Circular Torsion Bars with Circular Inclusions

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Seminar of Division of Structural Engineering in HRE in NTOU

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Outlines

- Review of the papers
  - Null field formulation
  - Results
- Applications to derive Green's function
  - Problem statement
  - Numerical example
- Conclusion

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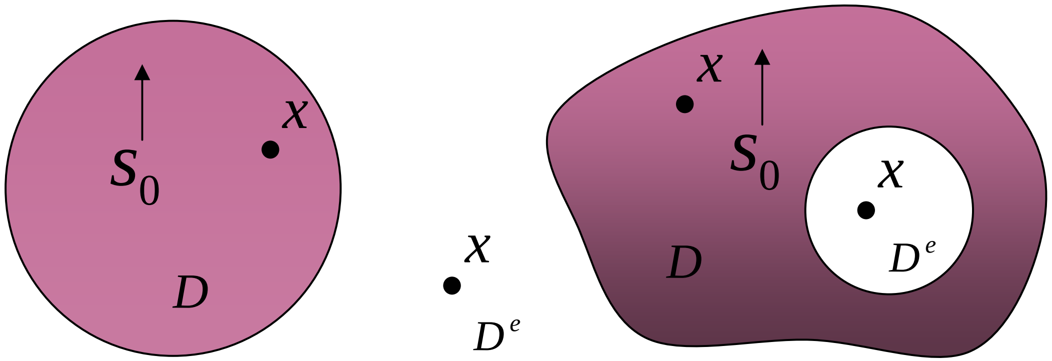
Reference method



where

$$U(s,x) = \ln|x-s| = \ln r$$

$$T(s,x) = \frac{\partial U(s,x)}{\partial n_s}$$

$$t(s) = \frac{\partial u(s)}{\partial n_s}$$

$$2\pi u(x) = \int_B T(s,x)u(s)dB(s) - \int_B U(s,x)t(s)dB(s) + U(s_0, x)$$

$$0 = \int_B T(s,x)u(s)dB(s) - \int_B U(s,x)t(s)dB(s) + U(s_0, x)$$

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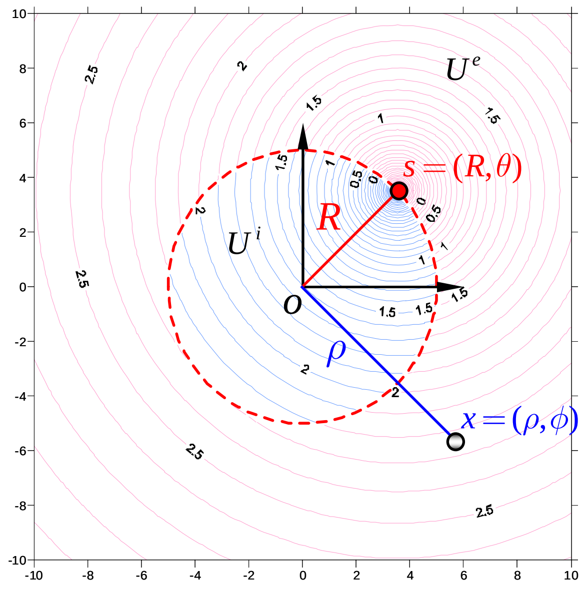
Reference method

$$U(s,x)=\begin{cases} U^{i}(R,\theta;\rho,\phi)=\ln R-\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{\rho}{R}\right)^{m}\cos m(\theta-\phi),\ R\geq\rho \\ U^{e}(R,\theta;\rho,\phi)=\ln \rho-\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{R}{\rho}\right)^{m}\cos m(\theta-\phi),\ \rho>R \end{cases}$$

$$T(s,x)=\begin{cases} T^{i}(R,\theta;\rho,\phi)=\frac{1}{R}+\sum_{m=1}^{\infty}\left(\frac{\rho^{m}}{R^{m+1}}\right)\cos m(\theta-\phi),\ R>\rho \\ T^{e}(R,\theta;\rho,\phi)=-\sum_{m=1}^{\infty}\left(\frac{R^{m-1}}{\rho^{m}}\right)\cos m(\theta-\phi),\ \rho>R \end{cases}$$

$$u_{k}(s)=a_{0}^{k}+\sum_{n=1}^{\infty}(a_{n}^{k}\cos n\theta_{k}+b_{n}^{k}\sin n\theta_{k}),\ s\in B_{k}$$

$$\frac{\partial u_{k}(s)}{\partial n_{s}}=p_{0}^{k}+\sum_{n=1}^{\infty}(p_{n}^{k}\cos n\theta_{k}+q_{n}^{k}\sin n\theta_{k}),\ s\in B_{k}$$



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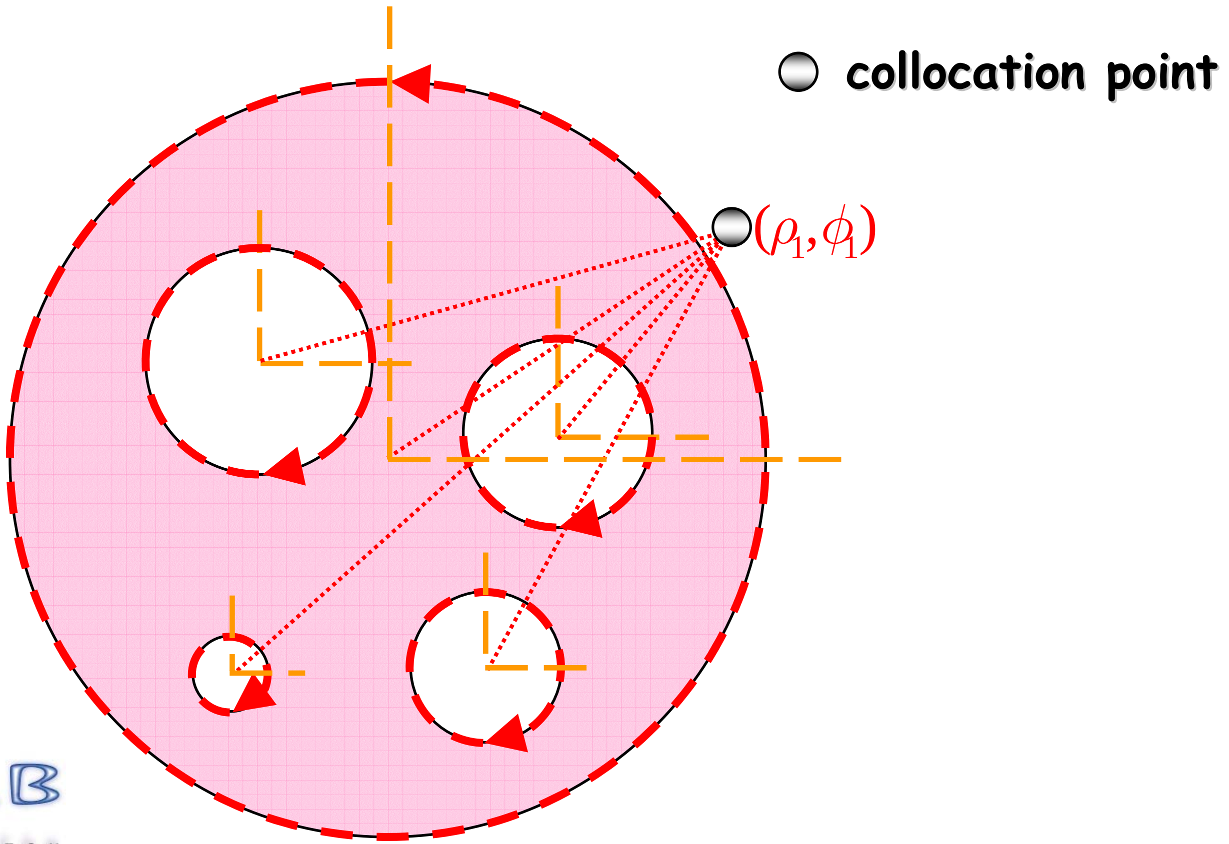
Adaptive observer system

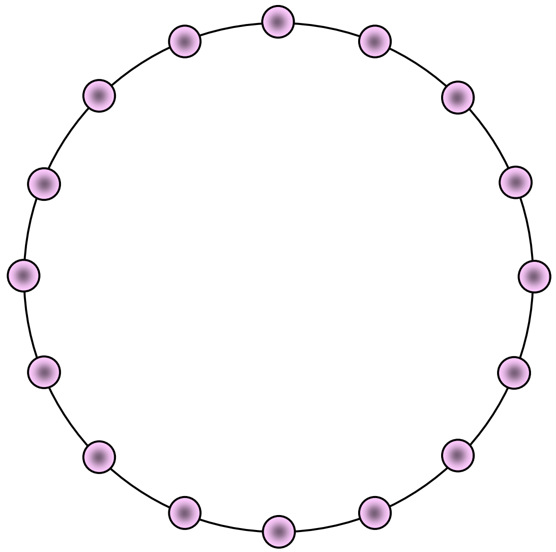


Fourier expansion and collocation points

$$u(s) = a_0 + \sum_{n=1}^{M} (a_n \cos n\theta + b_n \sin n\theta)$$

$$t(s) = p_0 + \sum_{n=1}^{M} (p_n \cos n\theta + q_n \sin n\theta)$$

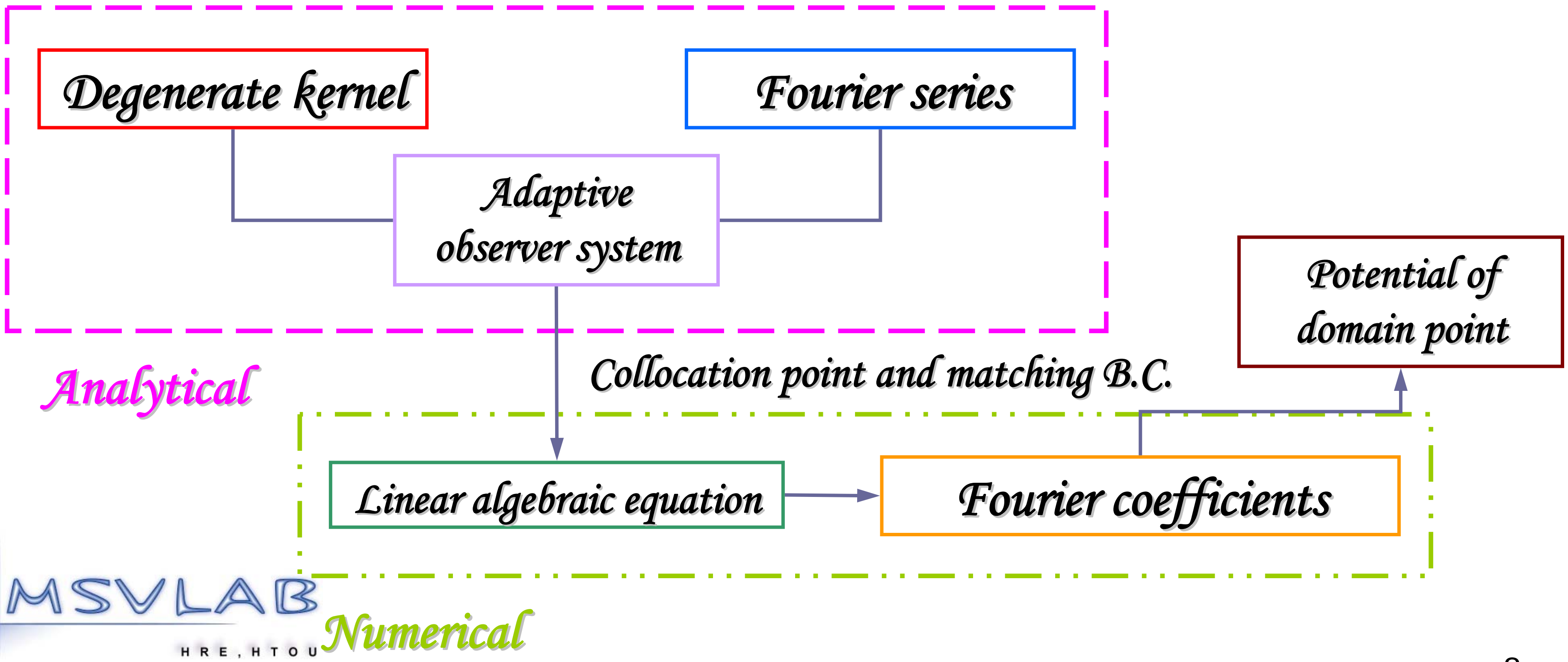
2M+1 unknown Fourier coefficients



collocation point

Flowchart of present method

$$0 = \int_B [T(s,x)u(s) - U(s,x)t(s)]dB(s)$$



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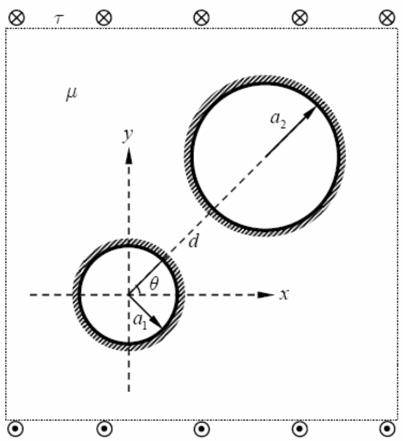
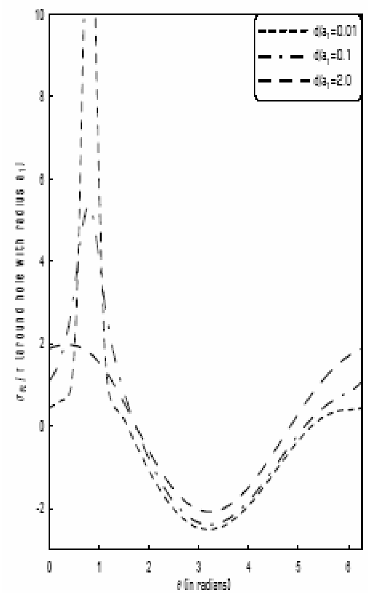
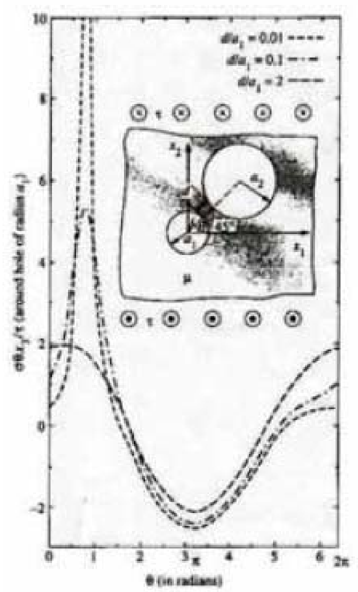
Reference example

Two holes lie on the line making 45 degree joining the two centers making 45 degree





The stress $\sigma_{\theta z}$ around the hole of radius a_1 with various values of d/a_1



Honein's data

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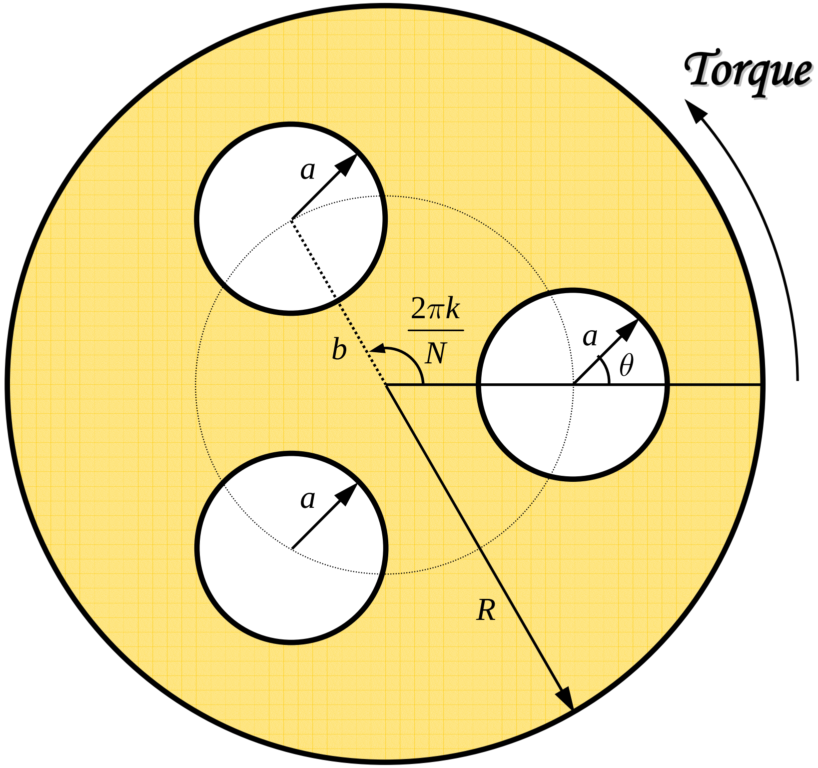
Reference example



The warping function φ

$$\nabla^2\varphi(x) = 0, \quad x \in D$$

Boundary condition

$$\frac{\partial\varphi}{\partial n} = x_k \sin\theta_k - y_k \cos\theta_k \quad \textit{on} \quad B_k$$

where

$$x_i = b\cos\frac{2\pi i}{N}, \quad y_i = b\sin\frac{2\pi i}{N}$$

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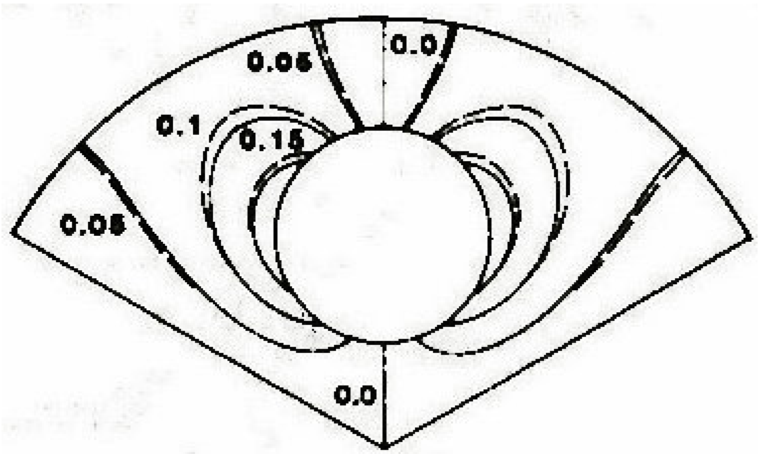
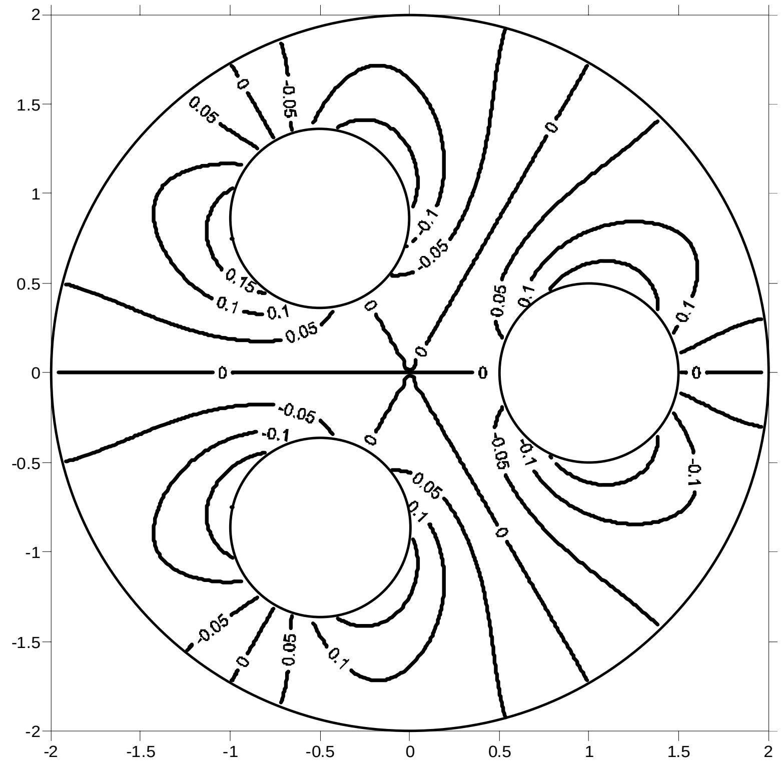
Axial displacement with three circular holes

Dashed line: exact solution
Solid line: first-order solution



Caulk's data (1983)
ASME Journal of Applied Mechanics



Present method (M=10)

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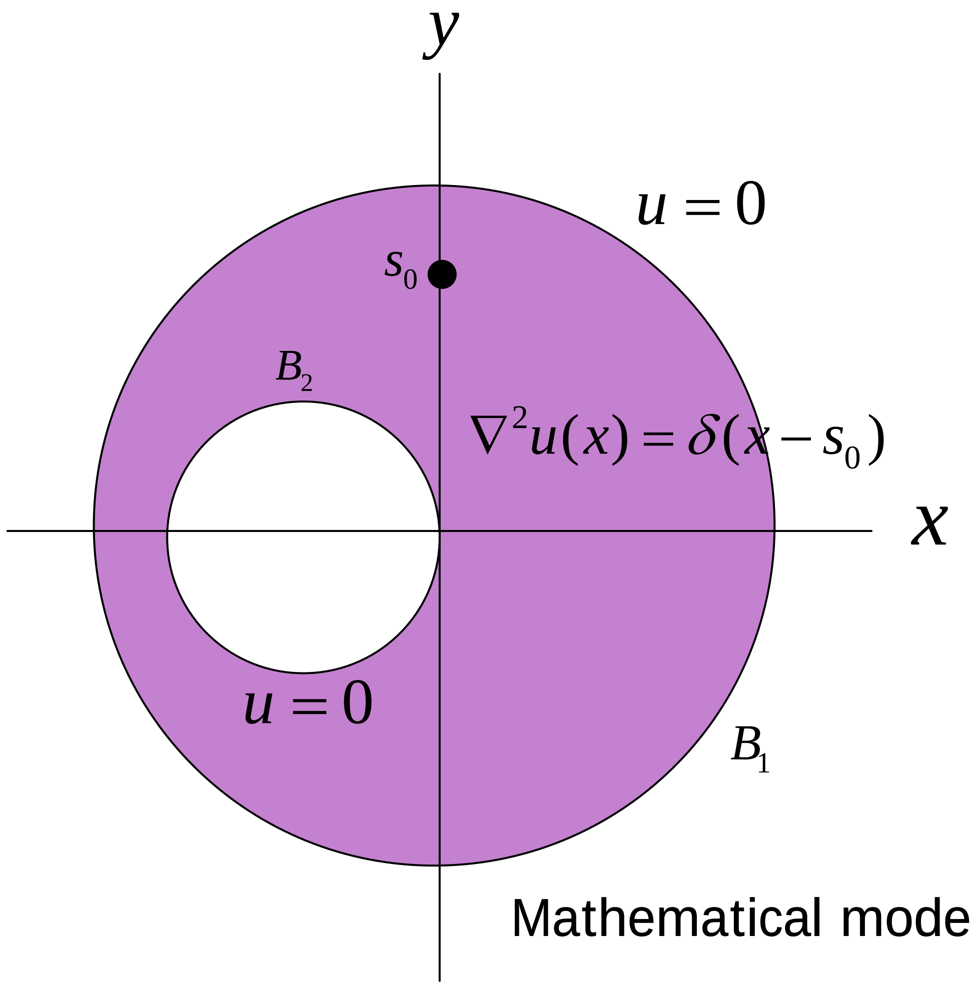
Problem statement



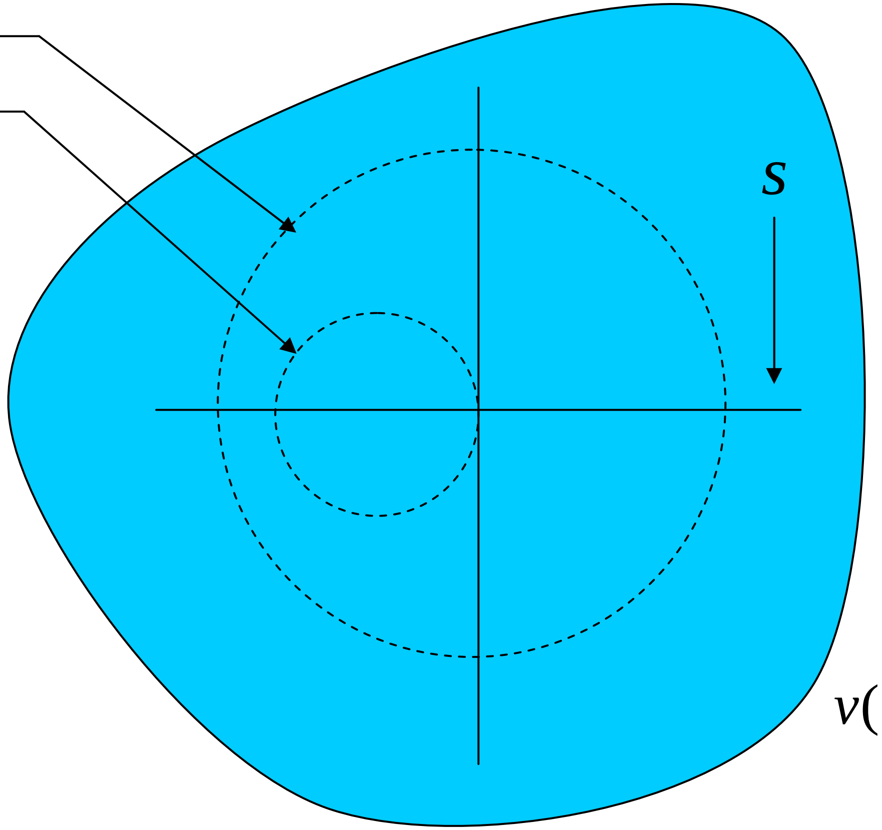
Mathematical model (PDE + BC)

Problem statement

Take free body

Fundamental solution

$$\nabla^2 U(s,x) = 2\pi\delta(x-s)$$



$$v(x) = U(s,x)$$

Reference method

$$\iint_D (u\nabla^2 v - v\nabla^2 u)\,dD = \int_B \left(u\frac{\partial v}{\partial n} - v\frac{\partial u}{\partial n}\right) dB$$

$$2\pi u(s) = \int_{B_1+B_2} T(s,x)u(s)\,dB(s) - \int_{B_1+B_2} U(s,x)t(s)\,dB(s) + U(s_0,x)$$

U T **Degenerate kernels**

u t **Fourier series**

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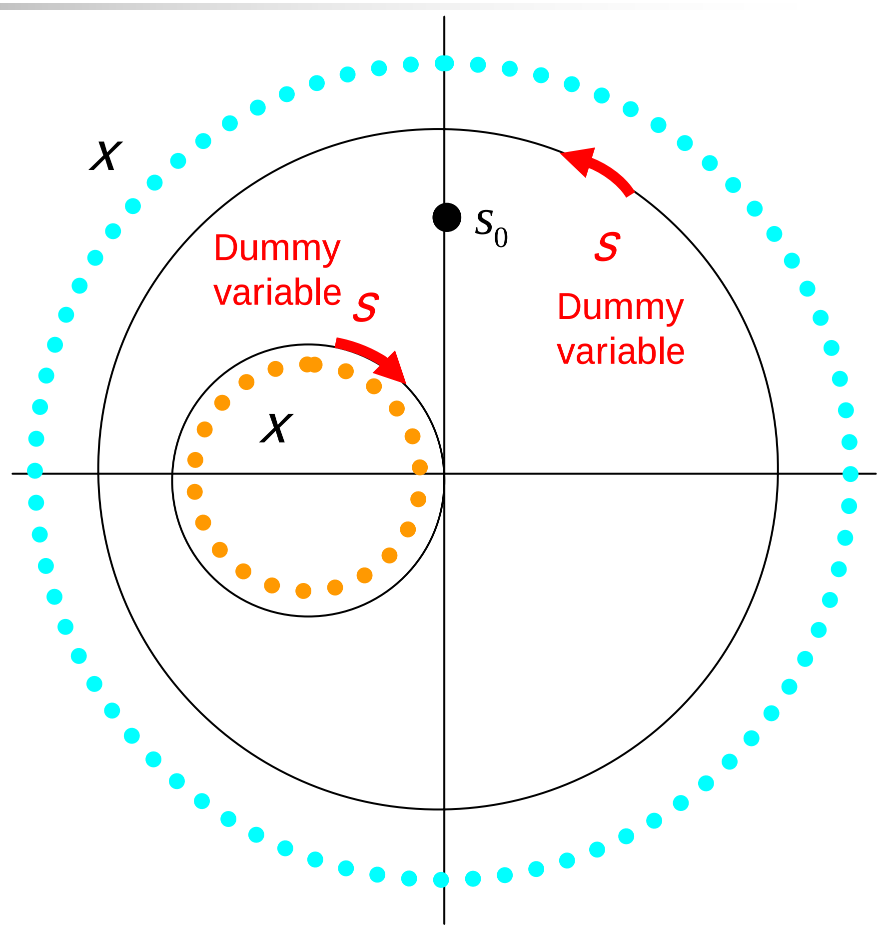
Problem statement

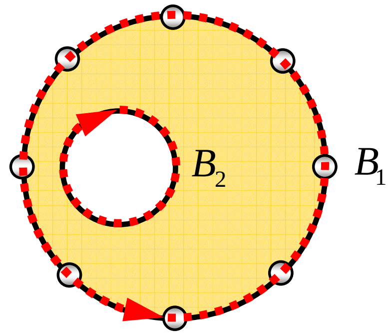
$$0=\int_{B_1}(T^e(s,x)\overset{0}{u(s)}-U^e(s,x)t(s))dB(s)$$
$$+\int_{B_2}(T^e(s,x)\overset{0}{u(s)}-U^e(s,x)t(s))dB(s)$$
$$+U(s_0,x)$$

$$0=\int_{B_1}(T^i(s,x)\overset{0}{u(s)}-U^i(s,x)t(s))dB(s)$$
$$+\int_{B_2}(T^i(s,x)\overset{0}{u(s)}-U^i(s,x)t(s))dB(s)$$
$$+U(s_0,x)$$



Linear algebraic equation

$$[\mathbf{U}]\{\mathbf{t}\}=\{\ln|r|\}$$



where

Index of collocation circle

$$[\mathbf{U}]=\begin{bmatrix}\boxed{U_{11}} & \boxed{U_{12}}\\ U_{21} & U_{22}\end{bmatrix}$$

Index of routing circle

$$\{t\}=\begin{Bmatrix} t_1^1\\ t_2^1\\ \vdots\\ t_n^1\\ t_1^2\\ t_2^2\\ \vdots\\ t_n^2\end{Bmatrix}$$

Column vector of Fourier coefficients

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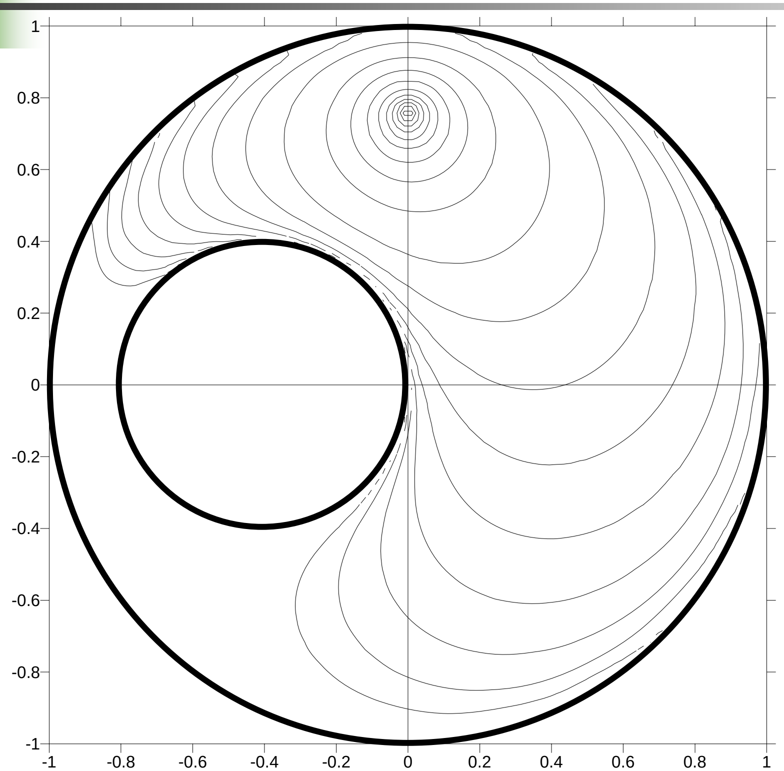
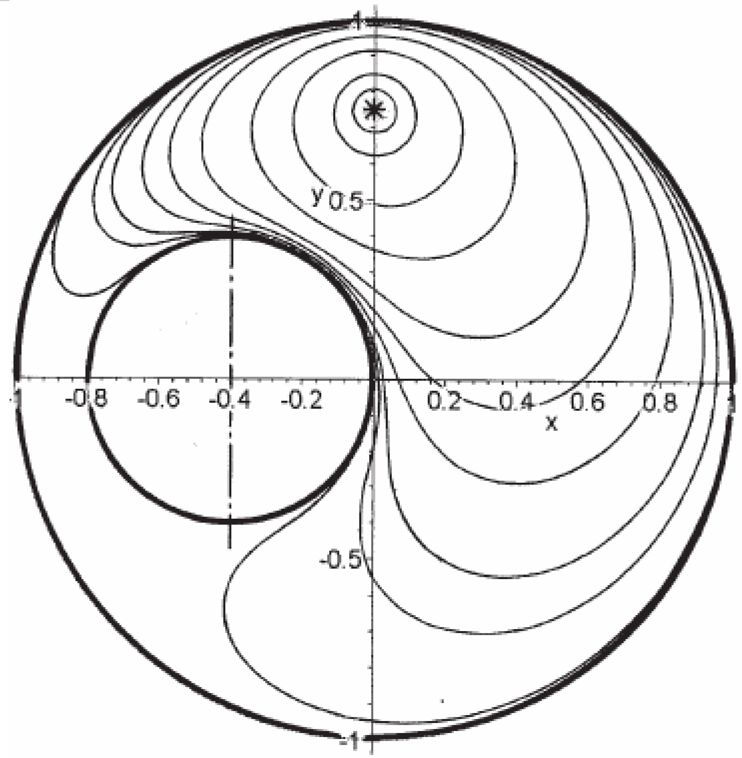
Numerical example



Present method



Green's function 2001

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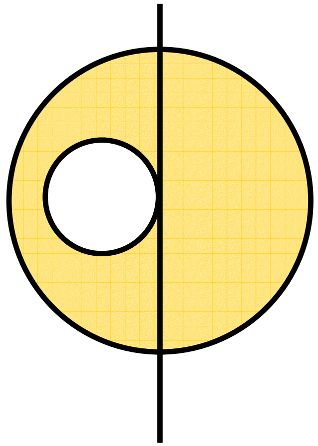
Numerical example

Field point, y	MCP			MMP			Present Method		
	k						Fourier term, n		
	10	20	50	10	20	50	10	20	50
0	-0.000280	-0.000128	-0.000067	-0.000107	0.000049	-0.000032	0 .000000	0.000000	0.000000
0.2	0.010667	0.010712	0.010781	0.010700	0.010779	0.010798	0 .010832	0.010832	0.010832
0.4	0.062359	0.062411	0.062443	0.062407	0.062435	0.062448	0 .062458	0.062462	0.062462
0.6	0.177534	0.177574	0.177585	0.177583	0.177590	0.177593	0.177597	0.177596	0.177596
0.8	0.317893	0.317902	0.317911	0.317907	0.317913	0.317914	0.318032	0.317915	0.317915
1.0	0.000014	0.000006	0.000002	0.000000	0.000000	0.000000	0.002014	0.000064	0.000000

MCP：Method of classical potentials

MMP：Method of modified potentials

k： Partitioning number



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Conclusion

- A semi-analytical method of the papers is formulated and verified by the Caulk's and Honein's examples.
- We apply this method to derive the Green function, and the results agree well with those of Melnikov data.

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學長走過 必留下足跡

1. **J. T. Chen**, **W. C. Shen**, **P. Y. Chen**, Analysis of Circular Torsion Bar with Circular Hole Using Null-field Approach, 2006, *CMES,* **12***,* 109-119*.*
2. **J. T. Chen, W. C. Shen, A. C. Wu**, Null-field Integral Equations for Field around Circular Holes under Antiplane Shear, 2006, *EABE*, **30**, 205-217.
3. **Yu. A. Melnikov and M. Yu. Melnikov**, Modified Potential as a Tool for Computing Green's Functions in Continuum Mechanics, 2001, *CMES*, **2**, 291-305.

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Acknowledgement of

NTOU/MSV family you are welcome to join us for research

The end

Thanks for your kind attentions.

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