Meshfree boundary integral equation method for calculating the conduction shape factor of exchanger tubes containing slits

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**Abstract**. Following the successful experience of applying the meshfree boundary integral equation method (BIEM) to determine the conduction shape factor of heat exchanger tubes, this paper extends to those containing slits. The main difference between the present method and the conventional boundary element method (BEM) is that the adaptive exact solution and Gaussian quadrature are simultaneously employed to technically calculate the singular integral free of the sense of Cauchy principal value in numerical implementation. When dealing with the boundary value problem containing a slit or so-called degenerate boundary, a rank-deficient influence matrix due to a degenerate boundary may occur. To overcome the rank-deficiency problem, we introduce the dual BIEM with the hypersingular boundary integral equation to obtain independent equations for collocation points on the slit. A feasible adaptive exact solution is also required for the problem with a degenerate boundary. Since the jump behaviour cannot be described by the previous adaptive exact solution using the Cartesian coordinates for the corresponding collocation point on the slit, we adopt the harmonic basis function in the elliptical coordinates to construct the new adaptive exact solution. After comparing available exact solutions of conduction shape factor, the obtained data of meshfree BIEM is consistent with those in the literature. However, the numerical instability due to the degenerate scale of an outer boundary is also observed. No matter what kind of slit is considered, the degenerate scale is dependent only on the size of outer boundary. Moreover, the boundary layer effect is also treated in the present method. In order to avoid the appearance of numerical instability due to a degenerate scale, regularized techniques are employed. Stable conduction shape factors for any size are obtained by using the proposed approach with regularized techniques.

**Keywords: conduction shape factor, meshfree boundary integral equation method, adaptive exact solution, Cauchy principal value, degenerate scale, slit, dual boundary integral equation method**

**1. Problem statements**

Cross-sections of elliptical and circular heat exchanger tubes containing a slit are shown in Figure 1. By considering no external heat source and no axial heat conduction, the governing equation of the temperature field satisfies the two-dimensional Laplace equation as shown below:

|  |  |
| --- | --- |
|  | (1) |

where *u*(**x**) is the temperature field at the point **x** on the cross-section, and *D* is the domain of cross-section.

To determine the conduction shape factor, the boundary conditions of the inner and outer surfaces are constant temperature (isothermal condition),

|  |  |
| --- | --- |
|  | (2) |

where and represent slit and outer boundaries of the cross-section, respectively, and represent the temperature on slit and outer boundaries, respectively. It is noted that a slit geometry belongs to a degenerate boundary. Therefore, adopting the meshfree dual BIEM to solve the problem with a degenerate boundary free of the domain decomposition technique is required.

|  |  |
| --- | --- |
|  |  |
| (a) An elliptical tube containing a focal slit | (b) A circular tube |

Figure 1 Cross-section of heat exchanger tubes containing a slit.

**2. Dual boundary integral formulation**

For solving a boundary value problem containing a degenerate boundary without the domain decomposition technique, the dual BIEM is one alternative. When a collocation point is located on an ordinary boundary (outer boundary, ) in the dual BIEM, either singular or hypersingular boundary integral equation can be used. For a collocation point on a degenerate boundary, both equations (singular and hypersingular) are required. Not only the singular integral in the Cauchy principal value sense but also the hypersingular integral in the Hadamard principal value sense can be technically calculated in terms of quadrature by using the adaptive exact solution and Gaussian quadrature. However, the rank-deficiency problem of influence matrices may occur when we only use either the singular or the hypersingular boundary integral equation for a collocation point on both sides of the degenerate boundary (slit, ) alone. Therefore, the dual BIEM is employed to obtain sufficient equations. The singular boundary integral equation is used for the collocation point on one side of the degenerate boundary, while the hypersingular boundary integral equation is employed on the other side.

According to the dual boundary integral formulation for a domain point are shown below:

|  |  |
| --- | --- |
|  | (3) |
|  | (4) |

where the kernel functions , 、、 and the boundary normal flux are defined as shown below:

|  |  |
| --- | --- |
|  | (5) |
|  | (6) |
|  | (7) |
|  | (8) |

and

|  |  |
| --- | --- |
|  | (9) |

respectively, in which, and are unit outward normal vectors of the source point and the field point ,respectively. When the field point is outside the domain, we have the null-field dual BIE,

|  |  |
| --- | --- |
|  | (10) |
|  | (11) |

where is the two-dimensional plane and . If the field point is located on the real boundary , we may encounter the singular integral due to the kernel functions and as well as the hypersingular integral due to the kernel function . By introducing the limiting process and the bump contour approach to calculate singular and hypersingular integrals, we have the following dual BIE from Eqs. (10) and (11) for the boundary point,

|  |  |
| --- | --- |
|  | (12) |
|  | (13) |

where is the solid angle of the boundary point, the, are the Riemann, Cauchy and Hadamard principal values, respectively.

**2.1 Skillful calculation of singular integrals** **using the adaptive exact solution**

The field in the infinite domain also satisfies the Laplace equation with respect to variable **s** as given below:

|  |  |
| --- | --- |
|  | (14) |

where is the corresponding infinite domain.

The dual boundary integral equation of the adaptive exact solution is shown below:

|  |  |
| --- | --- |
|  | (15) |
|  | (16) |

where the degenerate boundary is the union of two sides, and , and are defined as shown below:

|  |  |
| --- | --- |
|  | (17) |
|  | (18) |

Subtracting Eq. (15) from Eq. (12) and collocating , we obtain

|  |  |
| --- | --- |
|  | (19) |

By similarly subtracting Eq. (16) from Eq. (13) and collocating , we have Eq. (20)

|  |  |
| --- | --- |
|  | (20) |

When the collocation point is equal to or **,** the kernel functions have singularities for both situations (). In other words, when the point **x** crosses the boundary, the values of and on both sides of the slit boundary are generally different. A special function to make and continuous through the slit boundary, to is required. Therefore, we can skillfully calculate the singular and hypersingular integrals when collocating **x** on the degenerate boundary.

First, four continuous conditions of boundary densities for deriving the adaptive exact solution for on the upside of the slit are given below:

|  |  |
| --- | --- |
|  | (21) |
|  | (22) |
|  | (23) |
|  | (24) |

According to Eqs. (21) to (24), and can be expressed as shown below:

|  |  |
| --- | --- |
|  | (25) |
|  | (26) |

whereand are shape functions which satisfy the governing equation in Eq. (14).

Substituting Eqs. (25) and (26) into Eqs. (21) to (24), we have:

|  |  |
| --- | --- |
|  | (27) |

It may have difficulty in the real implementation when the multi-valued functions are required to define above shape functions by using the Cartesian coordinates. For this reason, we introduce the exterior harmonic basis of elliptical coordinates to construct shape functions. In this way, four shape functions are obtained by the linear combination of the harmonic basis in the elliptical coordinates that are suitable for the exterior problem as shown below:

|  |  |
| --- | --- |
|  | (28) |

where in the ellipticall coordinates, and provides the solution representation for and

After choosing the base in Eq. (28) to satisfy Eq. (27), we have

|  |  |
| --- | --- |
|  | (29) |
|  | (30) |
|  | (31) |
|  | (32) |

where is the Jacobian term for the operator of normal derivative,

|  |  |
| --- | --- |
|  | (33) |

It is noted that *c* is the semi-focal length, and are the elliptic coordinates to describe the slit. Substituting Eqs. (21) to (24) into Eq. (19), we have

|  |  |
| --- | --- |
|  | (34) |

**3.** **Numerical result**

|  |  |
| --- | --- |
|  |  |
| (a) An elliptical tube  () | (b) A circular tube  *(*) |

Figure 2 Conduction shape factor versus the size, *a*, for a single slit

|  |  |
| --- | --- |
|  |  |
| (a) An elliptical tube with a confocal slit () | (b) A circular tube with a central slit () |

Figure 3 Conduction shape factor by using three regularization techniques.

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