Application of boundary integral quadrature method to torsion problems of the orthotropic bars and its treatment of degenerate scale problem

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Regarding the Saint-Venant torsion problem of orthotropic bars, the boundary integral quadrature method (BIQM) is employed to solve the stress function $\Phi(\mathbf{x})$ and warping function $w_a(\mathbf{x})$. We introduce the adaptive exact solution into the boundary integral eqation. Not only the singular integral in the sense of the Cauchy principal value can be novelty determined but also the calculation of solid angle on the boundary is free. After using the parametric form to represent the boundary contour and adopting the Gaussian quadrature for the boundary integral equation, the boundary integral equation is nothing more than an algebraic equation. Therefore, only collocating Gaussian points on the boundary are required to obtain the simultaneous equation. Finally, we calculate the torsional rigidity and the boundary shear stresses with different cross-sections by using the BIQM. After comparing the present results to these of references [3-5], agreement is made. In addtion, the numrical instability due to the degenerate scale for the orthotropic case is dealt with by uisng the combined Laplace integral equation formulation (CLEEF) method.

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