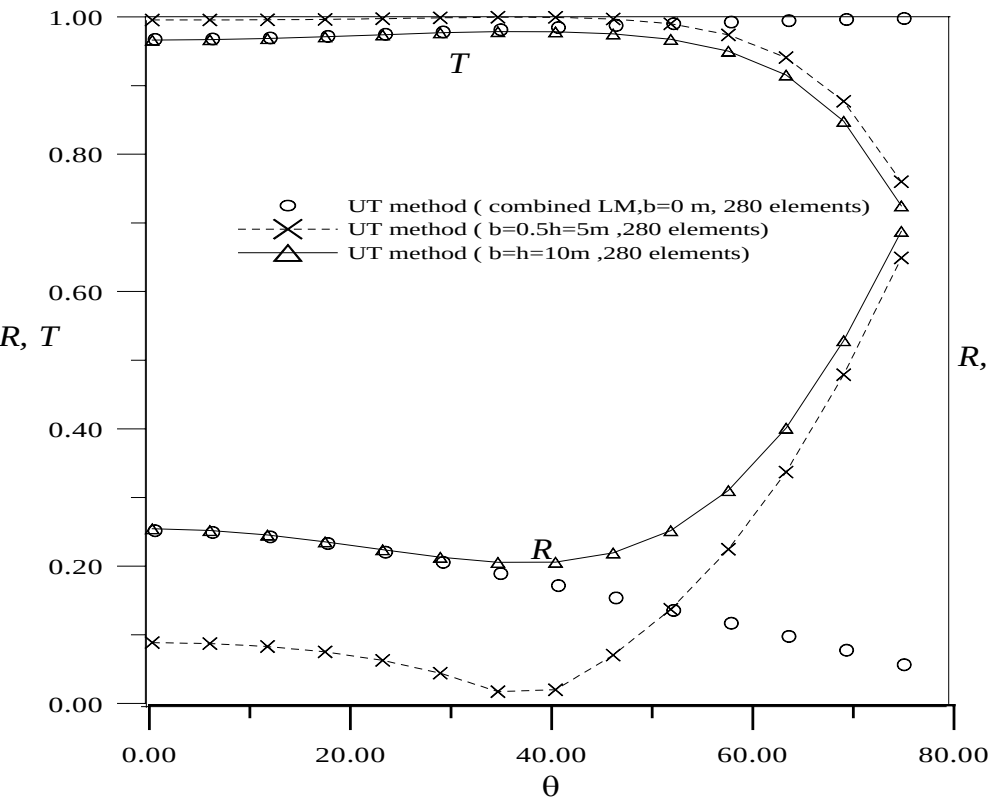


極薄厚度($b=0\text{m}$)不同方法
UT method, LM method, exact solution

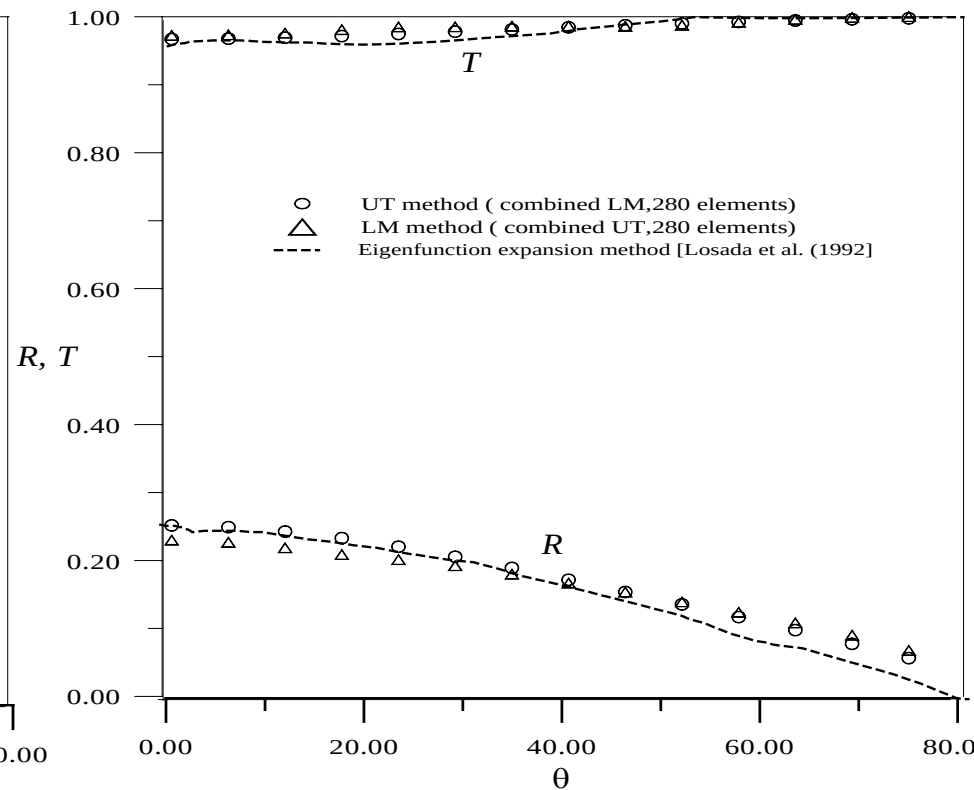


• 各種入射水波角度的消波效果， $kh = 2.1362$

不同潛堤厚度
($b=0\text{m}, 5\text{m}, 10\text{m}$)



極薄厚度($B=0\text{m}$)不同方法
UT method, LM method, exact solution





結論

- BEM僅需對邊界切割元素即可，可大大縮短計算與前處理時間。
- 奇異積分與超強奇異積分轉成定值及正規積分（獲得C.P.V.與H.P.V.）。
- 針對含或不含退化邊界之潛提的消波問題，發展一套對偶邊界元素程式。

結 束

敬 請 指 教

