Circular Torsion Bars with Circular Inclusions

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Seminar of Division of Structural Engineering in HRE in NTOU

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Outlines

- Review of the papers
  - Null field formulation
  - Results
- Applications to derive Green's function
  - Problem statement
  - Numerical example
- Conclusion

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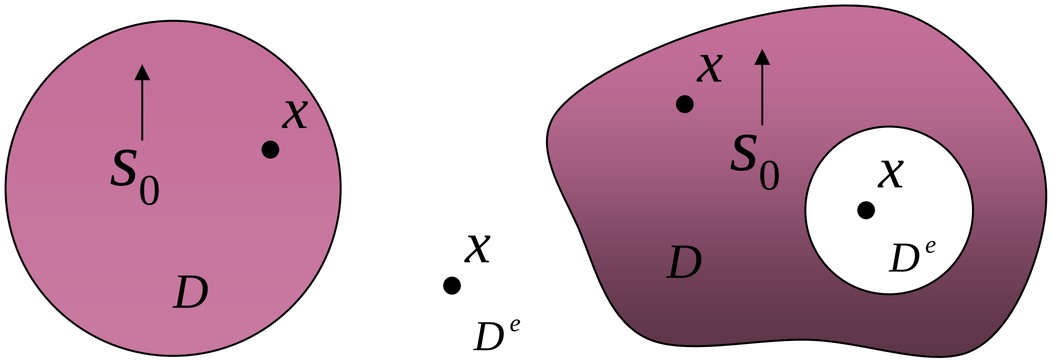
Reference method



where

$$U(s,x) = \ln|x-s| = \ln r$$

$$T(s,x) = \frac{\partial U(s,x)}{\partial n_s}$$

$$t(s) = \frac{\partial u(s)}{\partial n_s}$$

$$2\pi u(x) = \int_B T(s,x)u(s)dB(s) - \int_B U(s,x)t(s)dB(s) + U(s_0,x)$$

$$0 = \int_B T(s,x)u(s)dB(s) - \int_B U(s,x)t(s)dB(s) + U(s_0,x)$$

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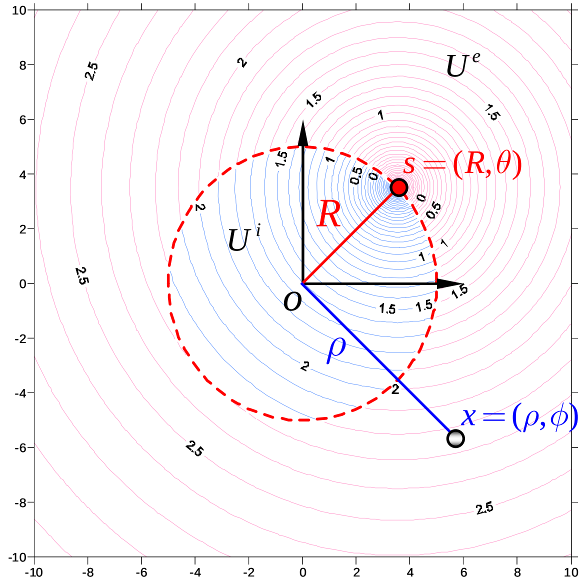
Reference method

$$U(s,x)=\begin{cases}U^i(R,\theta;\rho,\phi)=\ln R-\sum_{m=1}^{\infty}\frac{1}{m}(\frac{\rho}{R})^m\cos m(\theta-\phi),\ R\geq\rho\\ U^e(R,\theta;\rho,\phi)=\ln\rho-\sum_{m=1}^{\infty}\frac{1}{m}(\frac{R}{\rho})^m\cos m(\theta-\phi),\ \rho>R\end{cases}$$

$$T(s,x)=\begin{cases}T^i(R,\theta;\rho,\phi)=\frac{1}{R}+\sum_{m=1}^{\infty}(\frac{\rho^m}{R^{m+1}})\cos m(\theta-\phi),\ R>\rho\\ T^e(R,\theta;\rho,\phi)=-\sum_{m=1}^{\infty}(\frac{R^{m-1}}{\rho^m})\cos m(\theta-\phi),\ \rho>R\end{cases}$$

$$u_k(s)=a_0^k+\sum_{n=1}^{\infty}(a_n^k\cos n\theta_k+b_n^k\sin n\theta_k),\ s\in B_k$$

$$\frac{\partial u_k(s)}{\partial n_s}=p_0^k+\sum_{n=1}^{\infty}(p_n^k\cos n\theta_k+q_n^k\sin n\theta_k),\ s\in B_k$$



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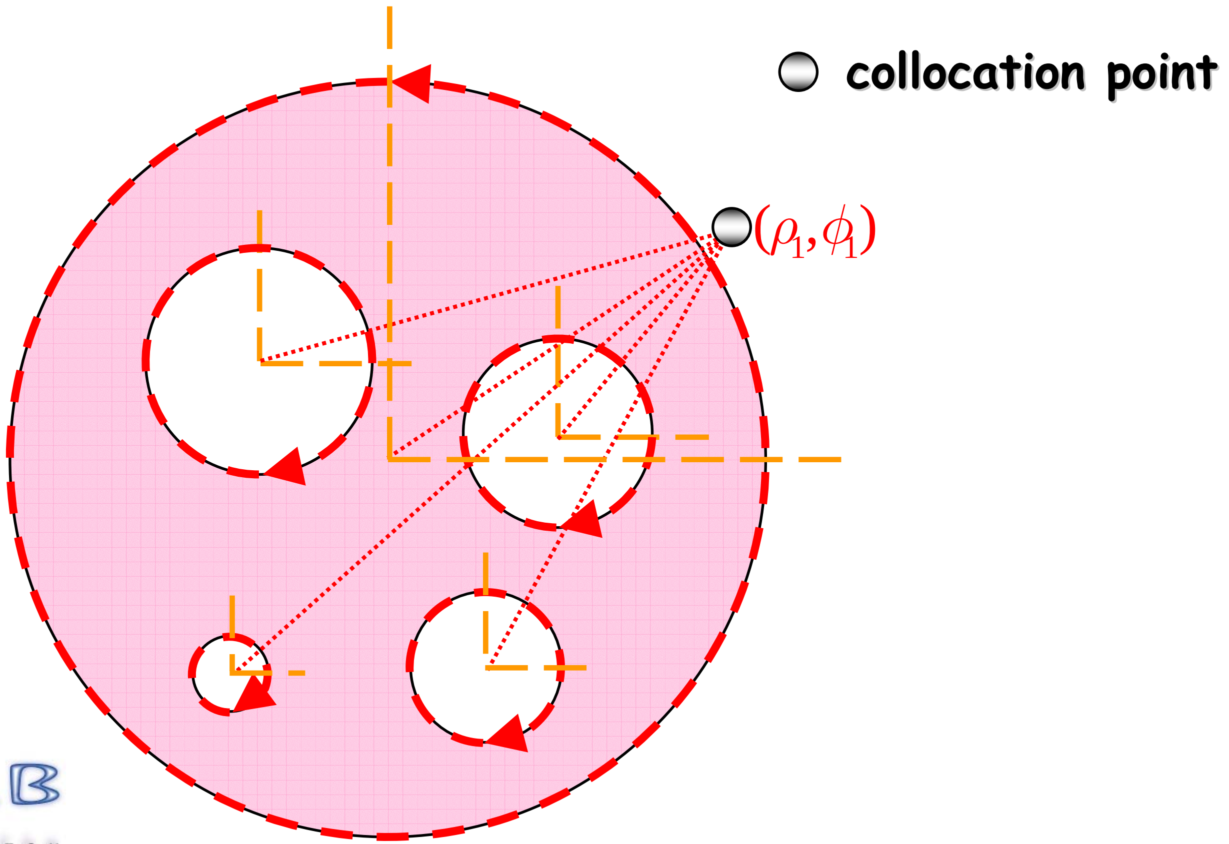
Adaptive observer system

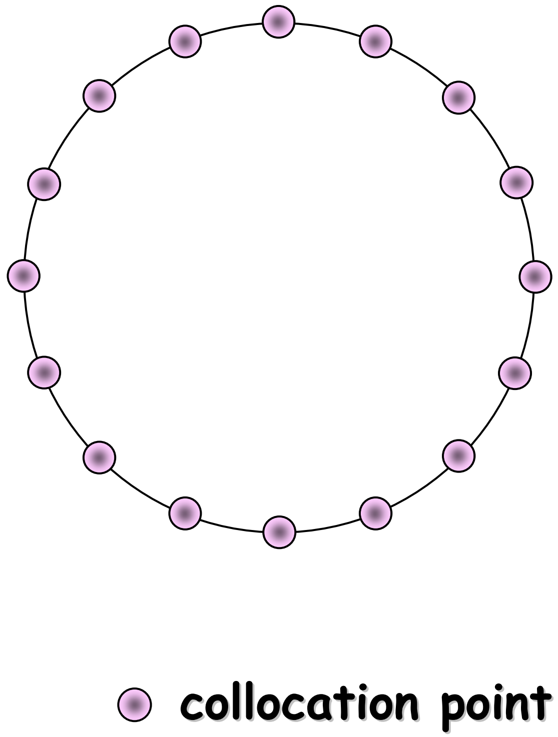


Fourier expansion and collocation points

$$u(s) = a_0 + \sum_{n=1}^{M} (a_n \cos n\theta + b_n \sin n\theta)$$

$$t(s) = p_0 + \sum_{n=1}^{M} (p_n \cos n\theta + q_n \sin n\theta)$$

2M+1 unknown Fourier coefficients



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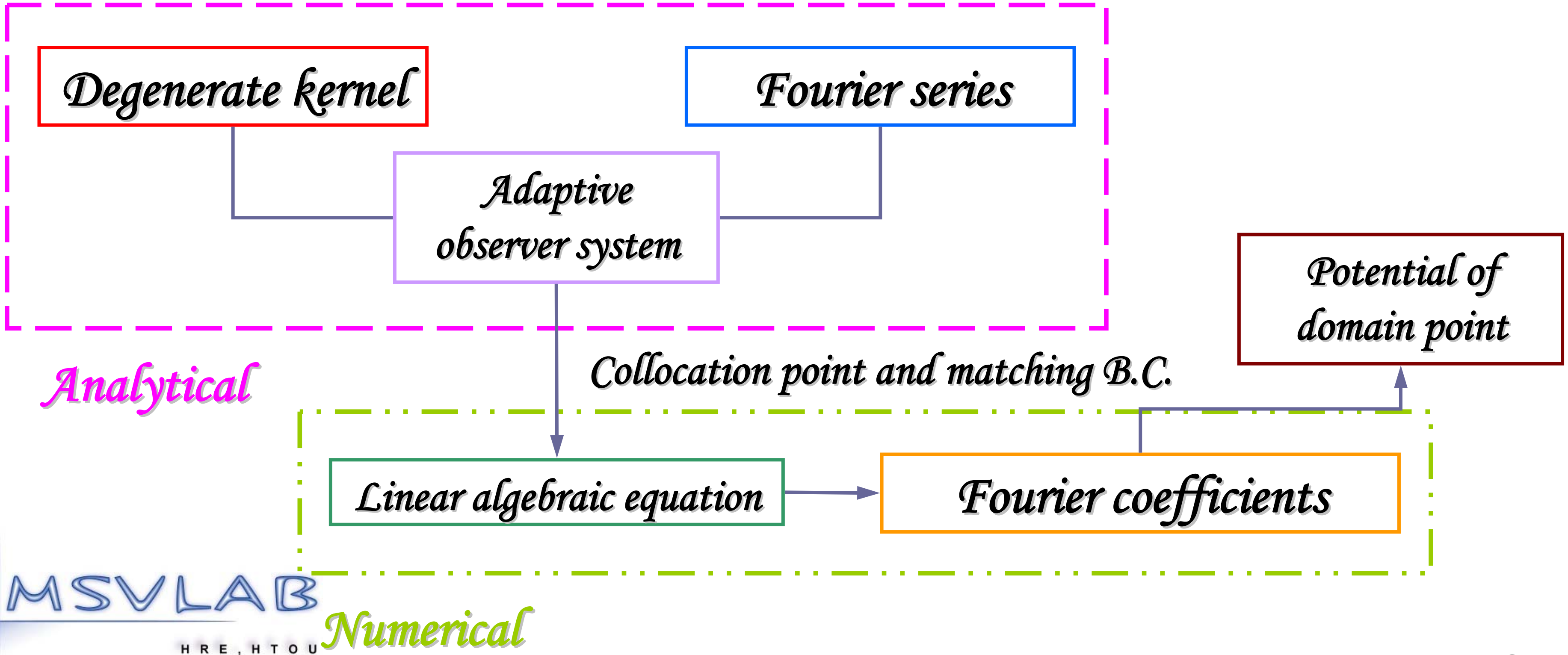
Flowchart of present method

$$0=\int_B [T(s,x)u(s)-U(s,x)t(s)]\,dB(s)$$



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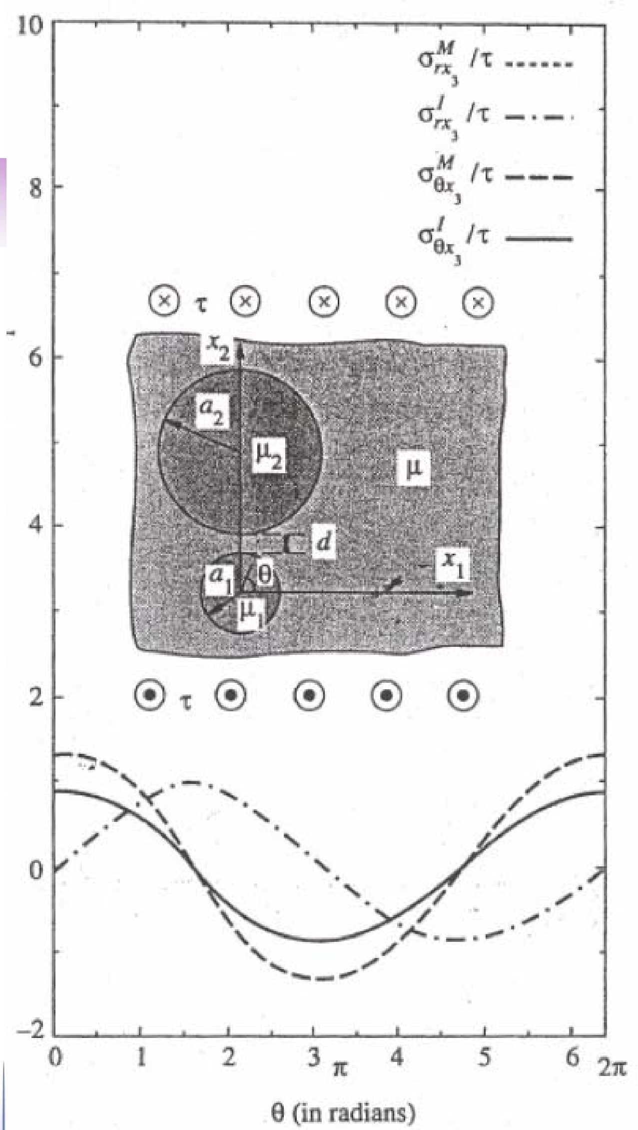
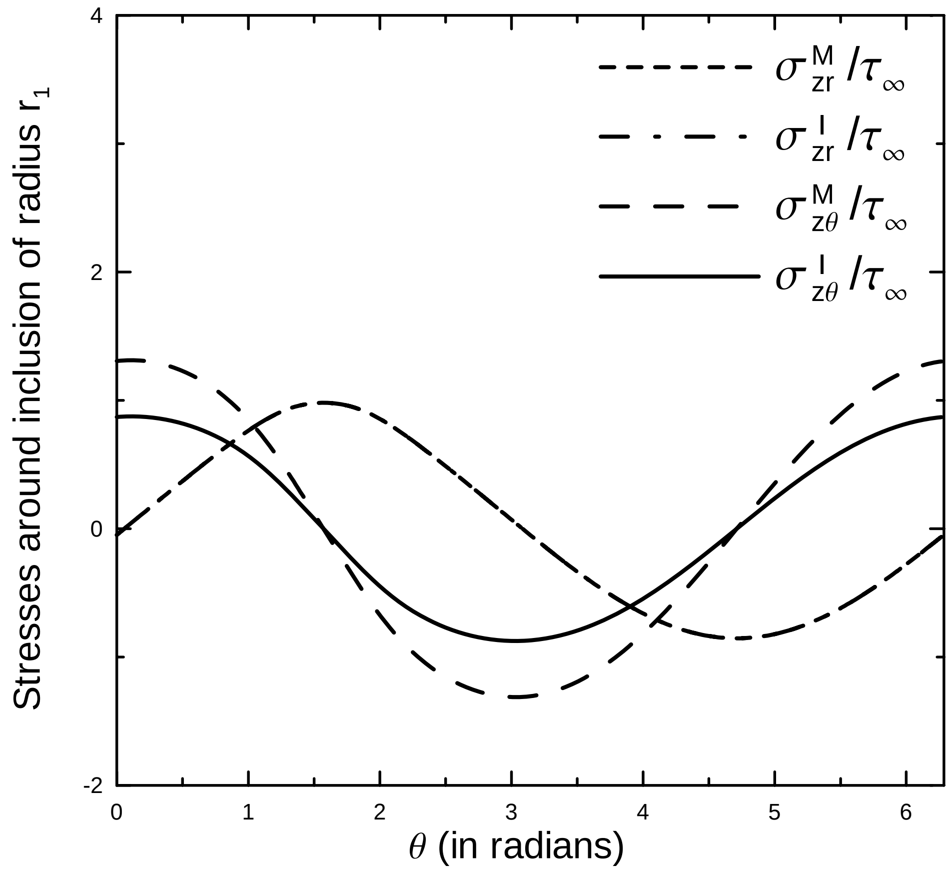
Example



$\sigma^{M}_{rx_3}/\tau$
$\sigma^{I}_{rx_3}/\tau$
$\sigma^{M}_{\theta x_3}/\tau$
$\sigma^{I}_{\theta x_3}/\tau$
τ
x_2
a_2
μ_2
μ
d
x_1
a_1
θ
μ_1
θ (in radians)



$\sigma^{M}_{zr}/\tau_\infty$
$\sigma^{I}_{zr}/\tau_\infty$
$\sigma^{M}_{z\theta}/\tau_\infty$
$\sigma^{I}_{z\theta}/\tau_\infty$
Stresses around inclusion of radius r_1
θ (in radians)

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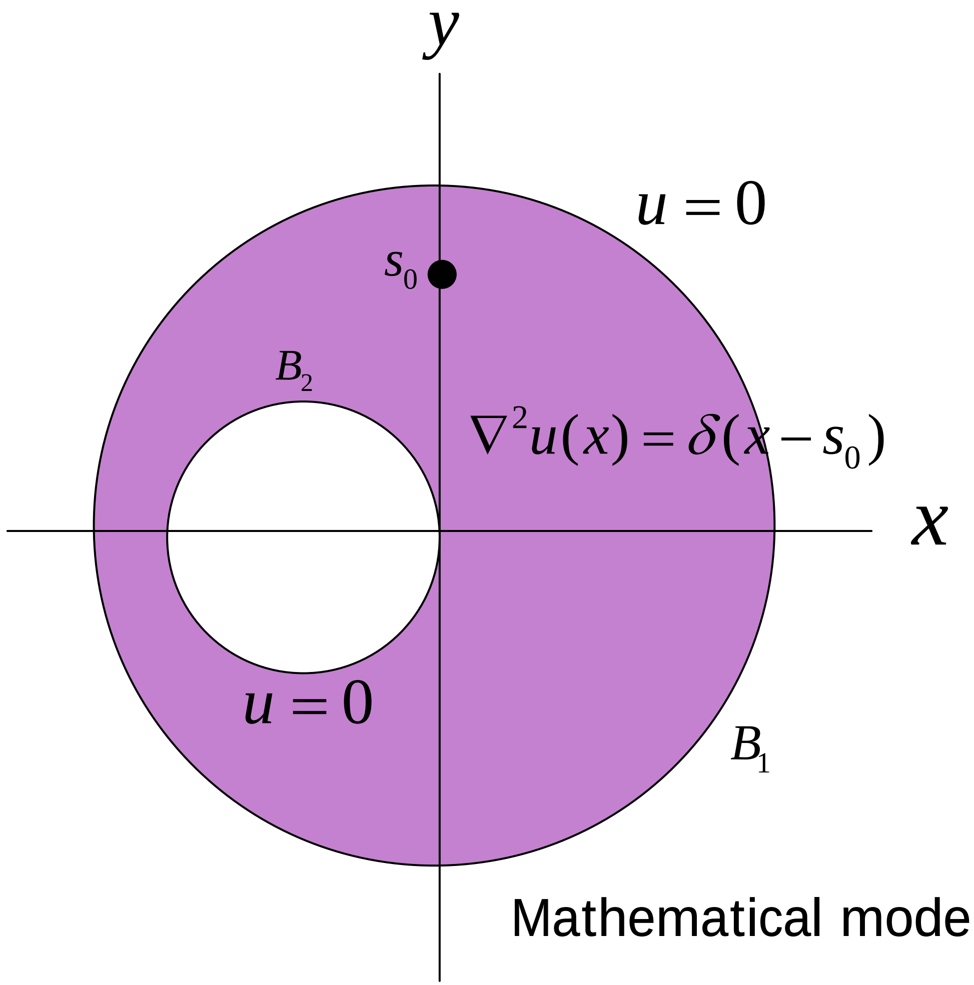
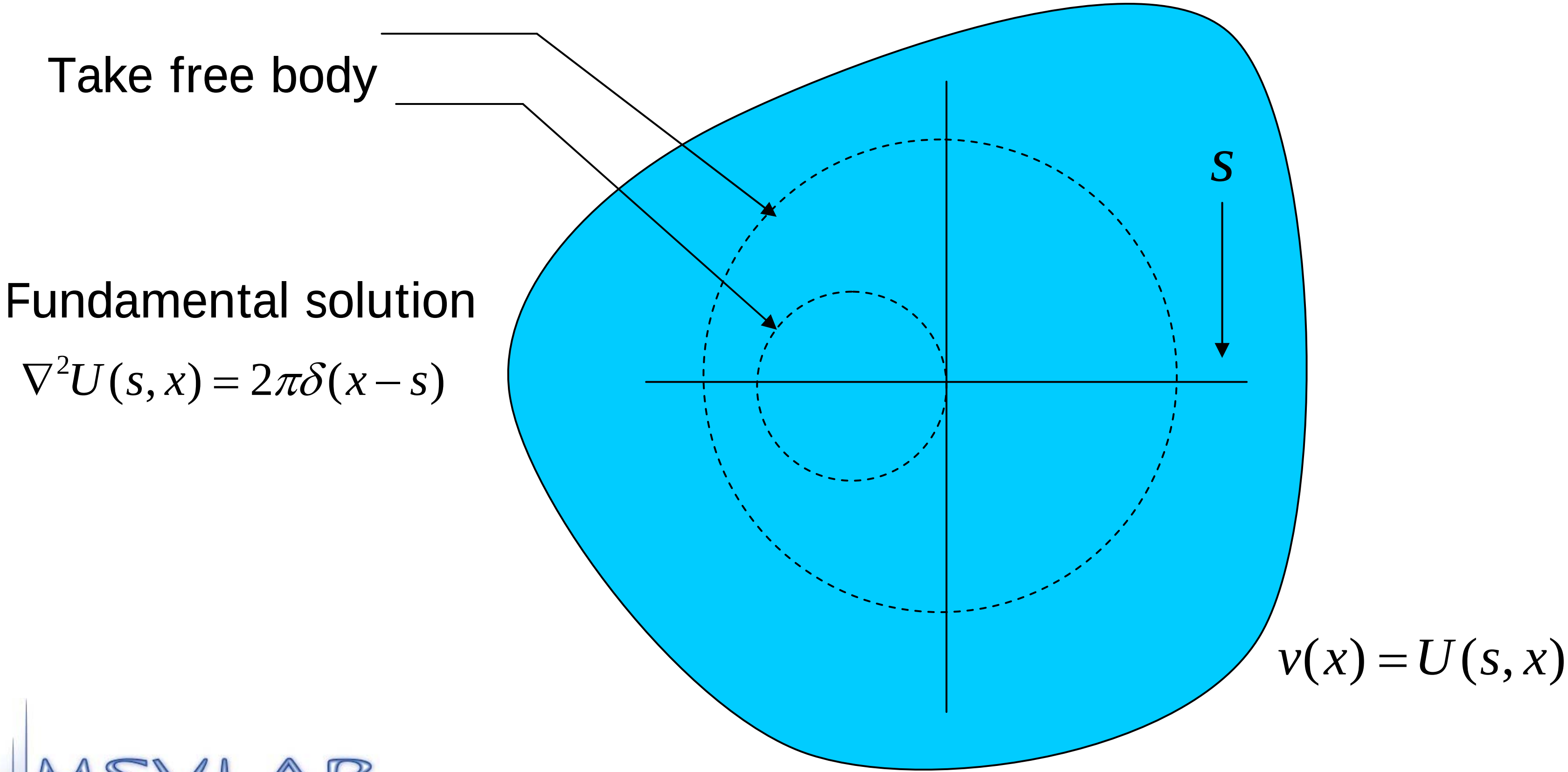
Problem statement



Mathematical model (PDE + BC)

Problem statement



Reference method

$$\iint_D (u\nabla^2 v - v\nabla^2 u)\,dD = \int_B \left(u\frac{\partial v}{\partial n} - v\frac{\partial u}{\partial n}\right) dB$$

$$2\pi u(s) = \int_{B_1+B_2} T(s,x)u(s)\,dB(s) - \int_{B_1+B_2} U(s,x)t(s)\,dB(s) + U(s_0,x)$$

U T **Degenerate kernels**

u t **Fourier series**

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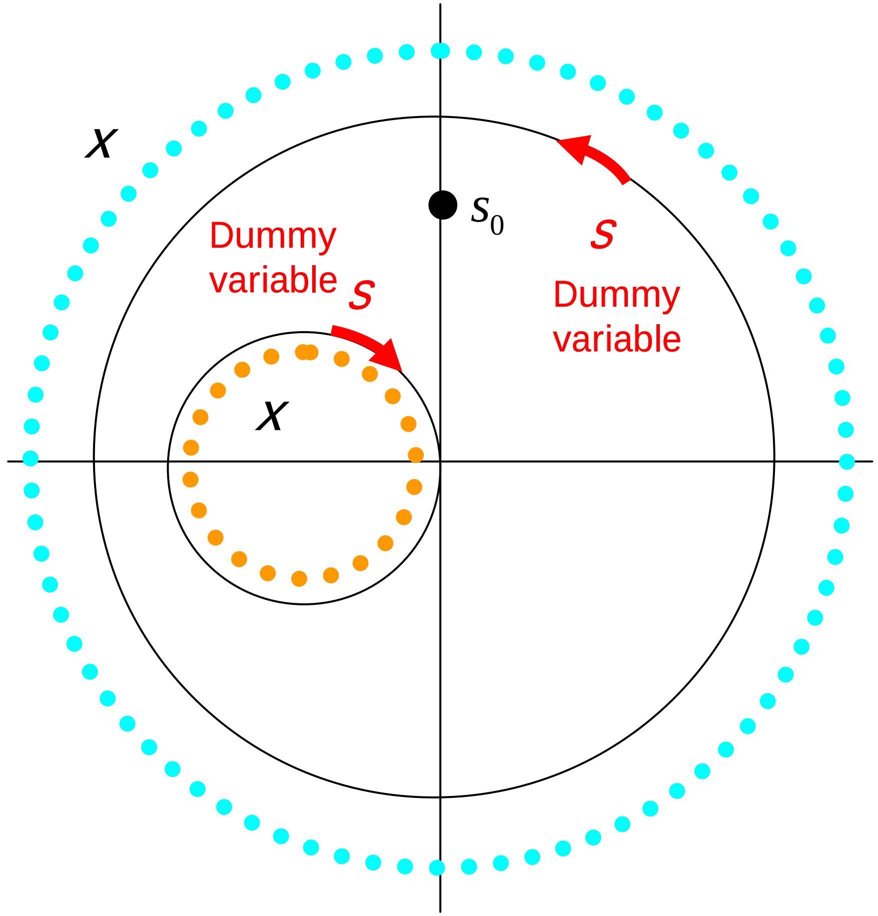
Problem statement

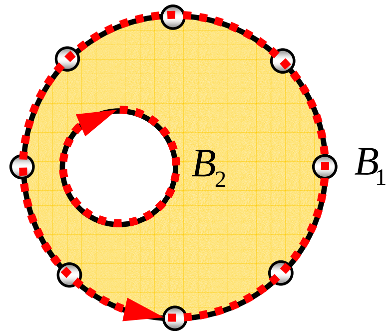
$$0=\int_{B_1}\left(T^e(s,x)\overset{0}{u}(s)-U^e(s,x)t(s)\right)dB(s)$$
$$+\int_{B_2}\left(T^e(s,x)\overset{0}{u}(s)-U^e(s,x)t(s)\right)dB(s)$$
$$+U(s_0,x)$$

$$0=\int_{B_1}\left(T^i(s,x)\overset{0}{u}(s)-U^i(s,x)t(s)\right)dB(s)$$
$$+\int_{B_2}\left(T^i(s,x)\overset{0}{u}(s)-U^i(s,x)t(s)\right)dB(s)$$
$$+U(s_0,x)$$



Linear algebraic equation

$$[\mathbf{U}]\{\mathbf{t}\}=\{\ln|r|\}$$



where

Index of collocation circle

$$[\mathbf{U}]=\begin{bmatrix}\boxed{U_{11}} & \boxed{U_{12}} \\ U_{21} & U_{22}\end{bmatrix}$$

Index of routing circle

$$\{t\}=\begin{Bmatrix} t_1^1 \\ t_2^1 \\ \vdots \\ t_n^1 \\ t_1^2 \\ t_2^2 \\ \vdots \\ t_n^2 \end{Bmatrix}$$

Column vector of Fourier coefficients

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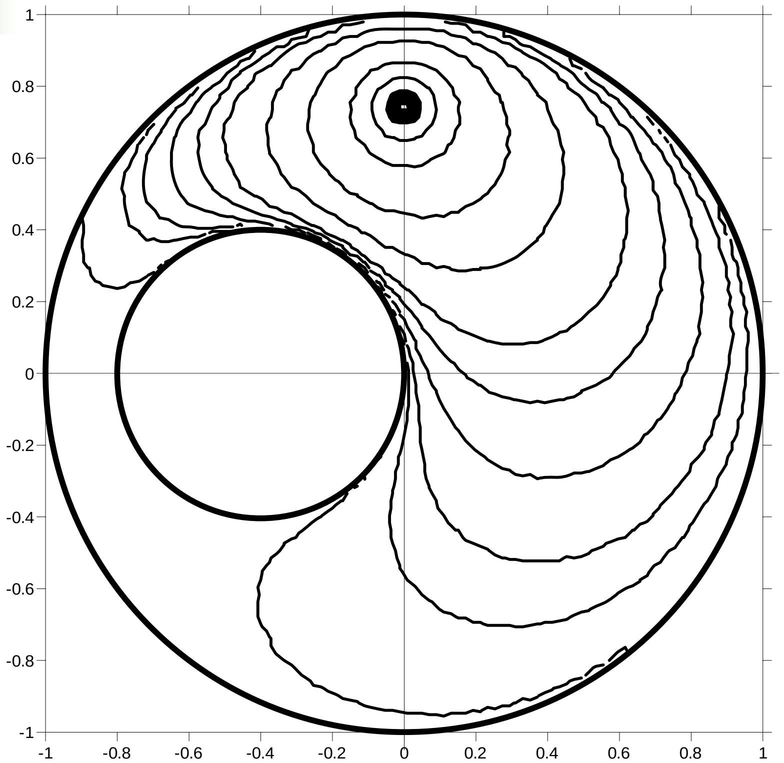
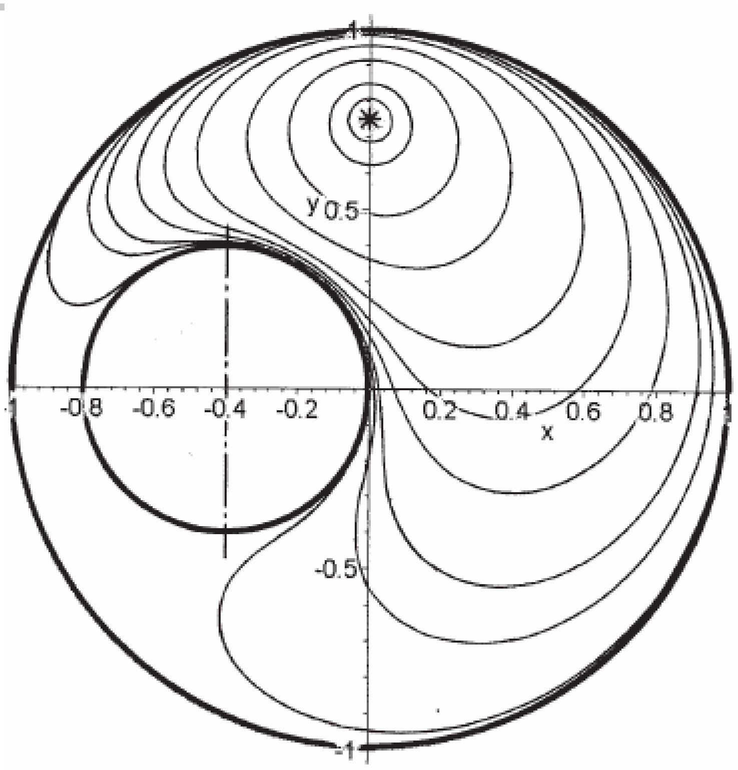
Numerical example (eccentric)



Present method



Green's function 2001

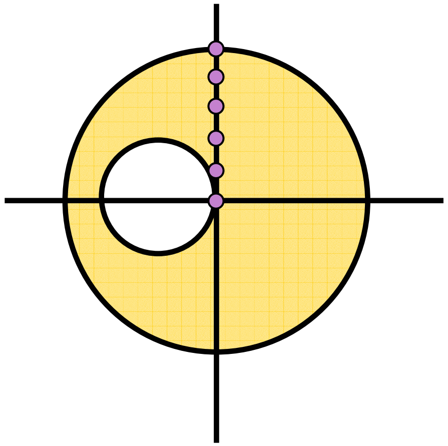
Numerical example (eccentric)

Field point, y	MCP			MMP			Present Method		
	k						Fourier term, n		
	10	20	50	10	20	50	10	20	50
0	-0.000280	-0.000128	-0.000067	-0.000107	0.000049	-0.000032	0 .000000	0.000000	0.000000
0.2	0.010667	0.010712	0.010781	0.010700	0.010779	0.010798	0 .010832	0.010832	0.010832
0.4	0.062359	0.062411	0.062443	0.062407	0.062435	0.062448	0 .062458	0.062462	0.062462
0.6	0.177534	0.177574	0.177585	0.177583	0.177590	0.177593	0.177597	0.177596	0.177596
0.8	0.317893	0.317902	0.317911	0.317907	0.317913	0.317914	0.318032	0.317915	0.317915
1.0	0.000014	0.000006	0.000002	0.000000	0.000000	0.000000	0.002014	0.000064	0.000000

MCP：Method of classical potentials

MMP：Method of modified potentials

k： Partitioning number



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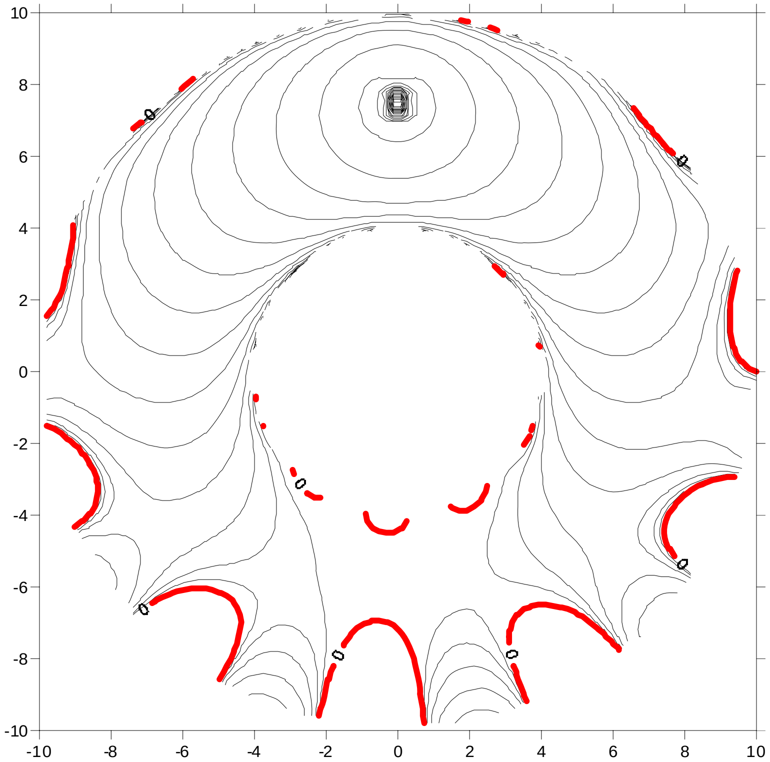
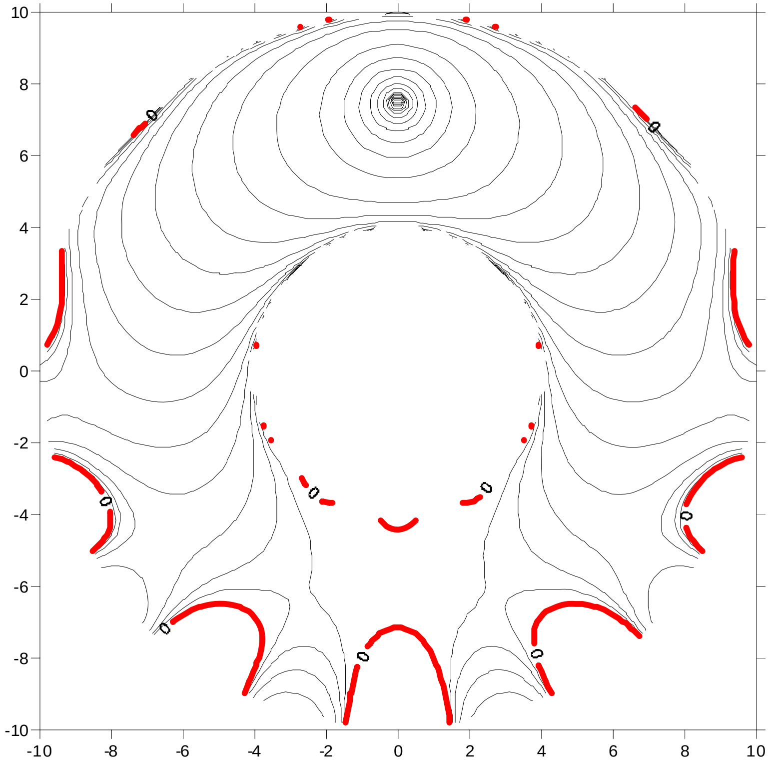
Numerical example (annular) m=10



Semi-analytical



analytical

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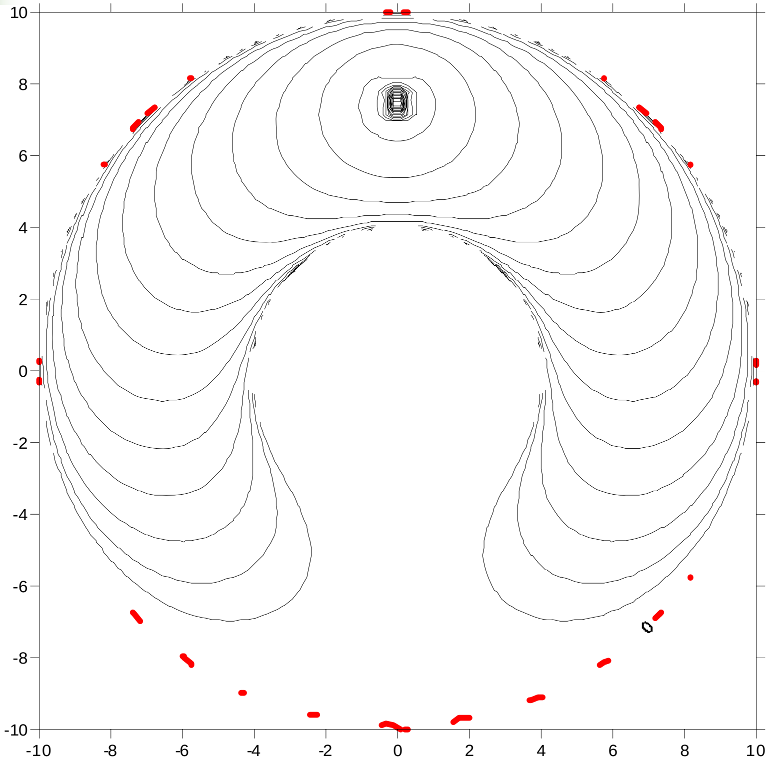
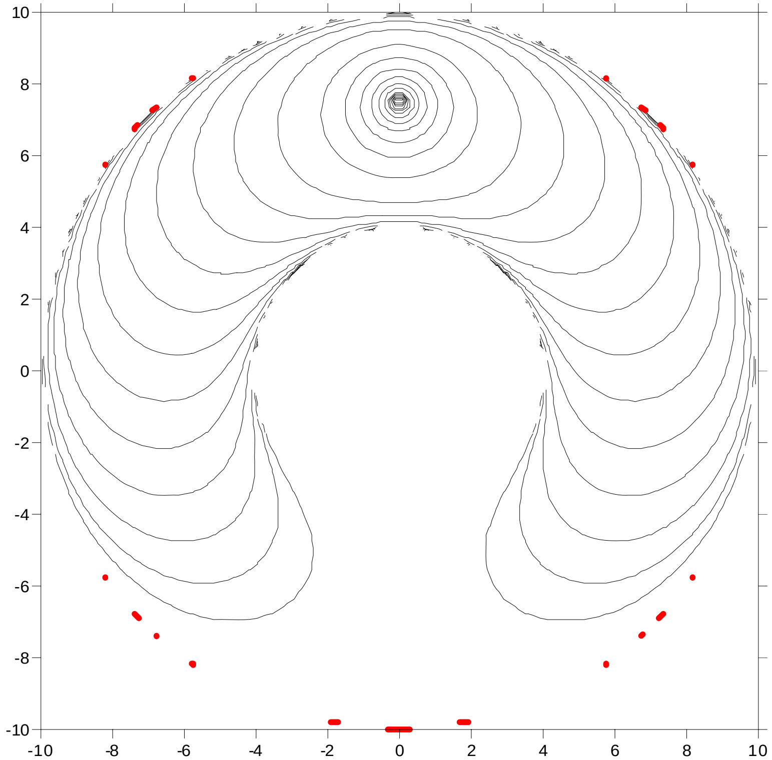
Numerical example (annular) m=30



Semi-analytical



analytical

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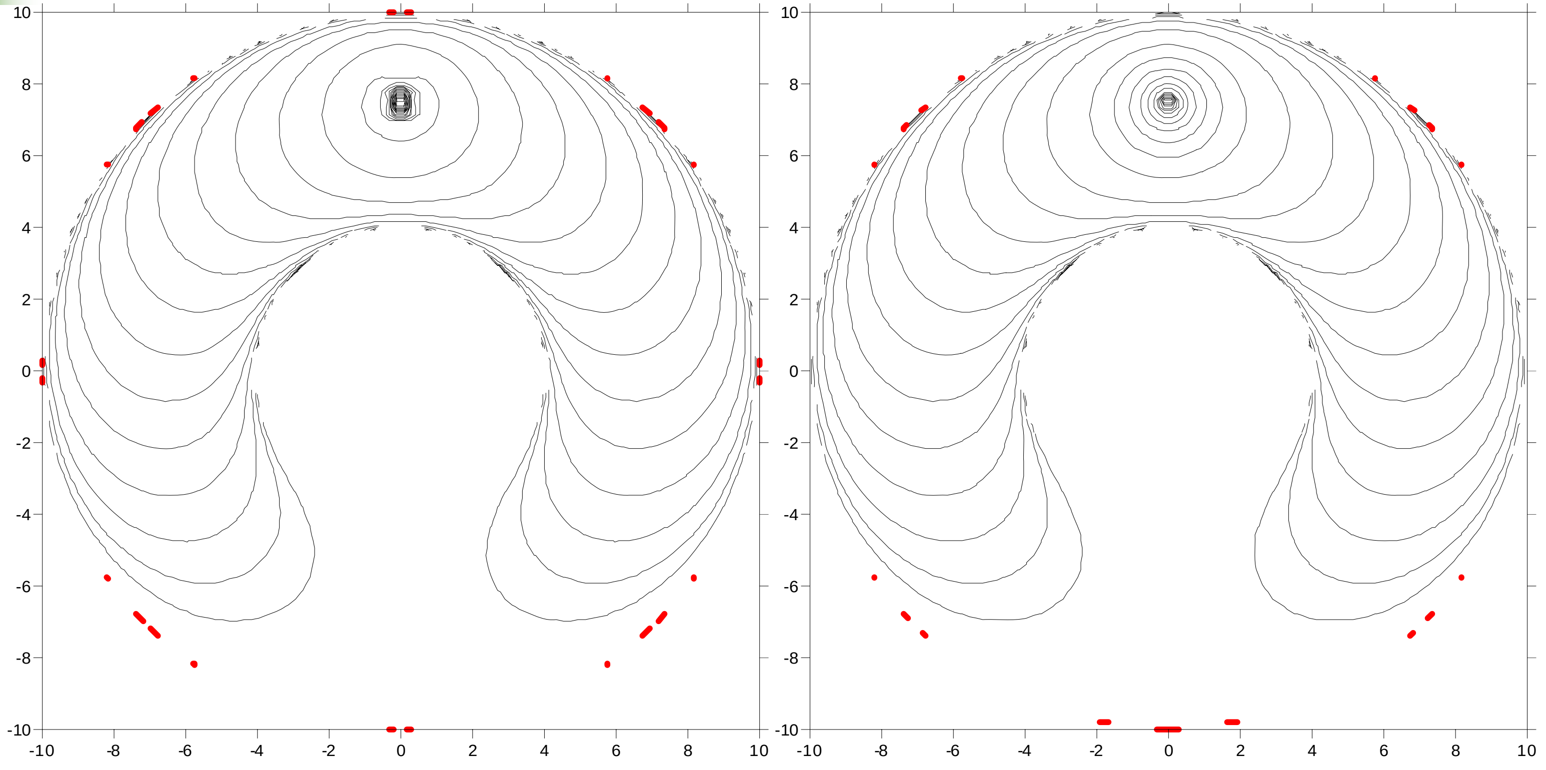
Numerical example (annular) m=50



Semi-analytical analytical

Conclusion

- A semi-analytical method of the papers is re-formulated and verified successfully by the Caulk's and Honein's examples.
- We apply the null-field integral equation method to derive the Green function for eccentric problems, and the results agree well with those of Melnikov data.
- Comparison of semi-analytical and analytical in annular case brings good results.

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References

1. **J. T. Chen**, **W. C. Shen**, **P. Y. Chen**, Analysis of Circular Torsion Bar with Circular Hole Using Null-field Approach, 2006, *CMES,* **12***,* 109-119*.*
2. **J. T. Chen, W. C. Shen, A. C. Wu**, Null-field Integral Equations for Field around Circular Holes under Antiplane Shear, 2006, *EABE*, **30**, 205-217.
3. **Yu. A. Melnikov and M. Yu. Melnikov**, Modified Potential as a Tool for Computing Green's Functions in Continuum Mechanics, 2001, *CMES*, **2**, 291-305.

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Acknowledgement of

NTOU/MSV family you are welcome to join us for research

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The end

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