

Recent development on the double-degeneracy mechanism of the BEM/BIEM

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BEM is an acceptable approach for simulating engineering problems. It is well known that four degenerate problems, degenerate scale, spurious eigenvalue, fictitious frequency and degenerate boundary, may occur by using the BEM/BIEM. However, only the degenerate scale and degenerate boundary may appear at the same time. This is so-called double degeneracy. Degenerate kernel is the key mathematical tool to understand the occurring mechanism. Objectivity of the degenerate kernel is examined. In this talk, the degenerate kernel is employed to analytically explain how the degenerate mechanism appears in the boundary integral formulation. It is found that only rigid line inclusion instead of a crack may have the possibility of double degeneracy on the same geometry. Even though the boundary density is polluted by the null space, the field solution may be correct. Not only the analytical derivation is proposed but also the numerical experiment is also performed. Anti-plane shear and two-dimensional elasticity problems are both addressed. Three possibility to appear double degeneracy degeneracy are also shown.

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**Biographical Record**

*Jeng-Tzong Chen, born in 1962, received a BS degree in Civil Engineering, an M.S. in Applied Mechanics, and a Ph.D. in Civil Engineering, respectively, in 1984, 1986 and 1994, from National Taiwan University, Taipei, Taiwan, R.O.C. He had worked as a research assistant in the Structural Division of the Department of Rocket and Missile System, Chung Shan Institute of Science and Technology, from 1986 to 1990. In 1994, he was invited to be an Associate Professor in the Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung, Taiwan, R.O.C. He was promoted to a full professor in 1998. Later in 2004, he was selected to be the Distinguished Professor. In 2007, he was selected as the Life-time Distinguished Professor. He is also the Professor of Department of Mechanical and Mechatronic Engineering of Taiwan Ocean University. In 2011, he won the MOE academic award and the ICACM Fellow Award. In 2023, he won the highest academic award of STAM, Sun F D medal. His major interest is computational mechanics. He had derived the theory of dual integral equations for boundary value problems with degenerate boundary. Prof. Chen also developed five dual BEM programs for the BVPs of Laplace equation, Helmholtz equation, bi-Helmholtz and modified Helmhotlz equation and Navier equation. Recently, he also employed the null field integral equations to solve BVPs with circular and/or elliptical boundaries including holes and inclusions. Besides, he focused on the nonuniqueness solution of integral equations in recent years. He wrote two books in Chinese on the dual BEM and the FEM using MSC/NASTRAN, respectively. He was ever invited to give plenary and keynote lectures, e.g., twice in World Congress on Computational Mechanics (WCCM4 (1998) in Buenos Aries and WCCM5 (2002) in Vienna), four times in ICOME 2006(Nanjing), 2009(Hefei), 2012(Kyoto) and 2015(Hangzhou), FEM/BEM 2003 in St. Petersburg, Russia, ICCES 2005 in Chennai, ICIP 2010 in Hong Kong, ACMFMS 2012 in Delhi, APCOM&ISCM III (2013) in Singapore and ICF 2013 in Beijing, BEM/MRM 36 (2014) in Dalian and IABEM 2014(Zhengzhou). In 2012, he was invited to deliver a plenary talk in ACMFMS (New Delhi, India). In 2018, he delivered plenary lectures in China twice, IWMM and cross strait meeting. In 2019, he delivered a semi-plenary talk in APCOM 2019 at Taipei. In 2016, 2019, 2021 and 2022, he was invited a plenary talk in ICCMS conferences in India. In addition, he is now the associate editor of Journal of Mechanics (JOM), Journal of Chinese Institute of Engineers (JCIE), and Engineering Analysis with Boundary Elements (EABE). Besides, he has been the associate editor of the editor of Journal of Marine Science and Technology. He won three times of Outstanding Research Awards from National Science Council, Taiwan. He also won the first Wu, Ta-You Memorial Award in 2002. He is currently the member of editorial board of many international SCI journals. Until now, he has published more than 247 SCI journal papers on the BEM and the FEM in technical Journals. More than 5000 citings from 2564 papers are found to cite Chen’s work. Two papers (ASME-AMR and ASCE-EMD) were both cited more than 483 and 432 times from the Google, respectively. Boundary element method is one focus of Professor Chen's research interests. Others may be categorized into two areas. One is vibration and acoustics, and the other is computational mechanics. In 2017, he was selected as the Fellow of STAM, R O C. In 2018, he was selected to be the first Distinguished Chair Professor in NTOU. In 2022, he is now a guest professor of National Taiwan University, an adjunct professor of National Cheng Kung University and Distinguished Chair Professor of National Taiwan Ocean University.*