

超奇異積分方程在含自由水面滲流問題之應用

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Applications of hypersingular equations to free-surface seepage problem

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Abstract

Hypersingular BEM Speed up rate of convergence

Easy mesh and remesh for free surface

1、Element mesh for FEM and BEM

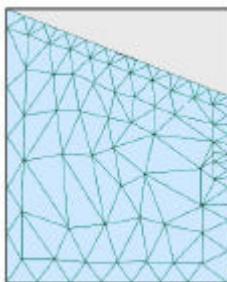


Fig. 1 Finite element mesh

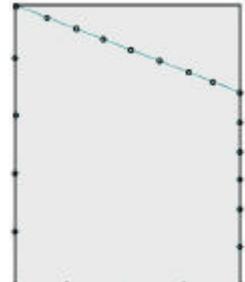


Fig. 2 Boundary element mesh

2、Numerical examples

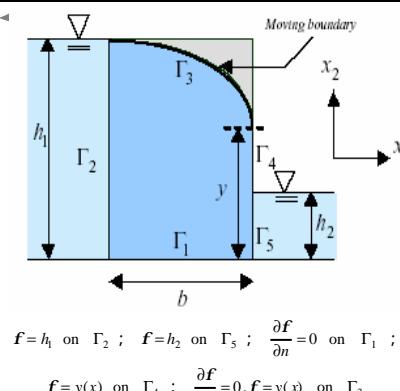


Fig. 3 Flow through rectangular dam

Case 1 : $h_1 = 24$, $h_2 = 4$, and $b = 16$

Case 2 : $h_1 = 1$, $h_2 = 0$, and $b = 1$

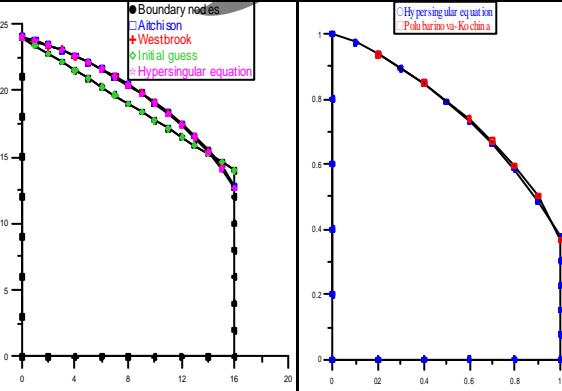


Fig. 4 Free surface of case 1
(Hypersingular equation)

Fig. 5 Free surface of case 2
(Hypersingular equation)

Table 1 Free surface obtained by different methods

Case 1								
x	2	4	6	8	10	12	14	16 (Separation point)
Aitchison (1972)	23.41	22.59	21.60	20.43	19.08	17.48	15.54	12.79
Westbrook (1985)	23.32	22.52	21.55	20.36	19.07	17.45	15.51	-
Present (Singular equation) (2004)	23.42	22.59	21.60	20.43	19.07	17.47	15.50	12.61
Present (Hypersingular equation) (2004)	23.40	22.52	21.57	20.39	19.02	17.39	15.39	12.68 (Best)

Case 2

x	0.2	0.4	0.6	0.8	1.0 (Separation point)
Polubarinova-Kochina (1962)	0.938	0.850	0.738	0.595	0.368
Present (Singular equation) (2004)	0.939	0.850	0.737	0.590	0.368
Present (Hypersingular equation) (2004)	0.937	0.847	0.732	0.584	0.379 (Best)

Table 2 Comparison for mesh and number of iterations

Case 1		
Method	Mesh	Number of iterations
FEM	17 x 5	49
Singular equation	39	14
Hypersingular equation	39	13 (Best)

Case 2		
Method	Mesh	Number of iterations
Singular equation	25	12
Hypersingular equation	25	9 (Best)

Table 3 Final position of separation point using different methods

References	Height
Polubarinova-Kochina (1962)	12.95
Cryer (1976)	12.7132
Ozis (1981)	12.7070
Westbrook (1985), FEM	NA
Bruch (1988), BEM, Linear element	12.98
Cabral (1991), BEM, B-spline	12.74
Present, (2004), BEM, constant element, Singular equation	12.61
Present, (2004), BEM, constant element, Hypersingular equation	12.68 (Best)

3、Conclusions

Friendly : easy mesh and remesh Efficient : fast rate of convergence Accurate, superior over other methods

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